$$\frac{1}{2}(\phi')^2 = U(\phi) \rightarrow \frac{d\phi}{dx} = \pm\sqrt{2U(\phi)}$$

$$\int_{x_0}^{x} dx = \pm \int_{\phi(x_0)}^{\phi(x)} \frac{d\phi}{\sqrt{2U(\phi)}}$$

$$U(\phi) = \frac{1}{2}\phi^2(\phi^2-1)^2$$

$\phi(-\infty) = 0, \pm 1$

$\phi(\infty) = 0, \pm, 1$

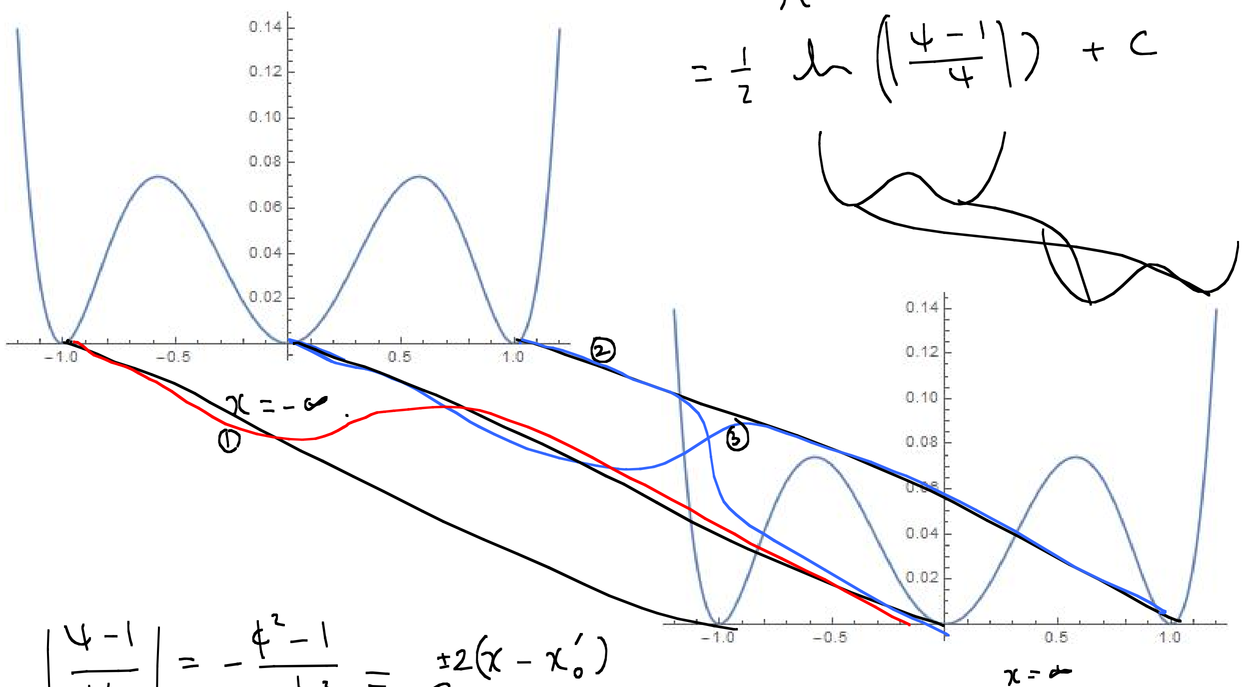
$$= \int \frac{d\phi}{\phi(\phi^2-1)}$$

$\phi^2 = \psi$

$$x - x_0 = \int \frac{\overbrace{\phi\, d\phi}^{\frac{1}{2}d\phi^2}}{\phi^2(\phi^2-1)} = \frac{1}{2}\int \frac{d\psi}{\psi(\psi-1)}$$

$$= \frac{1}{2}\int\left(-\frac{1}{\psi} + \frac{1}{\psi-1}\right)d\psi$$

$$= \frac{1}{2}\ln\left(\left|\frac{\psi-1}{\psi}\right|\right) + C$$



$$\left|\frac{\psi-1}{\psi}\right| = -\frac{\phi^2-1}{\phi^2} = e^{\pm 2(x-x_0')}$$

$$\frac{1}{\phi^2} - 1 = e^{\pm 2(x-x_0')}$$

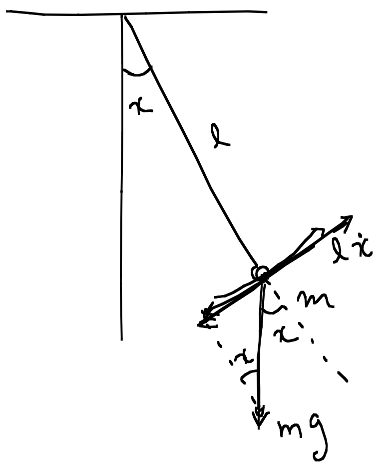
$$\phi = -\frac{1}{\sqrt{1+e^{2(x-x_0')}}} \quad ①$$

$x \to -\infty \quad \phi \to \pm 1$

$x \to \infty \quad \phi \to 0$

$$\phi = \frac{1}{\sqrt{1+e^{2(x-x_0')}}} \quad ②$$

$\leftrightarrow \quad x \to -x \quad ③$

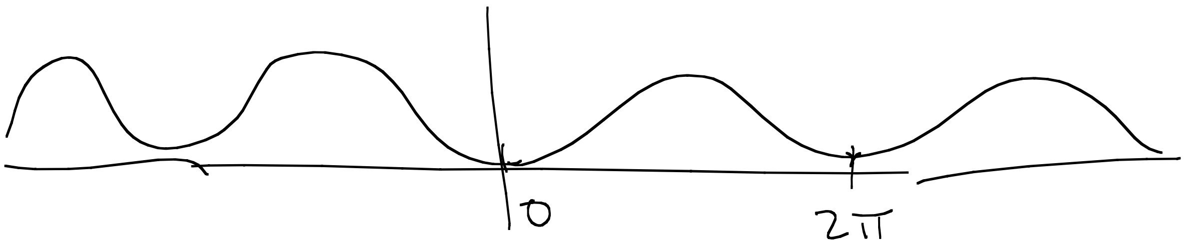


$$m l \ddot{x} = F = -mg\sin x - \gamma l \dot{x}$$

$$m\ddot{x} = -m \underbrace{\frac{g}{l}}_{\equiv\, \omega_0^2 = 1} \sin x - \gamma \dot{x}$$

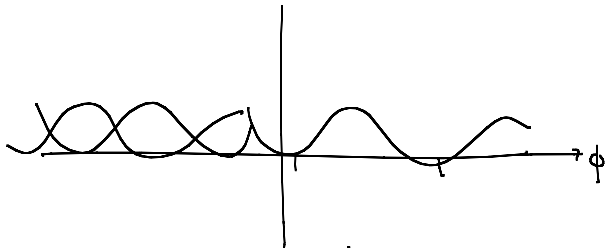
$$\ddot{x} + \cancel{\omega_0^2} \sin x + \underset{1}{\frac{\gamma}{m}}\, \dot{x} = 0$$

$U(\phi) = 1 - \cos\phi$



Sine-Gordon

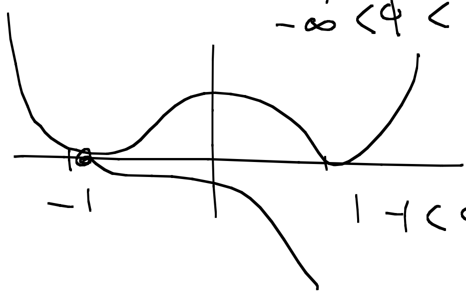
$V = 1 - \cos\phi$



$-\infty < \phi < \infty$

ϕ^4

$V = \frac{1}{2}(\phi^2 - 1)^2$



$-1 < \phi < 1$

Parametric plot $(x(t), y(t))$ $\quad t = 0, 2\pi$

plot $(x, y(x))$