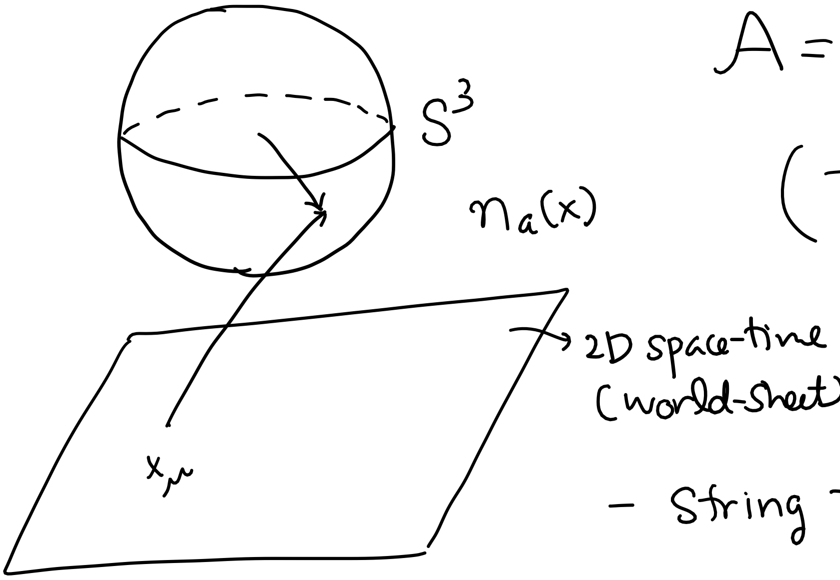
Seminar

Hot Sausage

with Balog, Ravanini

1. O(3) sigma model



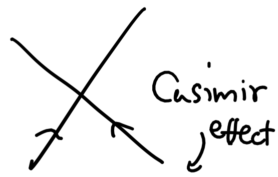
$$A = \frac{1}{2g}\int (\partial_\mu n_a)^2 d^2x + i\,\theta\, T$$

$$\left(T = \frac{1}{8\pi}\varepsilon^{abc} n_a \partial_\mu n_b \partial_\nu n_c \epsilon^{\mu\nu}\right)$$

- String target space: S^2, "$g \sim -\frac{2\pi}{t}$" (t: RG time)

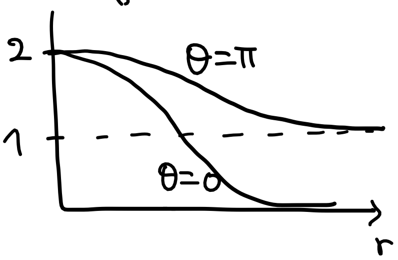
- Integrability is discovered at $\theta = 0, \pi$
- Haldane conjecture; $\begin{cases} \theta = 0 & \text{Integer spin } J \gg 1 \text{ of } (\vec{S}_i \cdot \vec{S}_{i+1}) \text{ spin.} \\ \theta = \pi & \frac{1}{2}\text{integer spin } \quad \text{"} \end{cases}$

- $\theta = 0$; Massive scattering theory
  - $m \sim e^{-\frac{2\pi}{g}}$ (g is used to give mass scale)
  - triplet of O(3); $(+, 0, -)$
  - $S(\theta) = \frac{\theta + 2\pi i}{\theta - 2\pi i} P_0 + (\times) P_1 + \frac{\theta - i\pi}{\theta + i\pi} P_2$ (P_0: $|\sigma| = 0$, P_1: $|\sigma| = 1$, P_2: $|\sigma| = 2$)

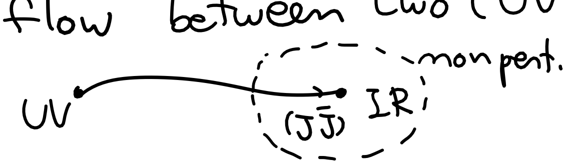


$E(r) = \frac{\pi\, C_{eff}}{6r}$

$C_{eff}(r)$



- $\theta = \pi$; Massless scattering theory
  - RG flow between two (UV, IR CFT)



  - IR CFT: $SU(2)_1$ WZW.
  - Away from IR; massless doublets L_σ, R_σ ($\sigma = 1, 2$)

$$e = \pm p \quad (L, R)$$

$$S^{(LL)} = S^{(RR)} = S^{(LR)} = \frac{\Gamma\,\Gamma}{\Gamma\,\Gamma}\,\frac{\theta \mathbb{1} - i\pi P}{\theta - i\pi}.$$

2. Sausage σ-Model [Fateev - Onofri - Zamolodchikov]

$$A[G] = \frac{1}{2}\int \underset{\substack{\uparrow \\ \text{target metric}}}{G_{ij}(x)}\, \partial_\mu x^i \partial^\mu x^j \, d^2\sigma$$

- not CFT. RG flow
- $$\frac{dG_{ij}}{dt} = -\frac{1}{2\pi} R_{ij} + \mathcal{O}(R^2)$$ small curvature expansion

$$\Rightarrow \frac{d\, e^{\Phi}}{dt} = \frac{1}{4\pi}\left(\frac{\partial}{\partial x^i}\right)^2 \Phi$$ Conformal coordinates $G_{ij} = e^{\Phi}\delta_{ij}$

fails as $t \to \infty$; IR nonpert.

- consider $\Phi(X, Y)$ $(0 \le X < 2\pi)$ $\quad \Phi = \Phi(Y)$ axial sym.

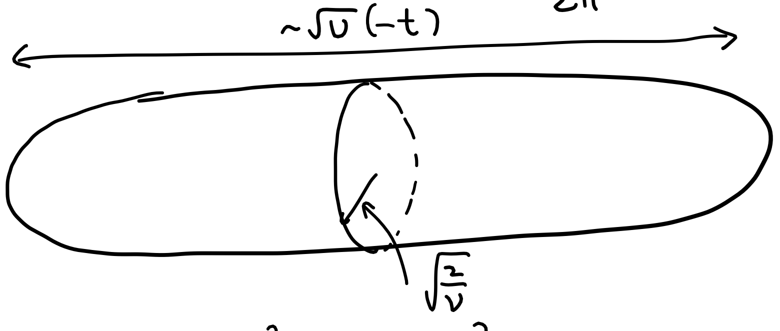
$$\frac{\partial \Phi}{\partial t} = \frac{1}{4\pi} e^{-\Phi} \frac{\partial^2 \Phi}{\partial Y^2} \to 0$$ as $\Phi \to -2|Y|$ as $Y \to \pm\infty$

$$A = \frac{1}{2}\int e^{\Phi(Y)} \left((\partial_\mu Y)^2 + (\partial_\mu X)^2 \right) + i\theta T$$

with $$e^{\Phi} = \frac{1}{2}\left(a(t) + b(t)\, \mathrm{ch}\, 2Y \right)$$

RG eq. $$\frac{da}{dt} = \frac{1}{2\pi} b^2 \qquad \frac{db}{dt} = \frac{1}{2\pi} ab \Rightarrow a^2 - b^2 \equiv \nu^2 \text{ (fixed const.)}$$

$$a(t) = -\nu \coth \frac{\nu(t-t_0)}{2\pi} \qquad b(t) = -\nu / \sinh \frac{\nu(t-t_0)}{2\pi}$$



only UV as $t \to -\infty$

- valid in UV

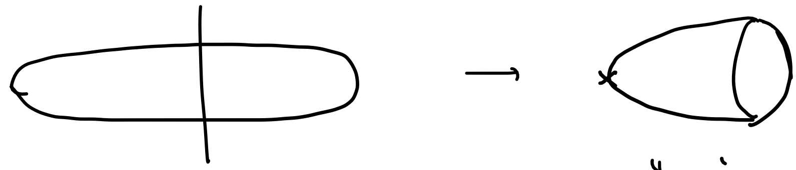
$\nwarrow$ sausage

- $$A_\nu = \int \frac{(\partial_\mu Y)^2 + (\partial_\mu X)^2}{a + b\cosh 2Y} d^2\sigma + i\theta T$$

$$\downarrow \quad n_a^2 = 1$$

$$\frac{(\partial_\mu n_a)^2}{1 - \frac{\nu^2}{2g^2(t)} n_3^2} \qquad g(t) = \frac{\nu}{2}\coth\frac{\nu(t_0 - t)}{4\pi}$$

- Mirror of CP^1

- 

2d black hole $\frac{SL(2)_k}{U(1)}$ CFT

"Cigar" $k=\frac{2}{\nu}$

- $(AdS/CFT)_\eta$ "η-deformation" or "sine-Liouville" $\cong$ $\mathcal{N}=2$ LFT

How to solve this model nonpert ?

$O(3)$ $\downarrow$

- Integrability
- S : trigonometric quantum group. $q=e^{\lambda}$ (ex) $S^{++}_{++}=\frac{\operatorname{sh}\lambda(\theta-i\pi)}{\operatorname{sh}\lambda(\theta+i\pi)}\xrightarrow{\lambda\to 0}\frac{\theta-i\pi}{\theta+i\pi}$

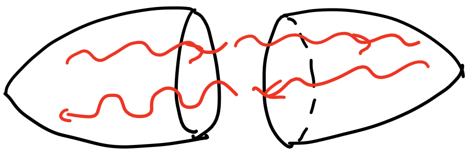
both for $\theta=0,\pi$

- $\boxed{\nu\longleftrightarrow\lambda\ ?}$

Reflection Amp of sLFT.

$$V_p^{\pm}\sim e^{(\frac{1}{2b}\pm iP)Y}$$

$$\langle V_p^{+}V_p^{-}\rangle\sim\frac{S(p)}{z^{2\Delta}}$$



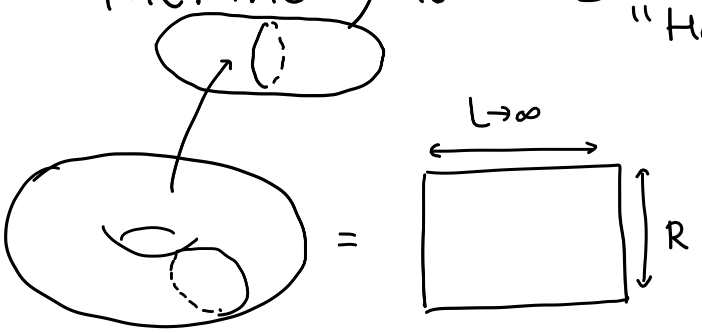
Quantization condition

$$S(p)S(-p)\,e^{iPL}=1\ \rightarrow\ c\sim 2-24p^2$$

$L\uparrow\ \sqrt{\nu}(-t)$ $\qquad 2-c\sim\frac{1}{t^2}$

$$\frac{\nu}{4\pi}=\lambda+O(\lambda^2)$$

- Thermodynamics for Sausage model "Hot Sausage"



channel duality

L : space $\quad R=\frac{1}{T}$ time → Thermody

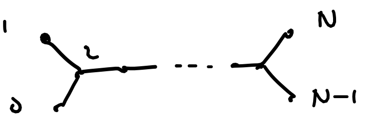
S-matrix $\updownarrow$

L : time $\quad R$ = space $\quad Z=e^{-LE_0(R)}$

TBA = Y-system

S-matrix $\rightarrow$ $\quad y_a^{+}y_a^{-}=\prod_b Y_b^{I_{ab}}$ → incidence matrix

$Y_a=1+y_a$



of D-type Dynkin

valid only $\lambda=\frac{1}{N}$ $\uparrow$

(ex) $y_2^{+}y_2^{-}=Y_0Y_1Y_2$

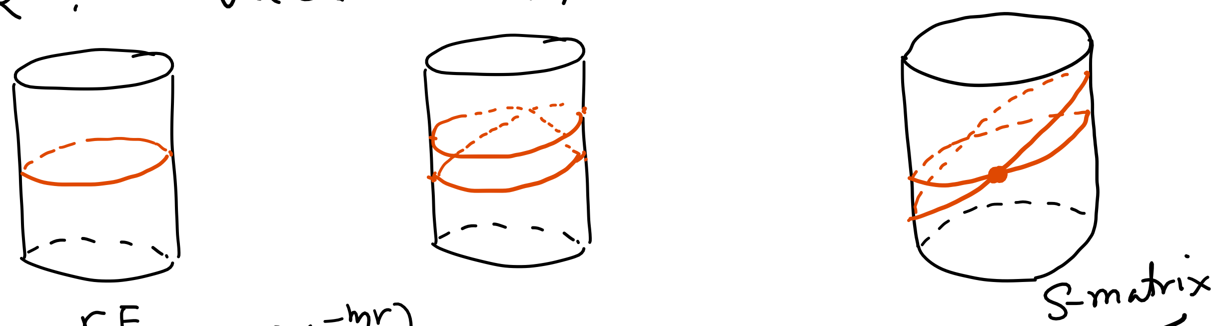
$$y_a^{\pm}(\theta)=y_a(\theta\pm\tfrac{\pi}{2}i)$$

How to analytically continue ?

- Baxter's Q or "T–Q" relation
- NLIE : only 3-functions & valid for any λ.

Check

① IR : $r \gg \frac{1}{m}$ vac. energy



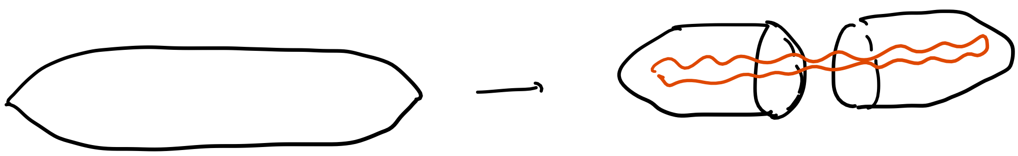
$\sim \int e^{-rE} dp \sim O(e^{-mr})$

$\sim e^{-2mr}$ double wrapping

② UV : $r \ll \frac{1}{m}$ (numerically solving NLIE)

$$C(r) \cong 2 - \frac{3\pi^2/2}{\lambda(\log mr + \delta)^2}$$

(cf) with



non pert. relation

$$\frac{v}{4\pi} = \frac{\lambda}{1-2\lambda} \quad \text{for all } \lambda$$