# Exact Correlators in Integrable AdS/CFT

Changrim Ahn
Ewha Womans University
Seoul, South Korea

contains work in progress with

Zoltan Bajnok (Budapest)

East Asia Joint Workshop at KEK, Japan

# Main goal of QFTs

Compute correlation functions non-perturbatively

$$\langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle$$

Generic, non-BPS operators

Dynamical with local coordinates dependence

Quantitatively exact

$$F(x_1,\ldots,x_N;\lambda)$$

Still unsolved problem!

No success so far

except …

# Conformal field theories

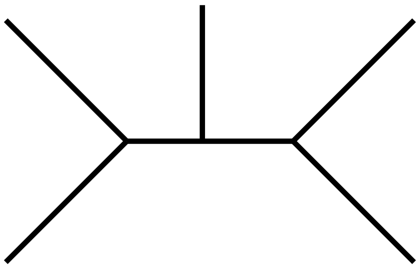
Conformal symmetry determines

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle = \frac{1}{|x_1 - x_2|^{2\Delta_{\mathcal{O}}(\lambda)}}$$

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle = \frac{C_{123}(\lambda)}{|x_1 - x_2|^{\Delta_1+\Delta_2-\Delta_3}\ldots}$$

$+$

Conformal bootstraps for higher-point correlators



Still, how can we compute these exactly ?

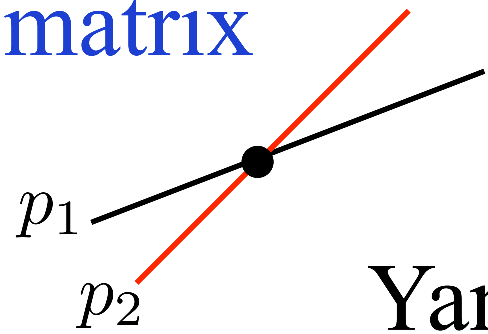
Need some nontrivial property

# Integrability

- Infinite conserved charges

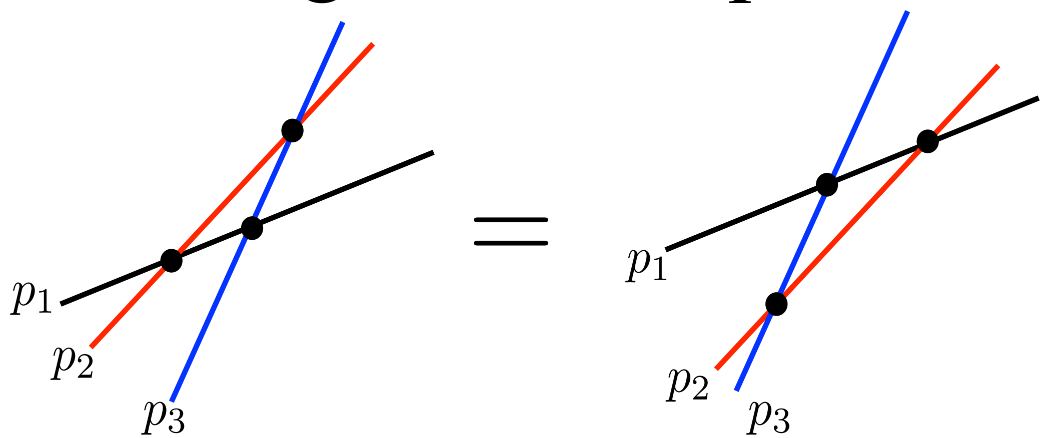
$$Q_n = \sum_j p_j^n = \sum_j {p'_j}^n \quad \rightarrow \quad \{p_j\} = \{p'_j\}$$

- S-matrix




$$S(p_1, p_2)$$

Yang-Baxter equation




$$\mathbf{S}_{12}(p_1,p_2)\,\mathbf{S}_{13}(p_1,p_3)\,\mathbf{S}_{23}(p_2,p_3) = \mathbf{S}_{23}(p_2,p_3)\,\mathbf{S}_{13}(p_1,p_3)\,\mathbf{S}_{12}(p_1,p_2)$$

# S-matrix at core of integrable AdS/CFT

- SYM side : scattering of fields on the spin chain

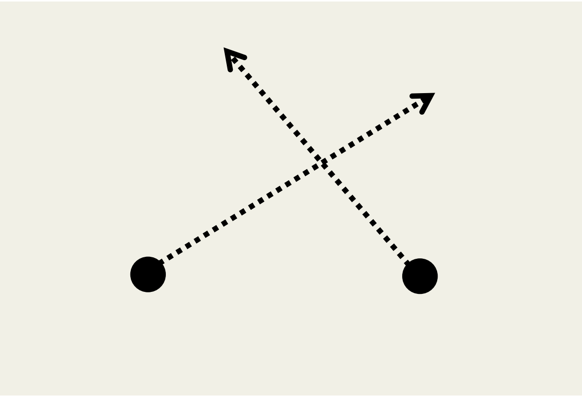
$$p_1 \qquad\qquad\qquad p_2$$

$$\mathrm{Tr}\left[\cdots ZZZZZZZZZ\chi_1 ZZZZZZ \cdots ZZZZZZ\chi_2 ZZZZZZZZZZZ\cdots\right]$$

$$L \to \infty$$

$$\left\{X, \overline{X}, Y, \overline{Y}, \chi^{\alpha}, D_{\mu}\right\}$$

- String side : scattering on the world sheet



# $\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$

$S_{su(2|2)} \otimes S_{su(2|2)}$ $\qquad$ $S_{su(2|2)} \oplus S_{su(2|2)}$ $\qquad$ $S_{su(1|1)} \otimes S_{su(1|1)}$

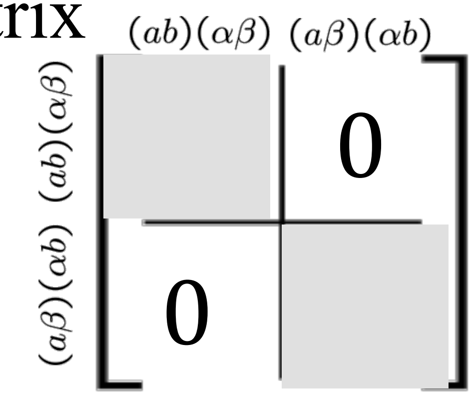
$d = 4$ $\qquad$ $d = 3$ $\qquad$ $d = 2$

N=4 SYM $\leftarrow\rightarrow$ $AdS_5 x S^5$

ABJM $\leftarrow\rightarrow$ $AdS_4 x CP^3$

$CFT_2$ $\leftarrow\rightarrow$ $AdS_3 x S^3 x M^4$

- $S_{\mathrm{su}(2|2)}$: 16 x 16 matrix




$$
\begin{aligned}
S_{a\,a}^{a\,a} &= A\,, \quad S_{\alpha\,\alpha}^{\alpha\,\alpha} = D\,,\\
S_{a\,b}^{a\,b} &= \frac{1}{2}(A-B)\,, \quad S_{a\,b}^{b\,a} = \frac{1}{2}(A+B)\,,\\
S_{\alpha\,\beta}^{\alpha\,\beta} &= \frac{1}{2}(D-E)\,, \quad S_{\alpha\,\beta}^{\beta\,\alpha} = \frac{1}{2}(D+E)\,,\\
S_{a\,b}^{\alpha\,\beta} &= -\frac{1}{2}\epsilon_{ab}\epsilon^{\alpha\beta}C\,, \quad S_{\alpha\,\beta}^{a\,b} = -\frac{1}{2}\epsilon^{ab}\epsilon_{\alpha\beta}F\,,\\
S_{a\,\alpha}^{a\,\alpha} &= G\,, \quad S_{a\,\alpha}^{\alpha\,a} = H\,, \quad S_{\alpha\,a}^{a\,\alpha} = K\,, \quad S_{\alpha\,a}^{\alpha\,a} = L
\end{aligned}
$$

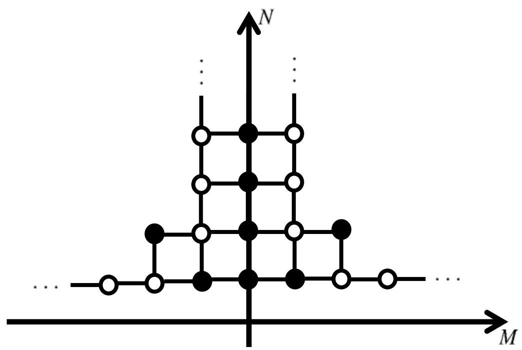
$$
\begin{aligned}
A &= S_0\frac{x_2^- - x_1^+}{x_2^+ - x_1^-}\frac{\eta_1\eta_2}{\tilde{\eta}_1\tilde{\eta}_2}\,,\\
B &= -S_0\left[\frac{x_2^- - x_1^+}{x_2^+ - x_1^-} + 2\frac{(x_1^- - x_1^+)(x_2^- - x_2^+)(x_2^- + x_1^+)}{(x_1^- - x_2^+)(x_1^-x_2^- - x_1^+x_2^+)}\right]\frac{\eta_1\eta_2}{\tilde{\eta}_1\tilde{\eta}_2}\,,\\
C &= S_0\frac{2ix_1^-x_2^-(x_1^+ - x_2^+)\eta_1\eta_2}{x_1^+x_2^+(x_1^- - x_2^+)(1 - x_1^-x_2^-)}\,, \qquad D = -S_0\,,\\
E &= S_0\left[1 - 2\frac{(x_1^- - x_1^+)(x_2^- - x_2^+)(x_1^- + x_2^+)}{(x_1^- - x_2^+)(x_1^-x_2^- - x_1^+x_2^+)}\right]\,,\\
F &= S_0\frac{2i(x_1^- - x_1^+)(x_2^- - x_2^+)(x_1^+ - x_2^+)}{(x_1^- - x_2^+)(1 - x_1^-x_2^-)\tilde{\eta}_1\tilde{\eta}_2}\,,\\
G &= S_0\frac{(x_2^- - x_1^-)}{(x_2^+ - x_1^-)}\frac{\eta_1}{\tilde{\eta}_1}\,, \qquad H = S_0\frac{(x_2^+ - x_2^-)}{(x_1^- - x_2^+)}\frac{\eta_1}{\tilde{\eta}_2}\,,\\
K &= S_0\frac{(x_1^+ - x_1^-)}{(x_1^- - x_2^+)}\frac{\eta_2}{\tilde{\eta}_1}\,, \qquad L = S_0\frac{(x_1^+ - x_2^+)}{(x_1^- - x_2^+)}\frac{\eta_2}{\tilde{\eta}_2}
\end{aligned}
$$

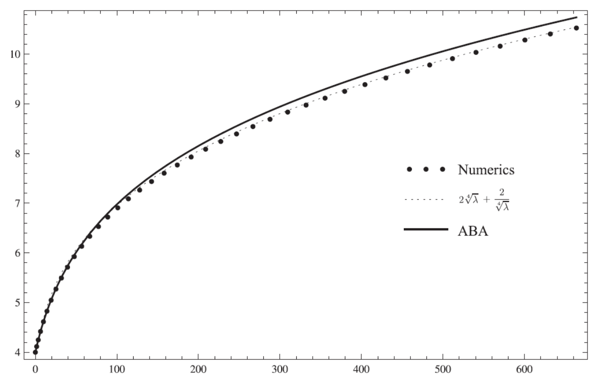
$$
\eta_1 = \eta(p_1)e^{ip_2/2}\,, \quad \eta_2 = \eta(p_2)\,, \quad \tilde{\eta}_1 = \eta(p_1)\,, \quad \tilde{\eta}_2 = \eta(p_2)e^{ip_1/2}
$$

# $\Delta$ from S-matrix

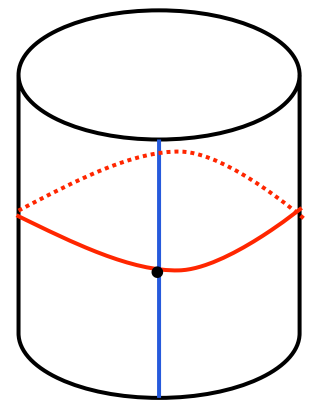
TBA, Y-systems, NLIE, QSC

$$\ln Y_{N,M} = s\star\left[\ln(1+Y_{N,M+1})+\ln(1+Y_{N,M-1})\right]-s\star\left[\ln(1+Y^{-1}_{N+1,M})+\ln(1+Y^{-1}_{N-1,M})\right]$$

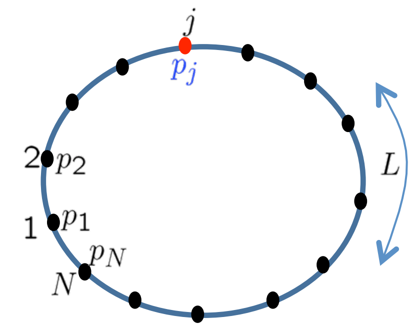



Lüscher correction



Bethe ansatz



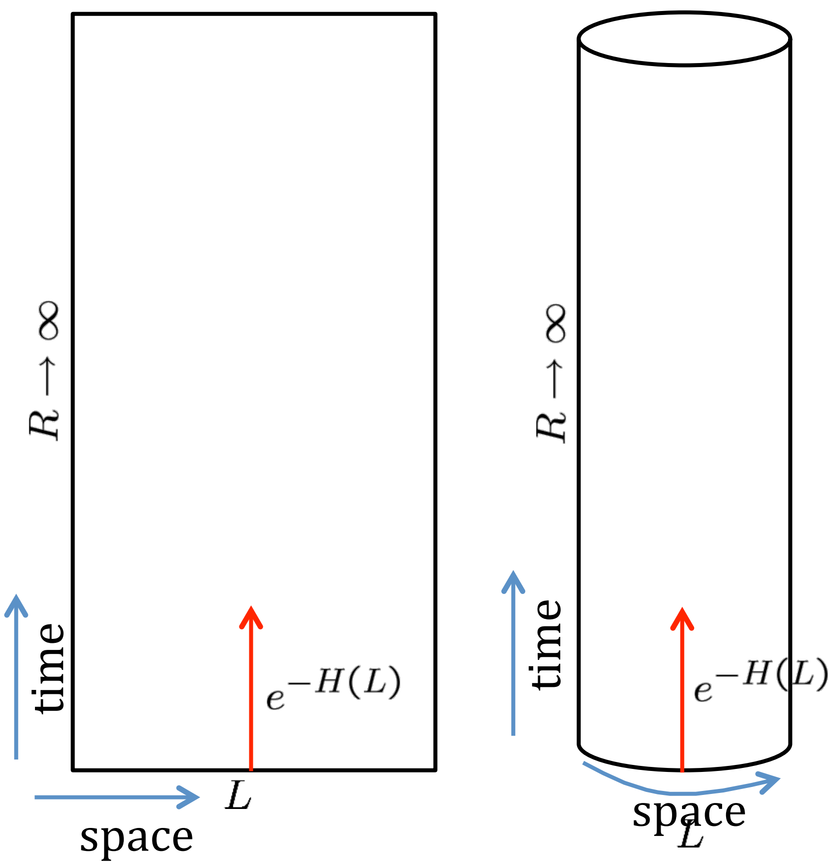

finite $L$, $\lambda$

$L \gg \lambda$

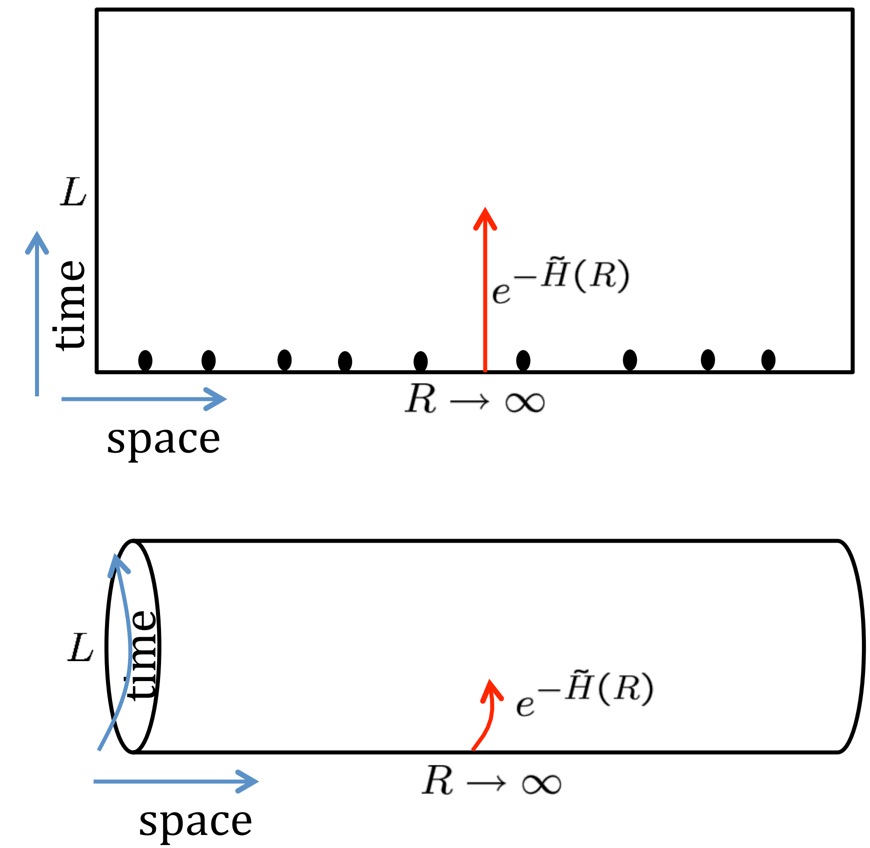
$L = \infty$

# Thermodynamic Bethe Ansatz (Al. B. Zamolodchikov)

## Physical space

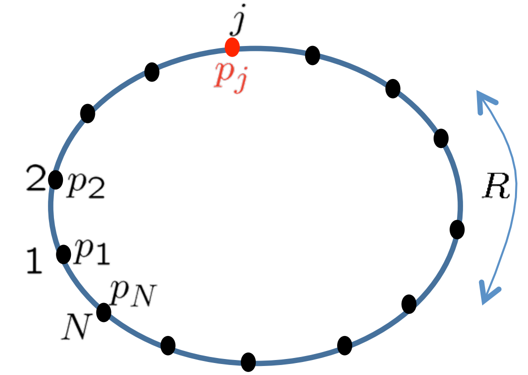



## Mirror space



# Channel Duality

- Mirror channel
  - Scatterings between asymptotic particles are valid since $R \to \infty$
  - Bethe ansatz equation from the PBC



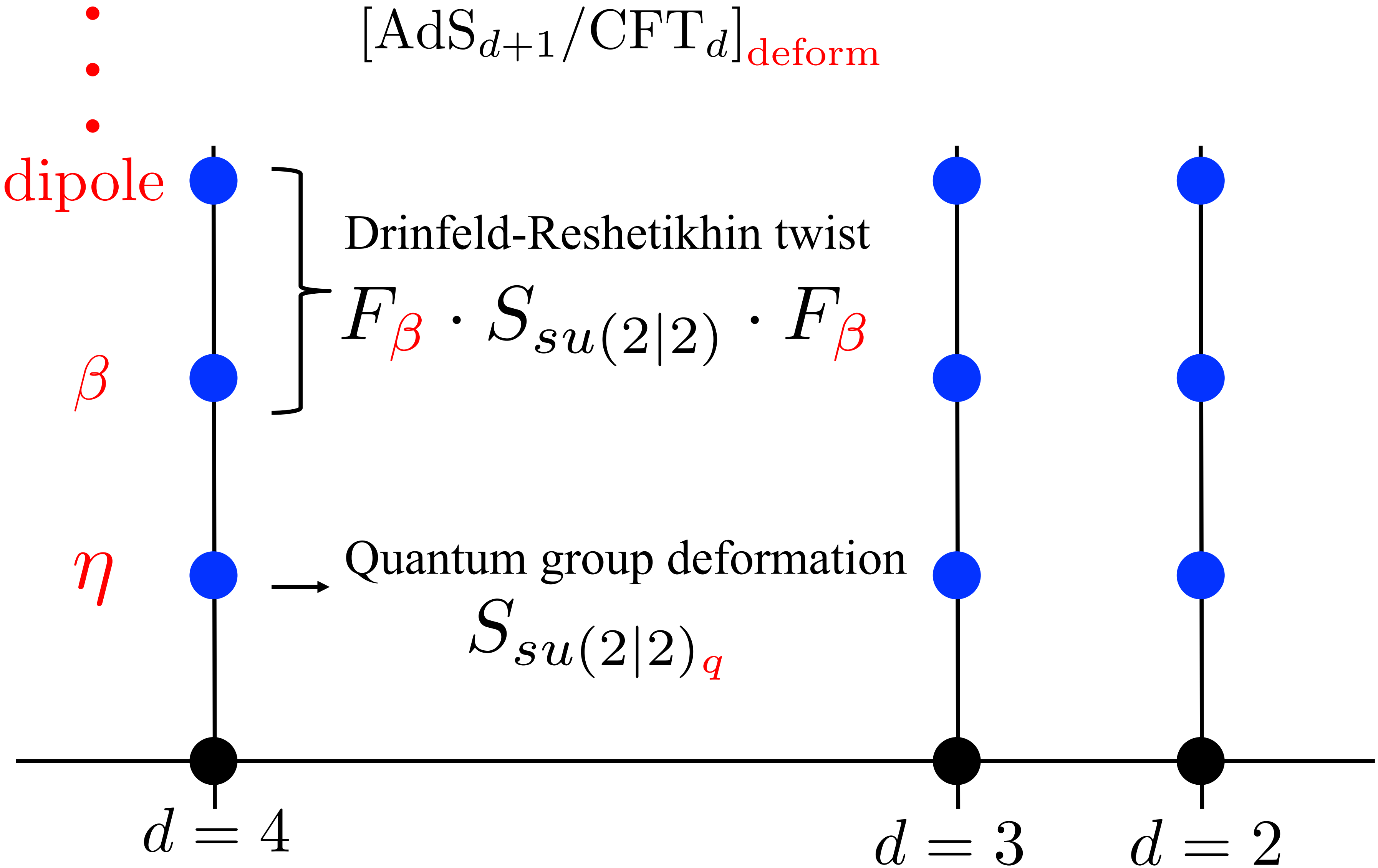

$$\widetilde{Z}(R,L) = \mathrm{Tr}\left[e^{-L\widetilde{H}(R)}\right]$$

- Physical channel
  - Partition function $Z(L,R) = \mathrm{Tr}\left[e^{-RH(L)}\right] \approx e^{-RE_0(L)}$ as $R \to \infty$

$$\widetilde{Z}(R,L) = Z(L,R) \quad \to \quad E_0(L) = -\frac{1}{R}\ln \widetilde{Z}(R,L) = \frac{L}{R}\widetilde{\mathcal{F}}(L)$$

- Minimize mirror free energy with the PBC
  - TBA equation

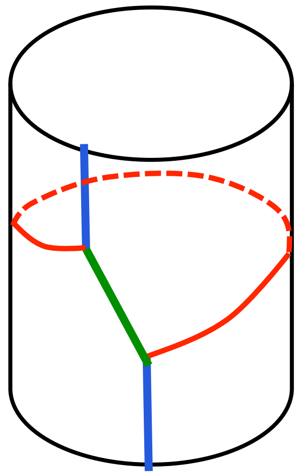
# Growing “Zoo” of Integrable AdS/CFT




+ Boundary (D-brane, Wilson-Loop,…)

# (ex) Lüscher corrections of Δ

- Strong coupling "μ"-term of $(AdS_5/CFT_4)_\eta$ [C.A. 2016]



$$q = e^{-\frac{\eta}{\sqrt{\lambda}\sqrt{1+\eta^2}}}$$

$$\delta\Delta = -\frac{8\sqrt{\lambda}(1+\eta^2)^{1/2}\sin^3\frac{p}{2}}{\sqrt{1+\eta^2\sin^2\frac{p}{2}}}\exp\left(-\frac{L}{\sqrt{\lambda}}\sqrt{\frac{1+\eta^2\sin^2\frac{p}{2}}{(1+\eta^2)\sin^2\frac{p}{2}}}\right)$$

match with string theory computation

# NEXT Challenge

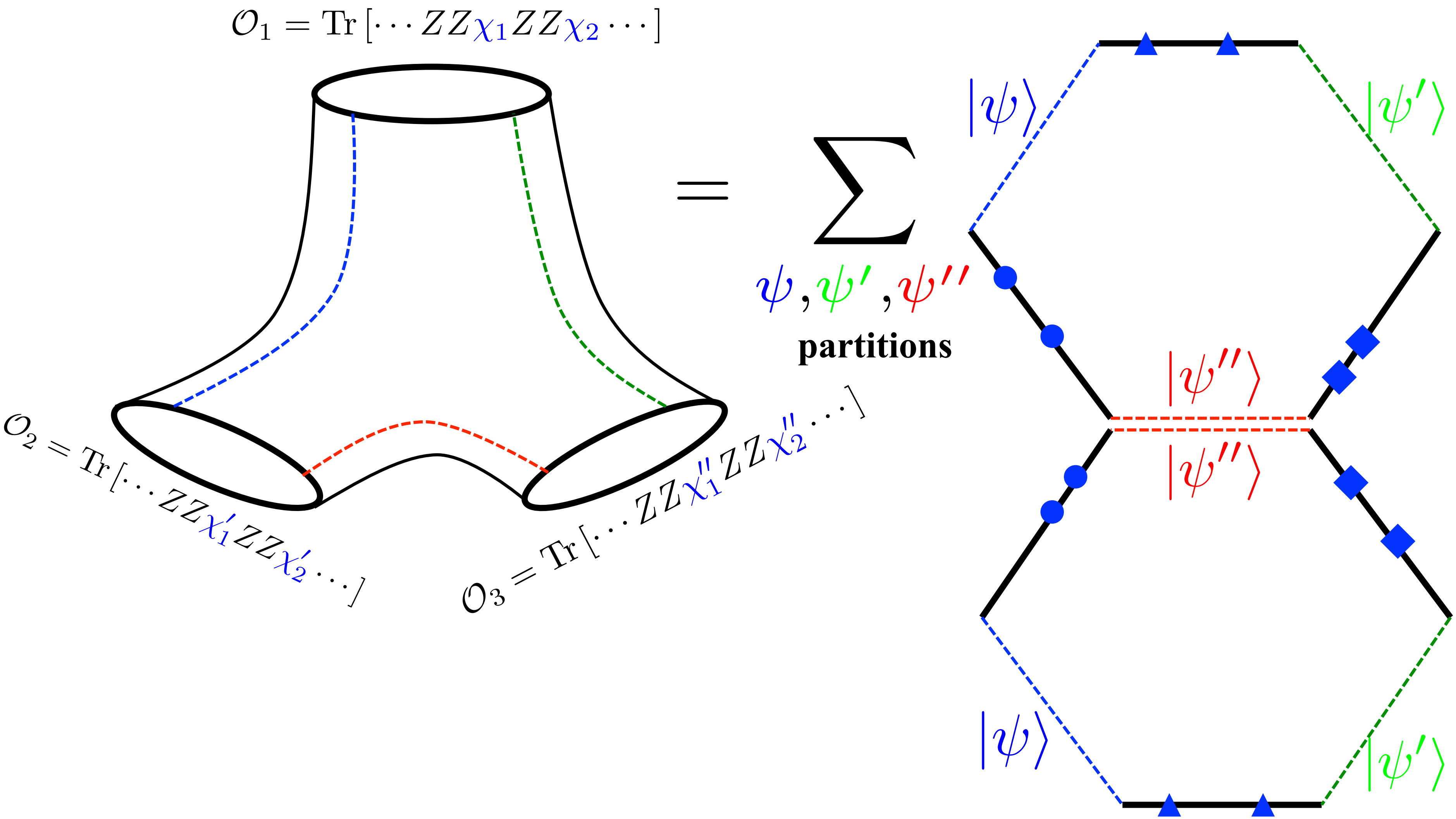
3-pt correlator or structure constant

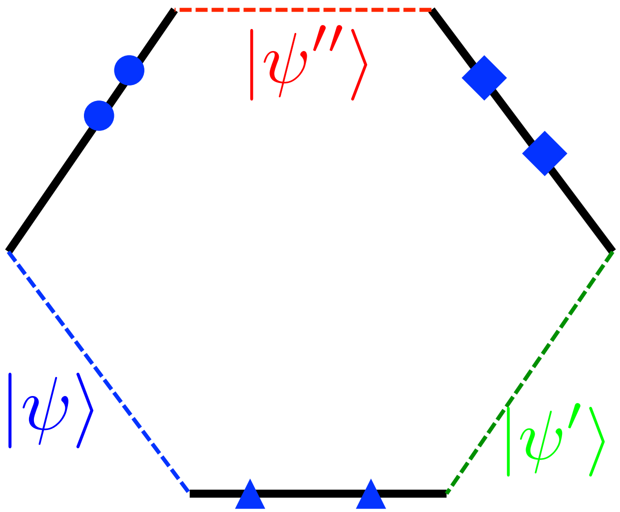
$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle = \frac{C_{123}(\lambda)}{|x_1 - x_2|^{\Delta_1+\Delta_2-\Delta_3} \dots}$$

from

Integrability (S-matrix)?

# Hexagon [Basso-Komatsu-Vieira]



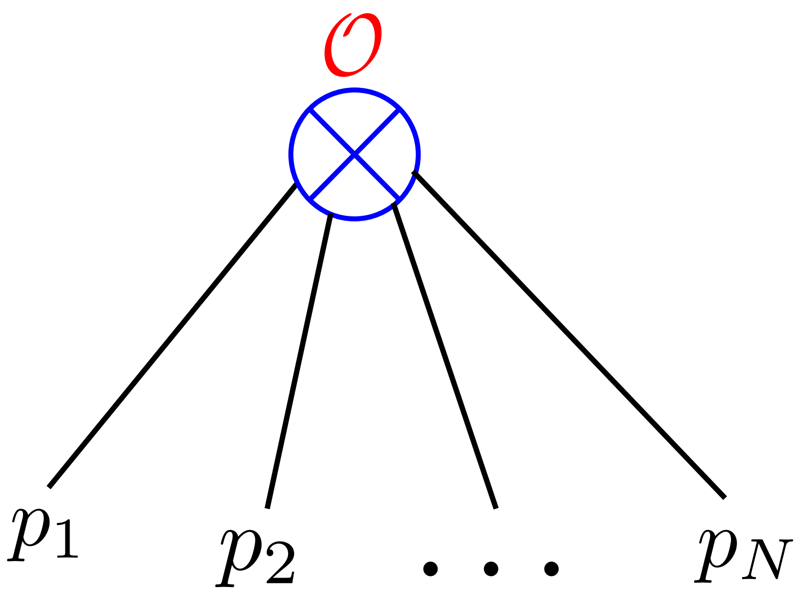



exact result in terms of S-matrix

- Sum over complete states $|\psi, \psi', \psi''\rangle$
- Sum over partitions
- Finite-size effects
- Effective in weak coupling
- Difficult to apply to strong coupling

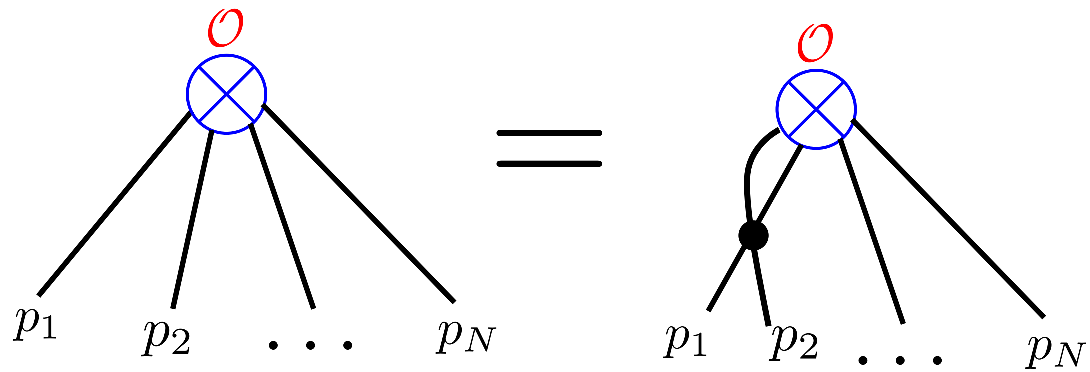
# Our approach: Form factor

- form factor: $$\langle 0|\mathcal{O}|\underbrace{p_1,\ldots,p_N}_{\text{asymp. Particle states } L\to\infty}\rangle$$

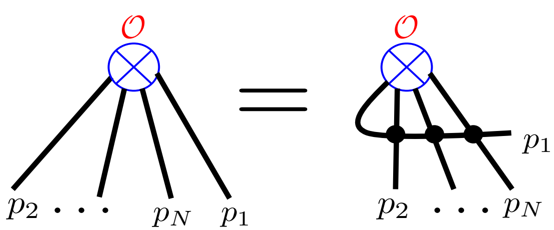


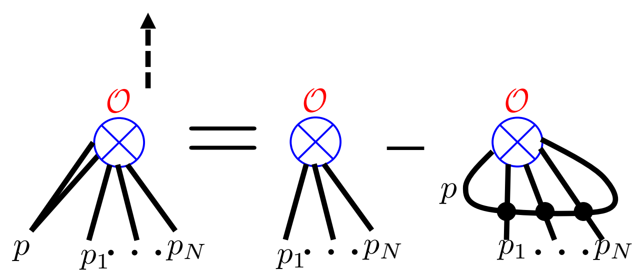
# Form Factor Axioms [Karowski,Weisz;Smirnov]

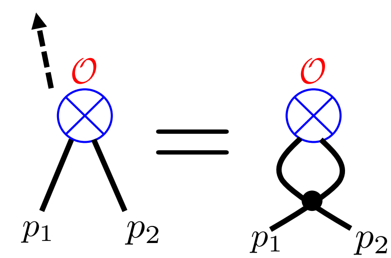
- Watson equation: $F(p_1, p_2, \cdots) = S(p_1, p_2) \cdot F(p_2, p_1, \cdots)$




$$F(p_1, \ldots, p_N) = P_{\mathrm{sym}}(p_1, \ldots, p_N) \cdot \prod_{i>j} F_{\mathrm{min}}(p_i, p_j)$$

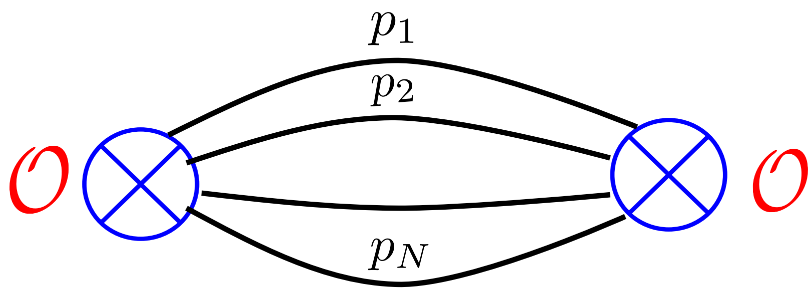


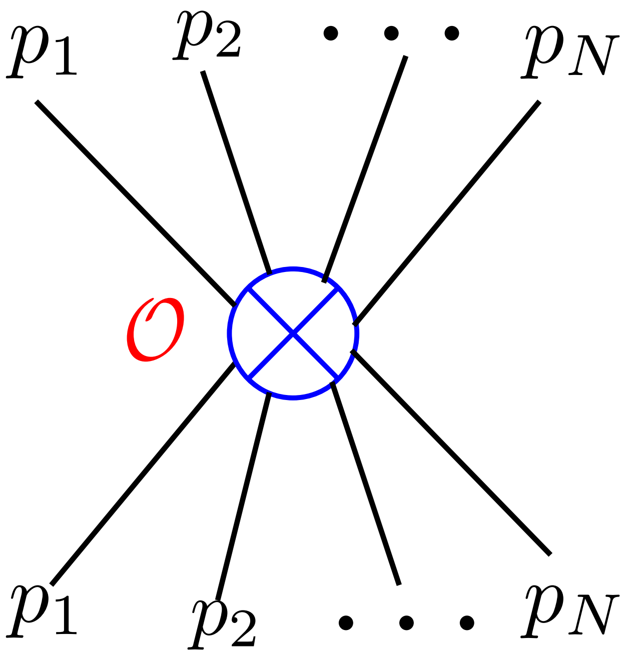
# Form factor expansion of correlators

$$\langle 0|\mathcal{O}(x)\mathcal{O}(0)|0\rangle = \sum_{N=0}^{\infty}\int\prod_i dp_i |F(p_1,\cdots,p_N)|^2 e^{iP\cdot x}$$




- Difficult for "non-diagonal" S-matrix
- Impossible to sum over internal states
- Not realistic to apply to AdS/CFT …

# Diagonal form factor approach [Bajnok, Janik]




$$\langle p_1,\dots,p_N|\mathcal{O}|p_1,\dots,p_N\rangle$$

$$\langle \underbrace{\mathcal{O}'(p)}_{p_1}\mathcal{O}\mathcal{O}'(p)\rangle$$

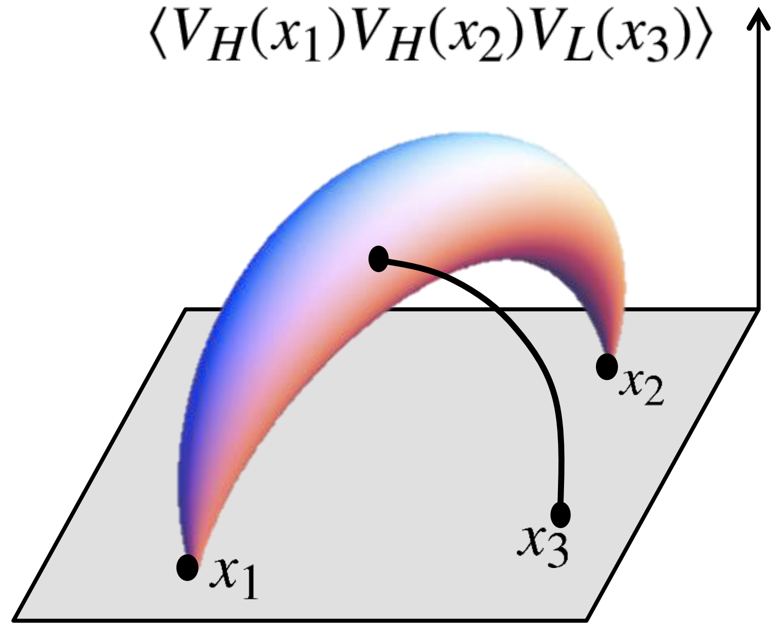
$$\mathrm{Tr}[\cdots ZZZZZZZZZ\overset{p_1}{\chi_1}ZZZZZZ\cdots ZZZZZZ\overset{p_2}{\chi_2}ZZZZZZZZZZ\cdots]$$

- Technical details to regularize
- Operators have finite-size : particle-states in finite volume

# Application to AdS/CFT: HHL [Bajnok,C.A.]

- Heavy ($\Delta \sim \sqrt{\lambda}$)-Heavy-Light 3-pt in large coupling limit

$$C_{HHL} = V_L[\Psi_H]$$

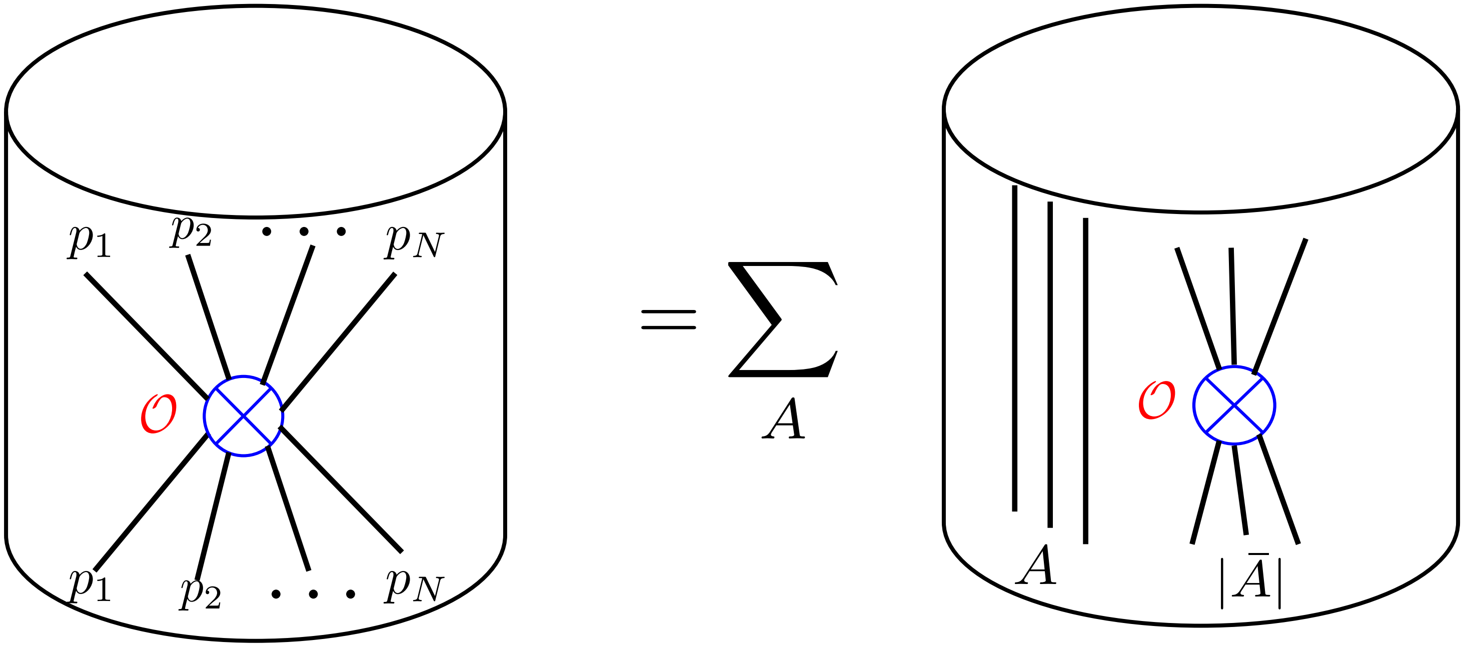



- Heavy: Giant magnon state

- Finite-size effect of structure constant [Bozhilov,C.A.]

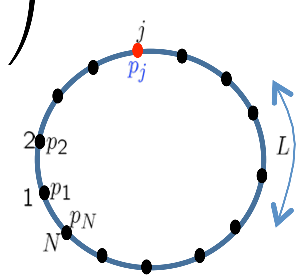
$$C = \sqrt{\lambda} \sin \frac{p}{2} - \left( 4\sqrt{\lambda} \sin^3 \frac{p}{2} + L \sin^2 \frac{p}{2} \right) e^{-\frac{L}{\sqrt{\lambda} \sin \frac{p}{2}}}$$

# Finite-size form factor [Pozsgay,Takacs]



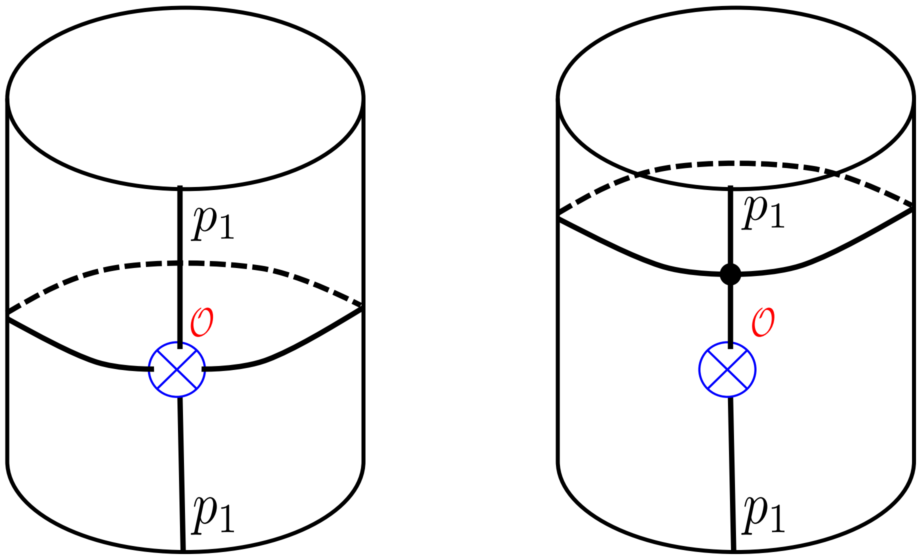

$${}_L\langle p_1,\cdots,p_N|\mathcal{O}|p_1,\cdots,p_N\rangle_L = \frac{1}{\rho_N}\sum_A \rho_A F_{|\bar{A}|}$$

$$\rho_A = \det\left(E(p_i)\frac{\partial}{\partial p_i}\underbrace{\left[p_k L - i\sum_{j\neq k\in A}\log S(p_k,p_j)\right]}_{\text{Log of Bethe-Yang equation}}\right)$$



# Form factor for HHL [Bajnok,C.A.]

$$ {}_L\langle p_2, p_1|\mathcal{O}|p_1, p_2\rangle_L = \frac{F_2(p_1,p_2)+\rho_1(p_1)F_1(p_2)+\rho_1(p_2)F_1(p_1)}{\rho_2(p_1,p_2)} $$




- Matches with the finite-size structure constant !

# Summary

- Integrability (S-matrix) is essential for exact correlators
- Integrable AdS/CFTs are multiplying
- Form factor approach is a way toward this goal
  - We are applying to many deformed cases in various dimensions
  - Challenge is to find FF at arbitrary coupling constant
  - HHH ?

Thank you for attention!