Low-energy effective theories for metals

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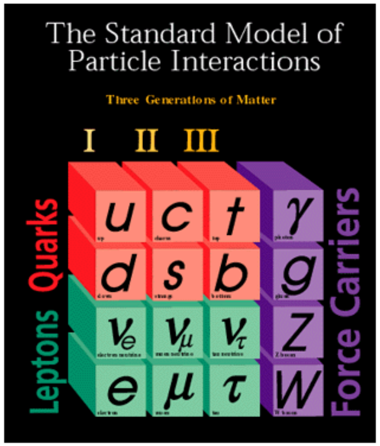
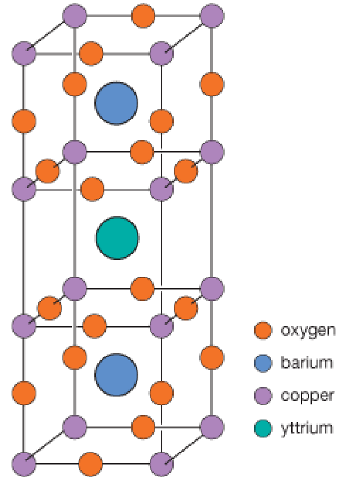
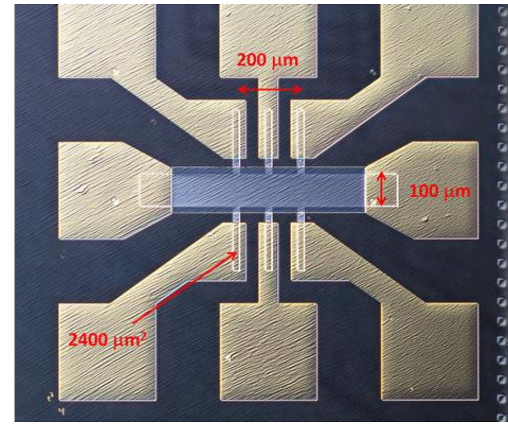
Emergent Phenomena







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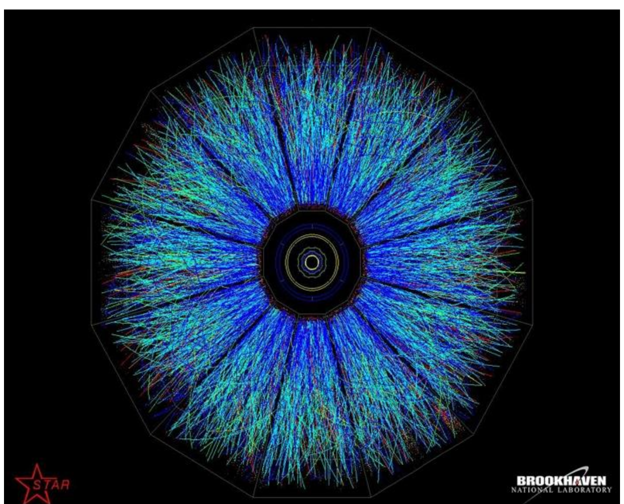
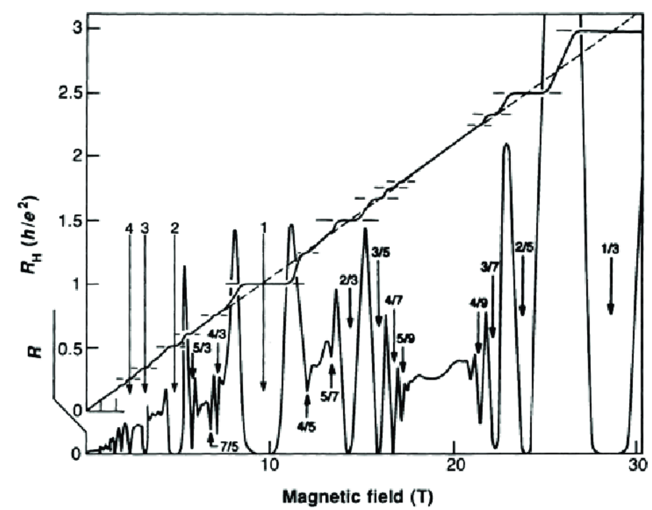




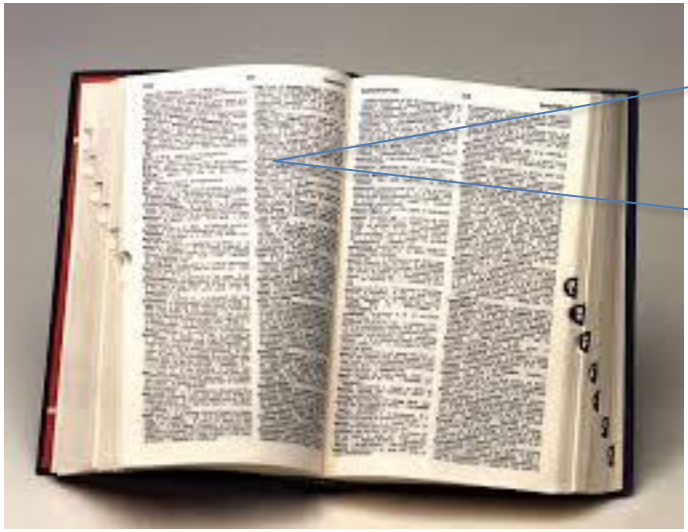
Quark-gluon plasma

High temperature superconductor

Fractional quantum Hall state

Mandate of low-energy theorists

Identify all phases of `matter' and characterize each phase in terms of a minimal set of low-energy parameters that determines all low-energy observables



Book of phases

Phase Y

- Low-energy parameters : $x_{1,}\ x_{2,\ \ldots}$
- Observables
  $O_i=f_i\,(x_{1,}\ x_{2,\ \ldots}) + ..$
- Symmetry : $g_{1,}\ g_{2,\ \ldots}$
- Basin of attraction : ..

Universality of low-energy physics

There exists a set of low-energy parameters defined through observables measured at energy scale $\mu \ll \Lambda$

$$S\left(\phi, \psi; \{g_j\}; \Lambda\right)$$

$$\mathbf{g}^{(m)}(\mu) = \Gamma_m\left(\{\omega_i^{(m)}, k_i^{(m)}\}; \{g_j\}; \Lambda\right), \quad \omega_i^{(m)} \sim \mu$$

Those low-energy parameters determine all observables at that energy scale and below within errors that vanish in powers of μ/Λ

$$\Gamma_n\left(\{\omega_i, k_i\}; \{g_j\}; \Lambda\right) = f_n\left(\{\omega_i, k_i\}; \{\mathbf{g}^{(m)}(\mu)\}\right) + (\mu/\Lambda)^a$$

$$\text{for} \quad \omega_i \leq \mu$$

*Extracting $f_n\left(\{\omega_i, k_i\}; \{\mathbf{g}^{(m)}(\mu)\}\right)$ is still highly non-trivial

For relativistic QFTs

- The space of low-energy parameters is composed of a finite number of marginal and relevant parameters that carry non-negative scaling dimensions
- RG flow within the finite-dimensional space

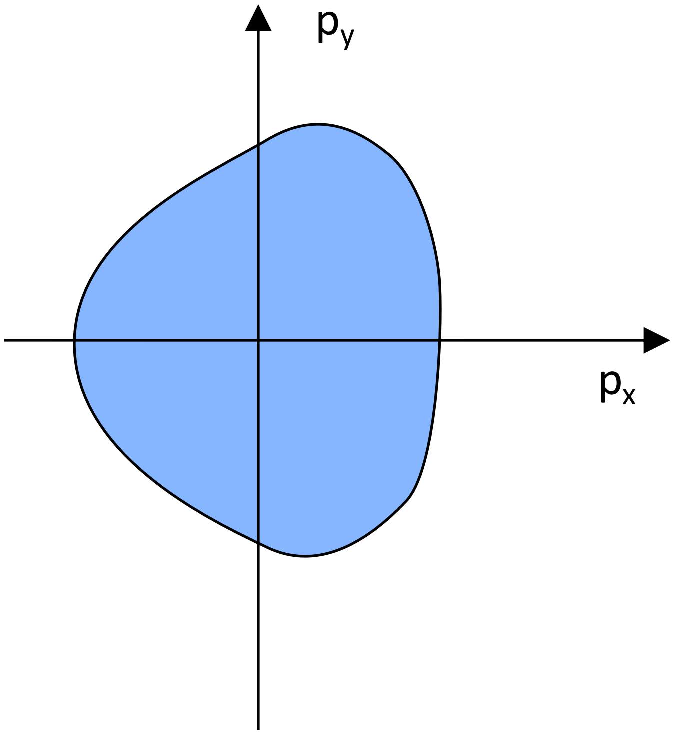
$$\mathbf{g}^{(m)}(\mu)$$

- Scale-invariant fixed points of the renormalization group flow represent universality classes

$$\lim_{\mu \to 0} \mathbf{g}^{(m)}(\mu)$$

Metals

A non-zero density of fermions form a droplet of occupied states in the momentum space



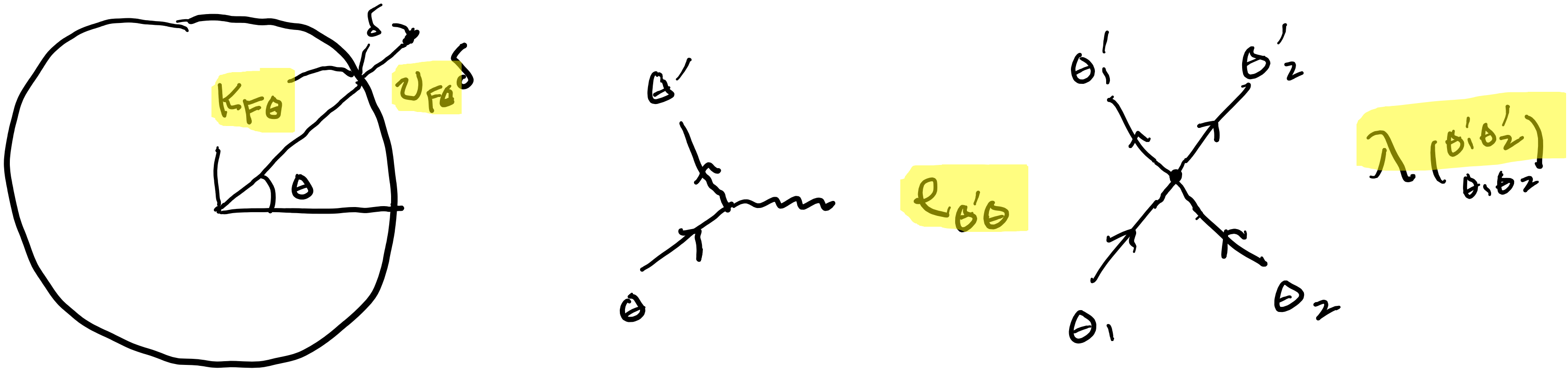
Infinitely many gapless modes that describe soft shape deformations of the droplet (particle-hole excitations near Fermi surface)

Metals subject to strong quantum fluctuations remain poorly understood

- Metals with vector flavors can behave like matrix models (non-trivial large N limit)
- Dynamical kinematic energy quenching
- **UV/IR mixing**

Theoretical challenge : infinite-dimensional space of low-energy theories

- Low-energy theories are characterized by Fermi momentum, Fermi velocity, couplings, which are functions of angles around FS

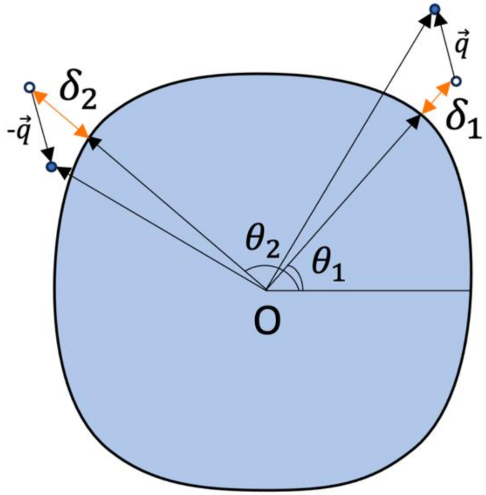


- To chart the space of metallic universality classes, one needs to track a functional renormalization group flow for a minimal set of coupling functions
  - Field-theoretic functional RG

Borges, Borissov, Singh, Schlief, SL (2023)

Theoretical challenge : absence of scale invariance

- There is no scale invariance in metals due to Fermi momentum, which is a large momentum(UV) scale but a low-energy(IR) scale
- Fermi momentum, measured in the unit of μ, grows incessantly under scale transformations



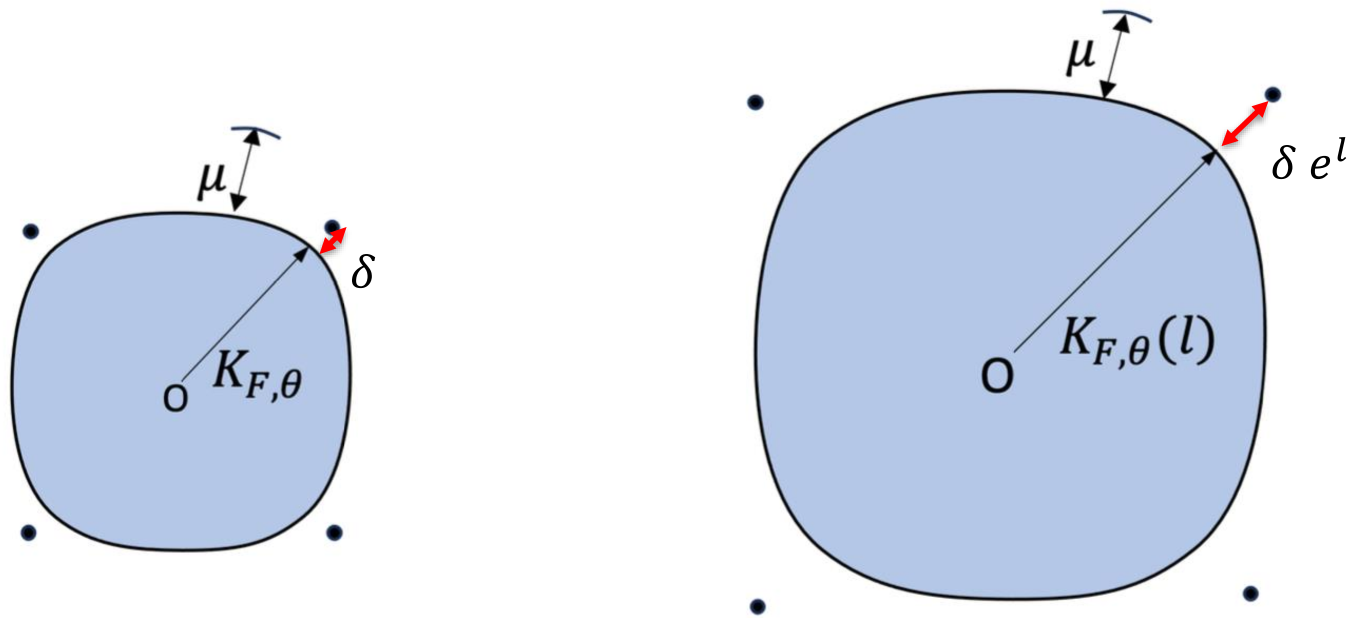
$$\Delta\theta \sim \frac{q}{K_F}$$

$$q \to q/b, \quad \theta \to \theta$$

$$K_F \to bK_F$$

Callan-Symanzik equation for metals

relates the vertex function of a theory with K_F to the vertex function with different K_F

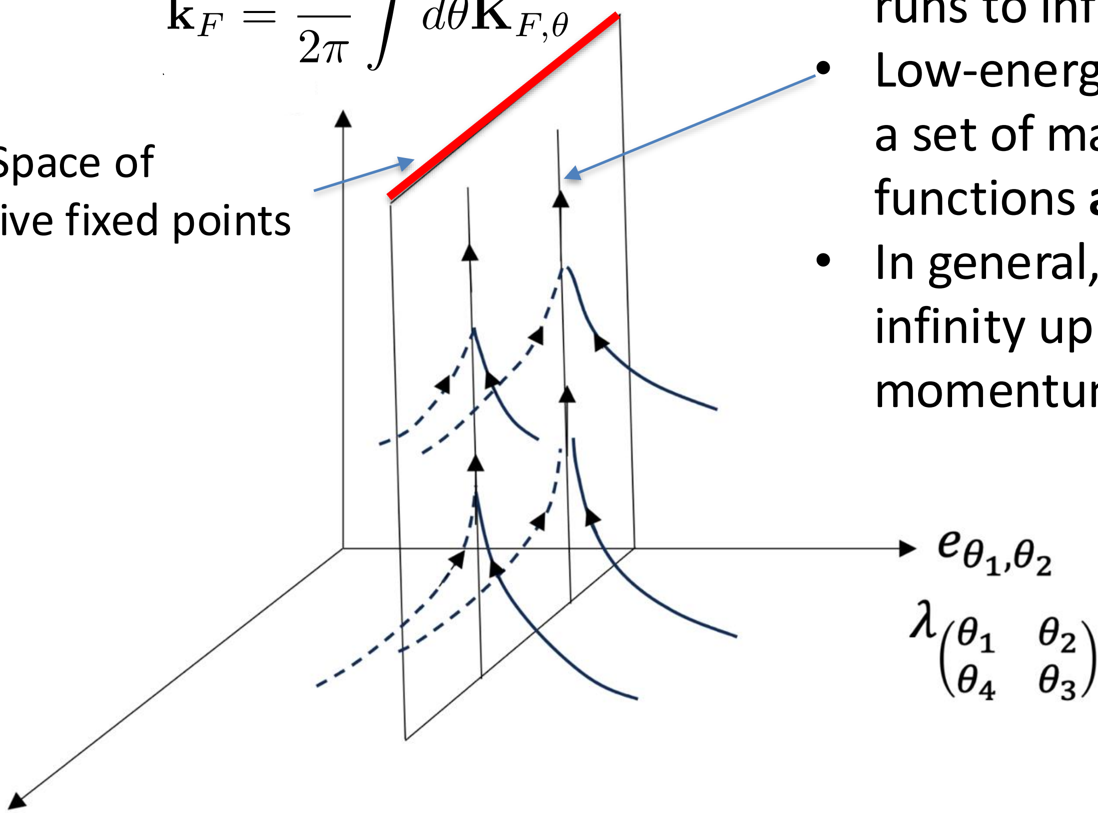


$$\Gamma^{(2m,n)}\left(\{\mathbf{k}_i(l)\};[e(l),v_F(l),K_F(l),\lambda(l)];\Lambda\right)=\Gamma^{(2m,n)}\left(\{\mathbf{k}_i\};[e,v_F,K_F,\lambda];\Lambda\right)$$
$$\times\exp\left\{\int_0^l dl'\left(-(2m+n-1)(d-1)z\left(l'\right)-n\eta_\phi\left(l'\right)-\sum_{i=1}^{2m}\eta_{\psi,\theta_i}\left(l'\right)+n\frac{(d-1)}{2}+m(d-2)+2\right)\right\}$$

Metallic Universality classes correspond to projective fixed points

- RG flow is attracted toward an one-dimensional manifold in which k_F runs to infinity
- Low-energy observables are fixed by a set of marginal/relevant coupling functions **and** Fermi momentum
- In general, one can not set k_F to infinity up front because large Fermi momentum limit can be singular

$$\mathbf{k}_F = \frac{1}{2\pi}\int d\theta \mathbf{K}_{F,\theta}$$



[Kukreja, Besharat, SL (2024)]

Physical consequences of projective nature of fixed points

- Mismatches between scaling dimension of couplings and their relevance
  - Coupling functions with negative dimensions can become marginal/relevant

- A lack of unique dynamical critical exponent
  - The vertex function in different kinematic regimes exhibit scale invariance under q $\rightarrow sq, \omega \rightarrow s^z\omega$ with different z

Example 0

Fermi Liquids

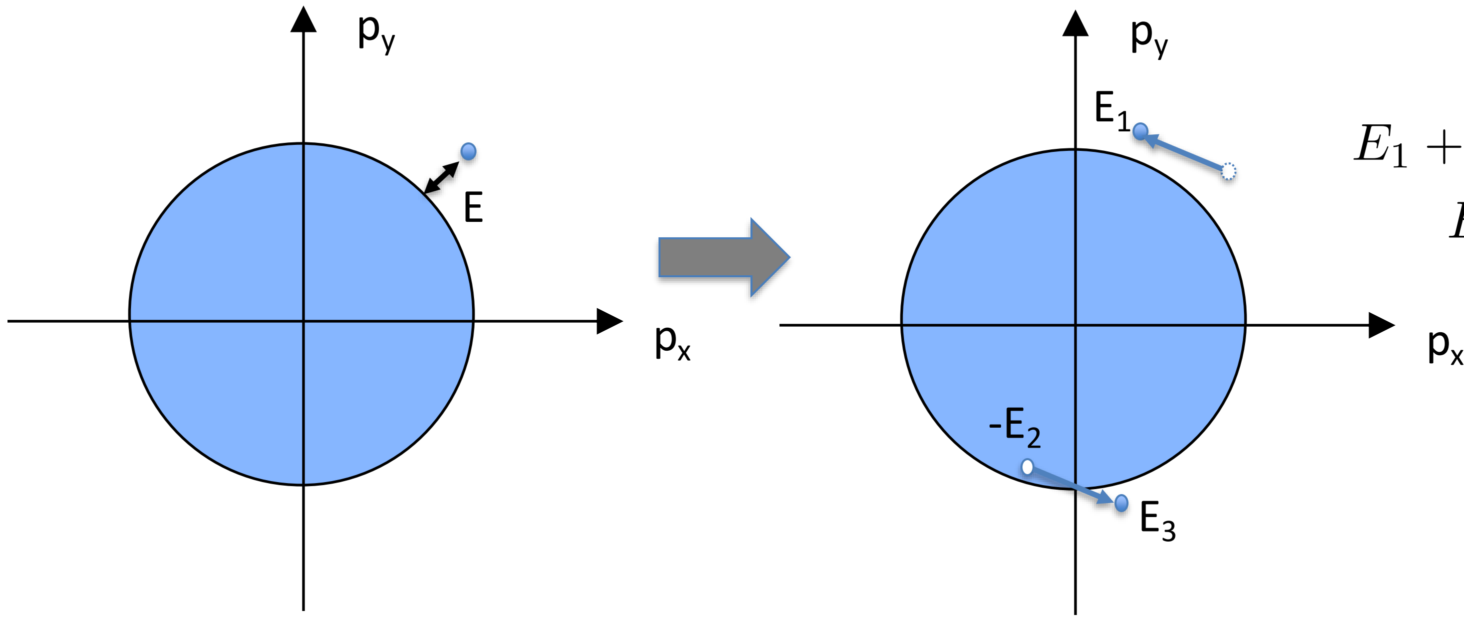
[Landau]

[Benfatto, Gallavotti] [Shankar] [Polchinski]

A finite density of fermions subject to a short-range interaction V

Particles close to the Fermi surface have long life-time

$$\frac{1}{\tau} \sim V^2 E^2$$



$$E_1 + E_2 + E_3 = E$$

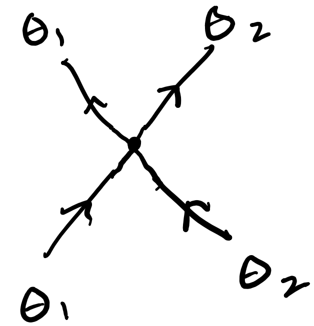
$$E_1, E_2, E_3 \geq 0$$

Low-energy eigenstates of interacting electrons are labeled by the occupation numbers of single-particle states

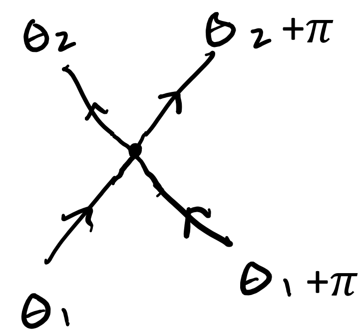
$$|n_{k_1,\sigma_1}, n_{k_2,\sigma_2}, \ldots >'$$

Fermi liquids

- Marginal functions :
  - Fermi momentum, Fermi velocity, Landau function (the forward scattering)



- Vanishing pairing interaction at the projective fixed point



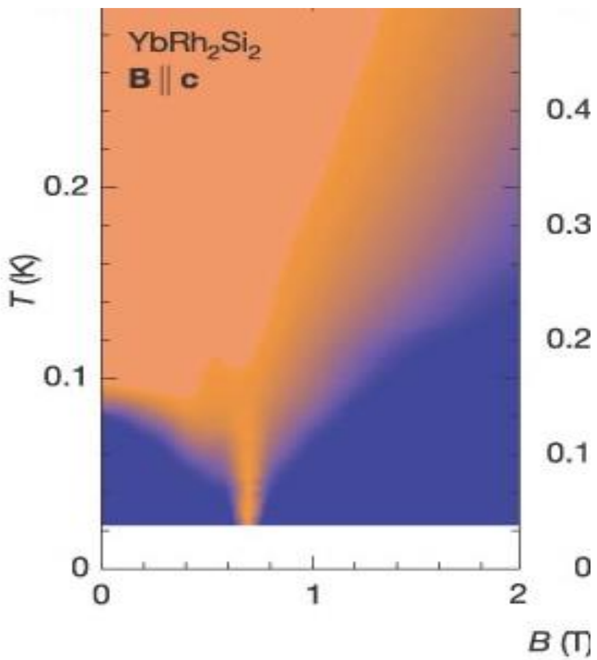
= 0

- Emergent symmetry : loop U(1)

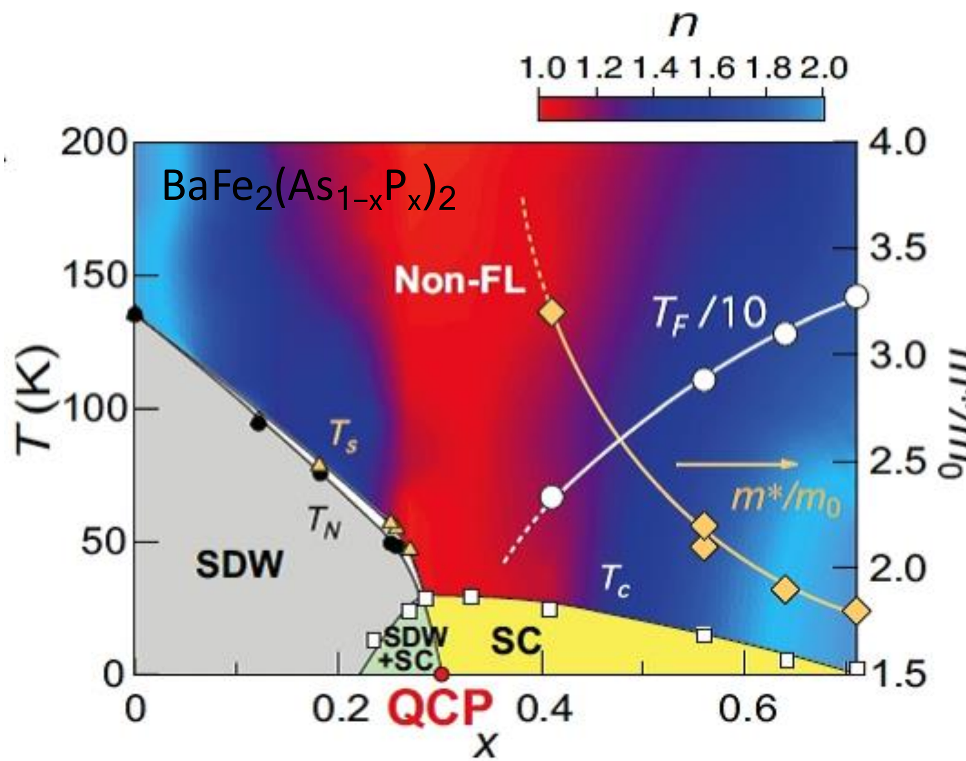
[Handane (93), Else, Thorgen, Senthil (22)]

$$\psi_j(\theta) \to e^{i\gamma(\theta)}\psi_j(\theta)$$

Non-Fermi liquids at Quantum Critical Points



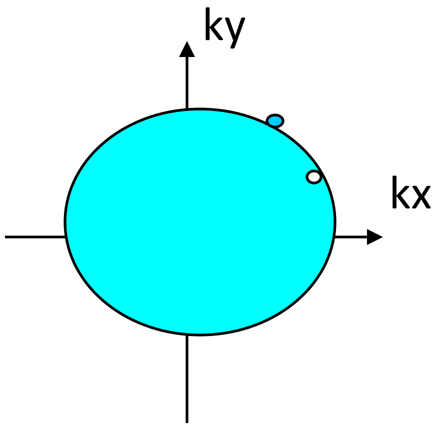
[Custers et al.(2003)]

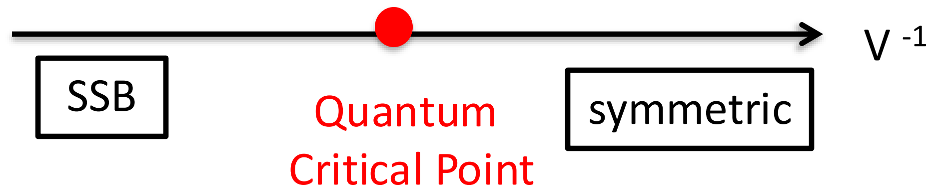


[Hashimoto et al. Science 336, 1554 (2012)]

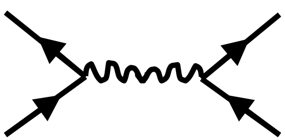
- Gapless collective boson coupled with particle-hole excitations creates qualitative deviations from Fermi liquids
- Strongly interacting metals realized in 2+1 dimension – the focus of this talk

Non-Fermi liquids @ QCP





- Gapless collective modes mediate interactions between electrons which are singular in the low-energy limit : no single-particle excitation with long lifetime



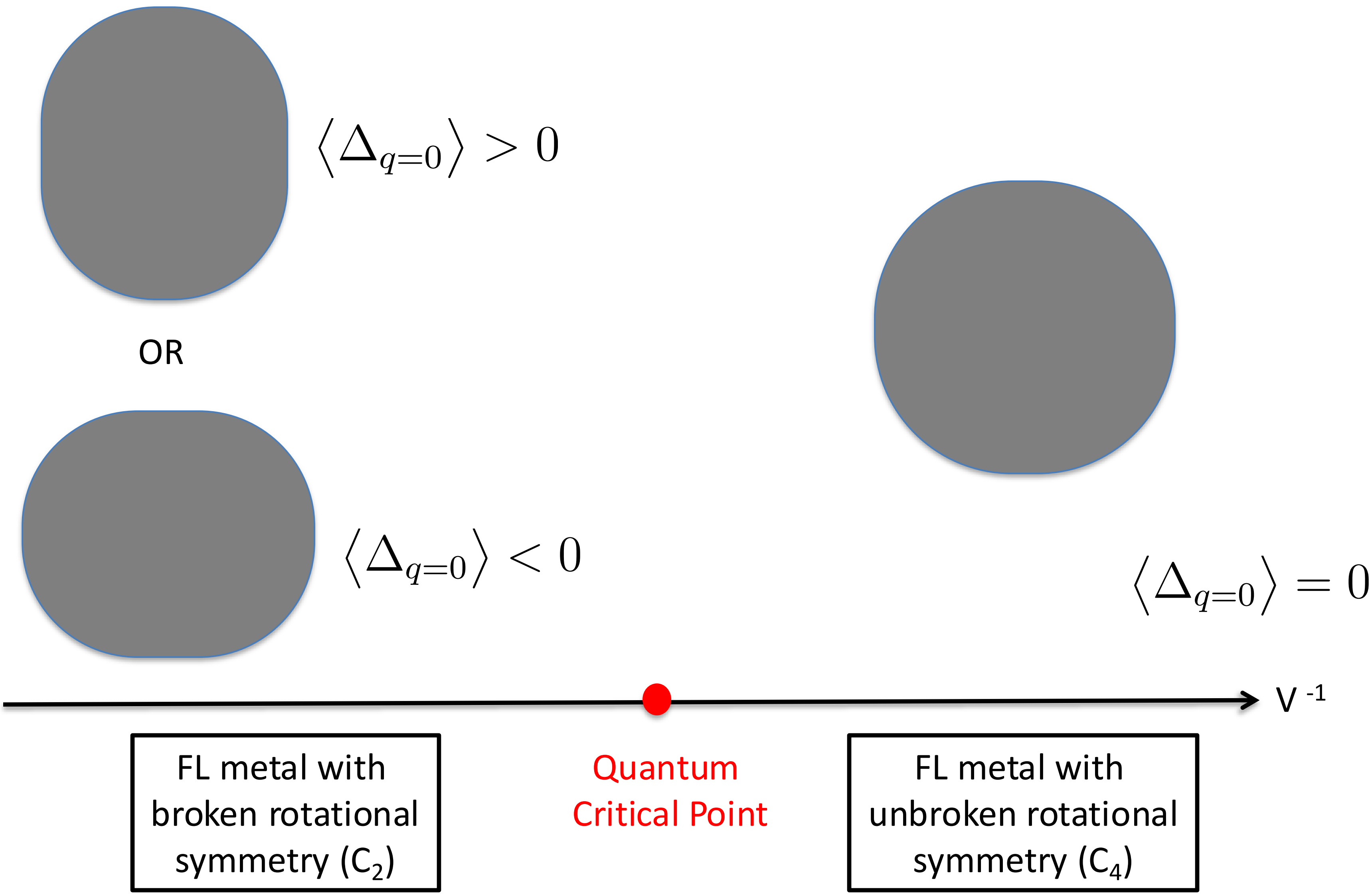
$$V(E) \sim E^{-\gamma}$$

$$\frac{1}{\tau} \sim V(E)^2 E^2 > E$$

- At quantum critical points, Fermi sea is subject to strong quantum fluctuations

Example 1

Ising-Nematic QCP



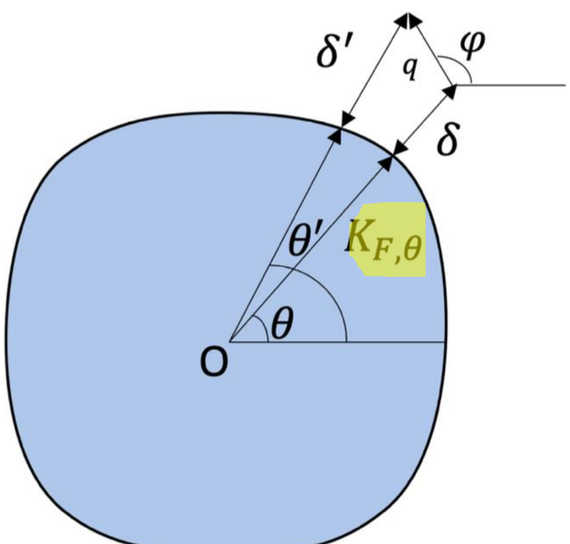
Theory of the Ising-nematic quantum critical metal

$$S = \int d_f^3\mathbf{k}\ \psi_j^\dagger(k_0,\delta,\theta)\left[ik_0 + v_{F,\theta}\delta\right]\psi_j(k_0,\delta,\theta) + \frac{1}{2}\int d_b^3\mathbf{q}\ |\mathbf{q}|^2\phi(-\mathbf{q})\phi(\mathbf{q})$$

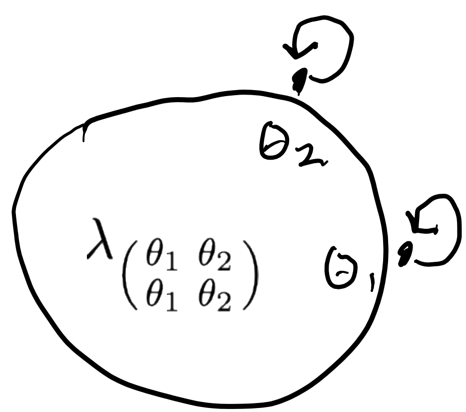
$$+ \frac{1}{\sqrt{N}}\int d_f^3\mathbf{k}\ d_b^3\mathbf{q}\ \boldsymbol{e}_{\Theta(\theta,\vec{q}),\theta}\phi(q_0,\vec{q})\psi_j^\dagger\left(k_0+q_0,\Delta(\delta,\theta,\vec{q}),\Theta(\theta,\vec{q})\right)\psi_j(k_0,\delta,\theta)$$

$$+ \int d_f^3\mathbf{k}\ d_f^3\mathbf{k}'\ d_b^3\mathbf{q}\ \boldsymbol{\lambda}_{\begin{pmatrix}\Theta(\theta,\vec{q}) & \theta' \\ \theta & \Theta(\theta',\vec{q})\end{pmatrix}}$$

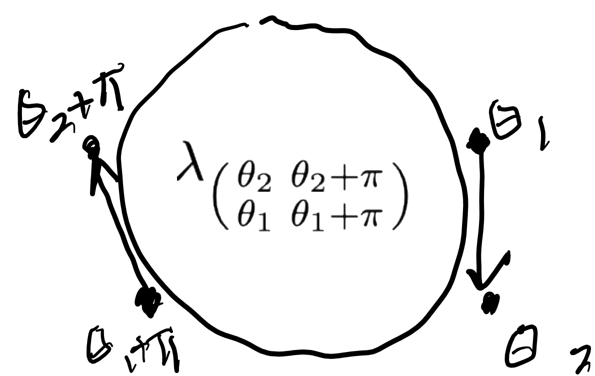
$$\psi_{j_1}^\dagger\left(k_0+q_0,\Delta(\delta,\theta,\vec{q}),\Theta(\theta,\vec{q})\right)\psi_{j_1}(k_0,\delta,\theta)\psi_{j_2}^\dagger(k_0',\delta',\theta')\psi_{j_2}\left(k_0'+q_0,\Delta(\delta',\theta',\vec{q}),\Theta(\theta',\vec{q})\right)$$



Forward scattering

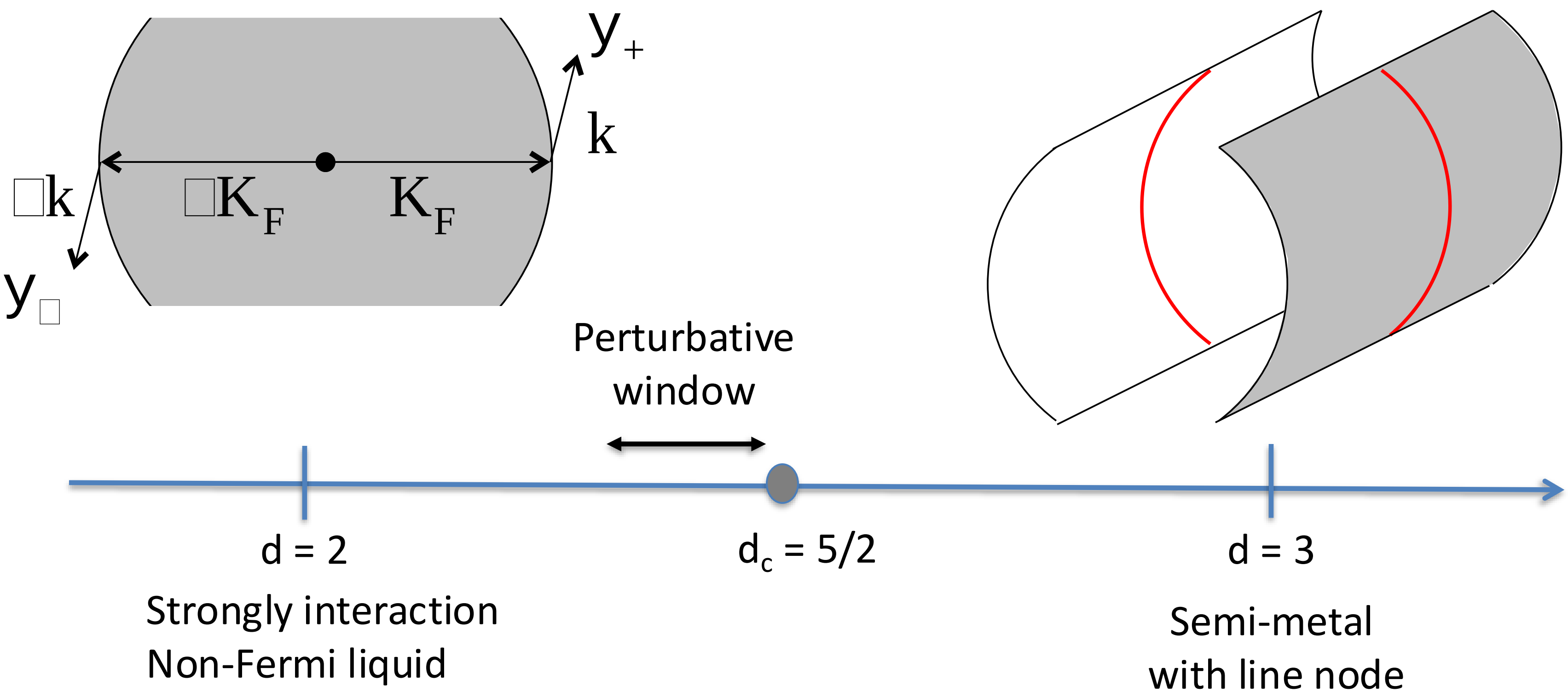


Pairing interaction



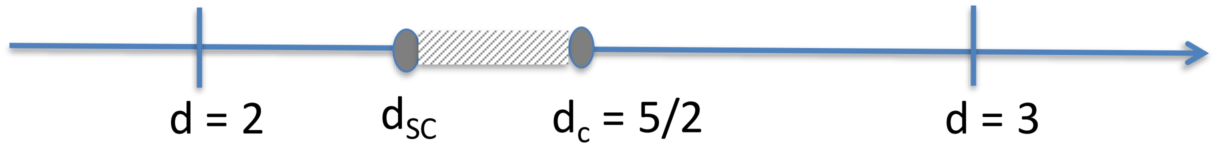
A dimensional regularization : an analytic continuation of the 2d metal to a semi-metal with line node in 3d

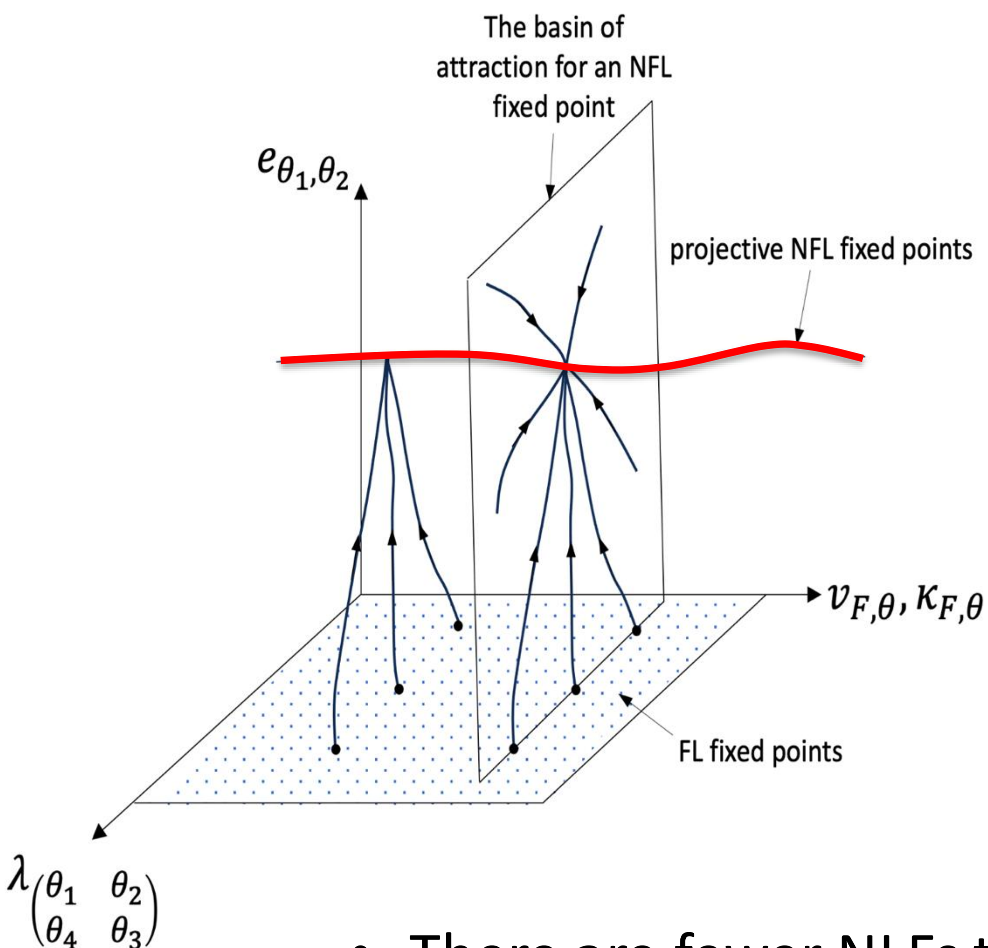
[D. Dalidovich, SL (2013)]



- Upper critical dimension : $d_c = 5/2$

$d > d_{SC}$





- Stable projective NFL fixed points
- Only two marginal parameters for the shape of FS and Fermi velocity

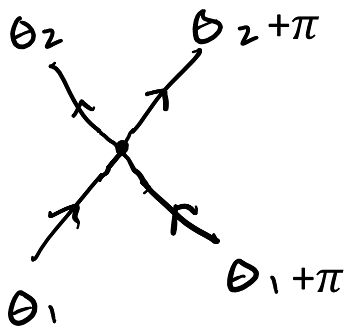
$$\kappa_{F,\theta}, v_{F,\theta}$$

- Landau function and pairing interaction are non-zero, but fixed by the marginal parameters and Fermi momentum

- There are fewer NLFs than FLs
- SC fluctuations are intrinsic parts of NFLs

Universal pairing interaction ($d > d_{SC}$)

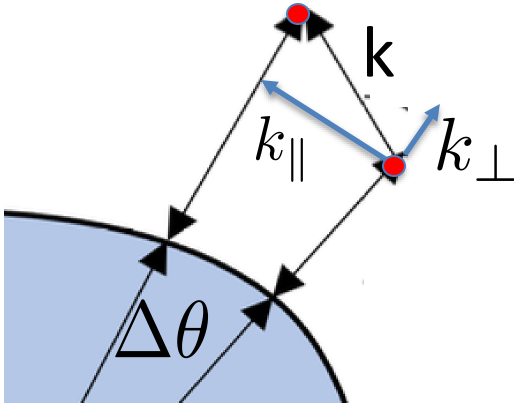
Dimensionless pairing interaction for Cooper pairs
with center of mass momentum $\vec{q}$ and energy ω



$$\Gamma^{(4)}_{\theta_1,\theta_2}(\vec{q}=0,\omega) \sim \left|\frac{\Delta\theta}{\theta_1-\theta_2}\right|^{2\Delta}$$

$$k_\perp \sim \omega^{1/z}, \quad k_\parallel^2 \sim k_\perp K_F \sim \omega^{1/z} K_F$$

$$\Delta\theta \sim k_\parallel / K_F \sim \sqrt{\omega^{1/z}/K_F}$$



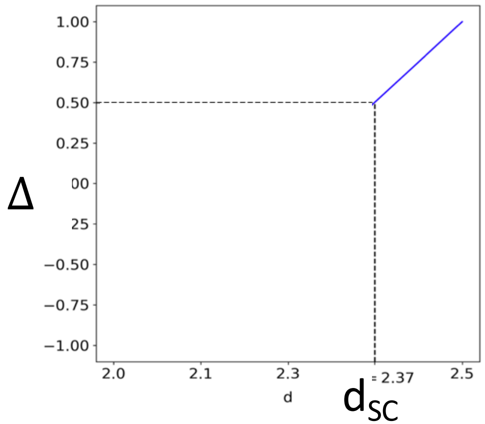
z : a dynamical critical exponent

Δ: universal exponent

(minus the scaling dimension of the four-fermion coupling function)

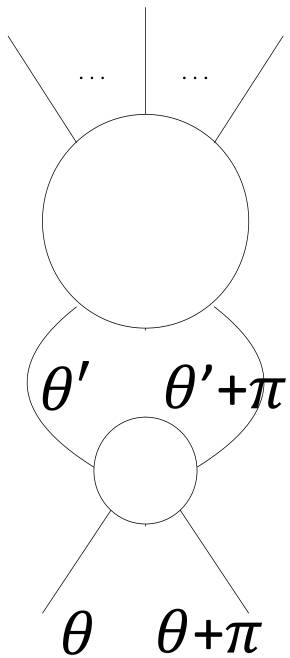
UV/IR mixing

$$\Gamma^{(4)}_{\theta_1,\theta_2}(\vec{q}=0,\omega) \sim \left| \left(\frac{\omega^{1/z}}{K_F}\right)^{1/2} \frac{1}{\theta_1-\theta_2} \right|^{2\Delta}$$



- Δ is a function of dimension
- Four-fermion coupling has a negative scaling dimension and *naively* irrelevant for $\Delta > 0$
- The pairing interaction becomes marginal for $\Delta = 1/2$
  - The growth of the number of patches compensates the decay of the interaction strength

$$\int \frac{d\theta'}{\Delta\theta}$$



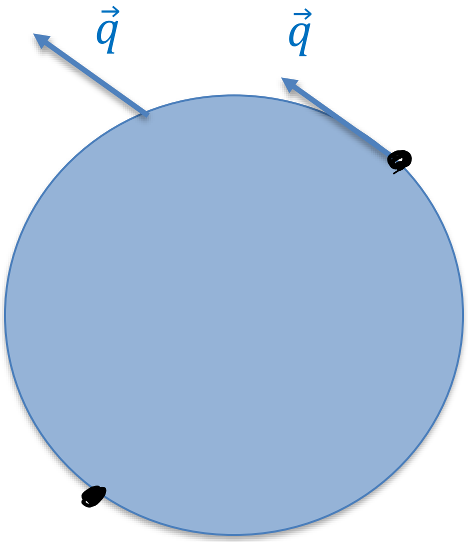
$$\Delta\theta = \sqrt{\omega^{1/z}/K_F}$$

Mismatch between scaling dimension and relevancy

Universal pairing interaction for $\vec{q} \neq 0$

Crossover function

$$\Gamma^{(4)}_{\theta_1,\theta_2}(\vec{q},\omega) \sim \left| \left(\frac{\omega^{1/z}}{K_F} \right)^{1/2} \frac{1}{\theta_1 - \theta_2} \right|^{2\Delta} \mathcal{V}_{\theta_1}(\vec{q})\mathcal{V}_{\theta_2}(\vec{q})$$

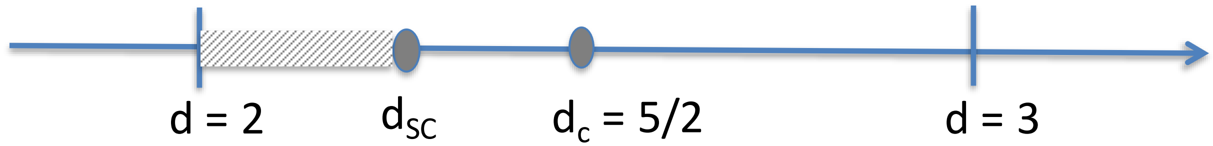


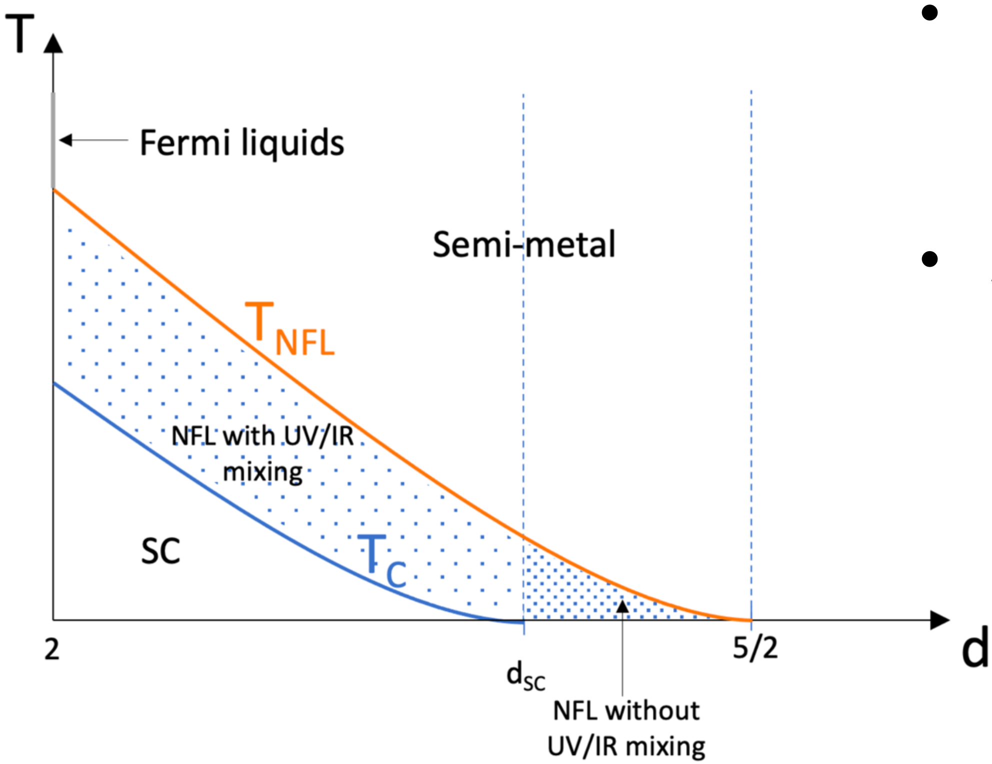
For large q,

$$\mathcal{V}_\theta(\vec{q}) \sim \begin{cases} \left(\frac{\omega^{1/z}}{q} \right)^{\frac{\eta_d}{2}}, & \vec{q} \text{ not parallel to FS at } \theta \\ \left(\frac{\omega^{1/z} K_F}{q^2} \right)^{\frac{\eta_d}{2}}, & \vec{q} \text{ almost parallel to FS at } \theta \end{cases}$$

No unique dynamical critical exponent that sets the relative scaling between q and ω

$d < d_{SC}$





- The non-Fermi liquids become unstable against superconductivity for $d<d_{sc}$
- A window of energy scale controlled by NFL quasi-fixed point with strong UV/IR mixing
  - In d=2, LU(1) is expected to be broken down to the OLU(1) [odd loop U(1)]

$$\psi_j(\theta) \rightarrow e^{i\gamma(\theta)}\psi_j(\theta)$$

$$\gamma(\theta+\pi) = -\gamma(\theta)$$

Summary

- The space of metallic universality classes is infinite dimensional
- Due to Fermi momentum, fixed points of metals are defined only projectively
  - Mismatch between scaling dimensions and relevancy of couplings
  - No unique dynamical critical exponent
- Universality classes of non-Fermi liquids being mapped out