COSMOLOGY JOURNAL CLUB

노트 제목 2009-12-13

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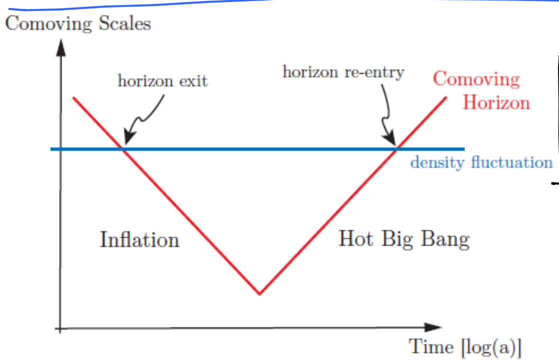
Lecture 3. Contact with observations

$\boxed{\text{Question}}$: Can inflation theory explain current and future observations?

Review of inflation theory (slow roll)



super	$k^{-1} > (aH)^{-1}$ or $aH > k$
	$\dot{\mathcal{R}} \cong 0$
sub	$k^{-1} \ll (aH)^{-1}$ or $aH \ll k$
	quantum fluctuation

single field slow roll

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\,\left[R - (\nabla\phi)^2 - 2V(\phi)\right]$$

with a gauge

$$\delta\phi = 0\,, \qquad g_{ij} = a^2[(1-2\mathcal{R})\delta_{ij} + h_{ij}]\,, \qquad \partial_i h_{ij} = h^i_i = 0\,.$$

metric fluctuation

① scalar perturbation

$$\boxed{S_{(2)} = \frac{1}{2}\int d^4x\, a^3\frac{\dot\phi^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i\mathcal{R})^2\right]}$$

or

$$\boxed{S_{(2)} = \frac{1}{2}\int d\tau d^3x\left[(v')^2 + (\partial_i v)^2 + \frac{z''}{z}v^2\right]}$$

with $v \equiv z\mathcal{R}\,,$ where $z^2 \equiv a^2\frac{\dot\phi^2}{H^2} = 2a^2\varepsilon$

$$\langle R_{\vec k} R_{\vec k'}\rangle = \frac{1}{z^2}\langle v_{\vec k} v_{\vec k'}\rangle = \frac{(2\pi)^3}{z^2}\delta(k+k')\,|v_k|^2 \qquad v_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1-\frac{i}{k\tau}\right)$$

$$= \frac{(2\pi)^3}{a^2\, 2k}\delta(k+k')\frac{H^2}{\dot\phi^2}\left(1+\frac{1}{k^2\tau^2}\right) \qquad a(\tau) = -\frac{1}{H\tau}$$

$$= (2\pi)^3\delta(k+k')\frac{H^2}{2k^3}\frac{H^2}{\dot\phi^2}\left(1+\cancel{k^2\tau^2}\right)$$

$$\equiv (2\pi)^3\delta(k+k')P_R(k) = (2\pi)^3\delta(k+k')\frac{2\pi^2}{k^3}\Delta^2_R(k)$$

$$\rightarrow \boxed{\underset{\text{"}\Delta_s^2}{\Delta_R^2(k)} = \frac{H_*^2}{(2\pi)^2}\frac{H_*^2}{\dot\phi_*^2}}$$ at t_* $\ni$: $aH(t_*) = k \rightarrow t_* = t_*(k)$

② tensor perturbation (Gravitational wave)

$$S_{(2)} = \frac{M_{pl}^2}{8}\int d\tau dx^3 a^2 \left[(h'_{ij})^2 - (\partial_l h_{ij})^2\right] \qquad v \equiv \frac{M_{pl}}{2} a h$$

$$\langle h_{\vec k} h_{\vec k'}\rangle = \frac{4}{a^2 M_{pl}^2}\langle v_k v_{k'}\rangle = \frac{4}{M_{pl}^2}(2\pi)^3\delta(k+k')\frac{H^2}{2k^3}(1+\cancel{k^2\tau^2})$$

$$\equiv (2\pi)^3 \delta(k+k') P_h(k) = (2\pi)^3\delta(k+k')\frac{2\pi^2}{k^3}\Delta_h^2(k)$$

$$\therefore \boxed{2\Delta_h^2 \equiv \Delta_t^2 = \frac{2}{\pi^2}\frac{H_*^2}{M_{pl}^2}}$$

t-to-s ratio : $r \equiv \dfrac{\Delta_t^2}{\Delta_s^2}$

③ let $\Delta_s^2(k) \equiv A_s\left(\frac{k}{k_*}\right)^{n_s-1}$, $\Delta_t^2(k) = A_t\left(\frac{k}{k_*}\right)^{n_t}$

$$\therefore n_s - 1 \equiv \frac{d\ln\Delta_s^2}{d\ln k} = \frac{d\ln\Delta_s^2}{dN}\frac{dN}{d\ln k} \qquad \varepsilon \equiv -\frac{\dot H}{H^2} = -\frac{d\ln H}{dN}$$

$$= \left(\frac{d\ln H^2}{dN} - \underbrace{\frac{d\ln\varepsilon}{dN}}\right)\frac{dN}{d\ln k} \qquad \Delta_s^2 = \frac{1}{8\pi^2}\frac{H^2}{M_{pl}^2}\frac{1}{\varepsilon}\Big|_{k=aH}$$

$$= \left(-2\varepsilon - 2(\varepsilon - \eta)\right)(1+\varepsilon)^{-1} \qquad k = aH \rightarrow \ln k = N + \ln H$$

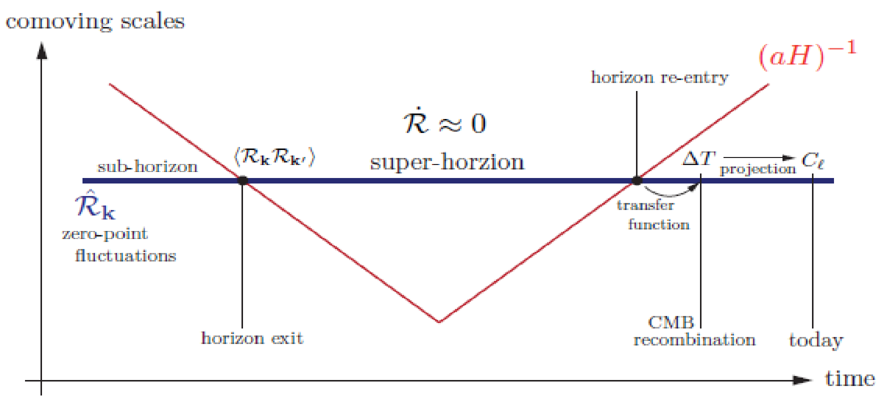
$$\rightarrow n_s - 1 \cong 2\eta_* - 4\varepsilon_*, \quad n_t \cong -2\varepsilon_*$$

④ Slow roll model : $\varepsilon \simeq \epsilon_V$, $\eta \cong \eta_V - \epsilon_V$

$$\therefore n_s - 1 = 2\eta_V^* - 6\epsilon_V^*, \quad n_t = -2\epsilon_V^*$$

$$\Delta_s^2 \approx \frac{1}{24\pi^2}\frac{V}{M_{pl}^4}\frac{1}{\epsilon_V}, \quad \Delta_t^2 = \frac{2}{3\pi^2}\frac{V}{M_{pl}^4} \rightarrow r = 16\epsilon_V^*$$

$$\therefore \boxed{r = -8n_t}$$



$$\mathcal{R}_{\vec{k}}(\tau_*)\, T_Q(k,\tau,\tau_*) = Q_{\vec{k}}(\tau)$$

"transfer function for Q"

[Claim] "observed quantity" Q fluctuations are originated by $\mathcal{R}$ of sub. and evolved by "unknown mechanism" T_Q

[Q: observed quantities] Power spectrum of

① CMB anisotropies [Temperature, polarization]

$$C_\ell^{XY} = \frac{2}{\pi}\int k^2 dk \underbrace{P(k)}_{P_{\mathcal{R}} \text{ or } P_h} \underbrace{\Delta_{X_\ell}(k)\,\Delta_{Y_\ell}(k)}_{\text{"transfer"}}$$

$$\Delta_{X\ell}(k) = \int_0^{\tau_0} d\tau \underbrace{S_X(k,\tau)}_{\text{Sources}} \underbrace{P_{X\ell}(k[\tau_0-\tau])}_{\text{Projection}}$$

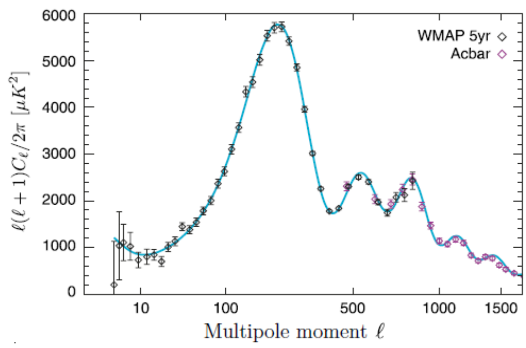
② LSS matter distribution : How about dark matter?

$$\delta \equiv \frac{\delta\rho}{\rho}$$

bias parameter b

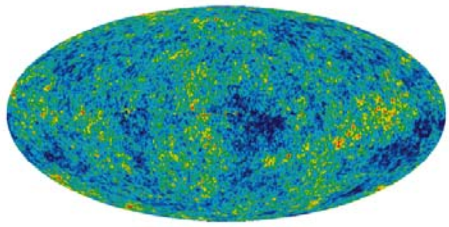
$$\delta_g = b\,\delta$$

$$\boxed{P_\delta(k,\tau) = \underbrace{\frac{4}{25}\left(\frac{k}{aH}\right)^4 T_\delta^2(k,\tau)}_{\text{transfer}} P_{\mathcal{R}}(k)}$$



① CMB anisotropies

1. Temperature



$$\Theta(\hat{n}) \equiv \frac{\Delta T(\hat{n})}{T_0} = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n}) \longleftrightarrow a_{\ell m} = \int \mathrm{d}\Omega\, Y^*_{\ell m}(\hat{n})\Theta(\hat{n})$$

define

$$C_\ell^{TT} = \frac{1}{2\ell+1}\sum_m \langle a^*_{\ell m} a_{\ell m}\rangle\,, \qquad \text{or} \qquad \langle a^*_{\ell m} a_{\ell' m'}\rangle = C_\ell^{TT}\delta_{\ell\ell'}\delta_{mm'}\,.$$

now let

transfer function

$$\boxed{a_{\ell m} = 4\pi(-i)^\ell \int \frac{\mathrm{d}^3k}{(2\pi)^3}\, \Delta_{T\ell}(k)\, \mathcal{R}_{\mathbf{k}}\, Y_{\ell m}(\hat{\mathbf{k}})}$$

$$\sum_m \langle a^*_{\ell m} a_{\ell m}\rangle \propto \int d^3k \int d^3k'\, \Delta_{T_\ell}(k)\, \Delta_{T_\ell}(k')\, \underbrace{\langle R_{\vec{k}} R_{\vec{k}'}\rangle}_{P_R(\vec{k})\,\delta(\vec{k}+\vec{k}')}\, \underbrace{\sum_m Y_{\ell m}(\hat{k}) Y_{\ell m}(\hat{k}')}_{P_\ell(\underbrace{\hat{k}\cdot\hat{k}'}_{-1})}$$

$$\Longrightarrow \qquad C_\ell^{TT} = \frac{2}{\pi}\int k^2 \mathrm{d}k\, \underbrace{P_{\mathcal{R}}(k)}_{\text{Inflation}}\, \underbrace{\Delta_{T\ell}(k)\Delta_{T\ell}(k)}_{\text{Anisotropies}}\,.$$

For large scale or small ℓ region ; $\underset{\lambda}{\overset{k^{-1}}{\wr\wr}} > (aH)^{-1}$ (super)

i.e. NOT re enter into sub $\therefore$ no transfer after reentry

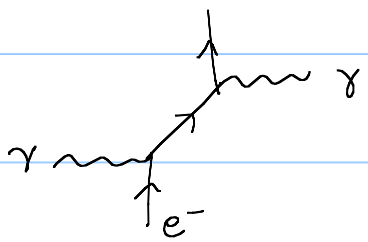
→ only primordial evolution → $\boxed{\Delta_{T\ell}(k) = \frac{1}{3} j_\ell(k[\tau_0 - \tau_{\rm rec}])}$

$$\therefore \quad C_\ell^{TT} = \frac{2}{9\pi}\int k^2 \mathrm{d}k\, P_{\mathcal{R}}(k)\, \underbrace{j_\ell^2(k[\tau_0-\tau_{\rm rec}])}_{\delta\text{-like peak at } \ell = k(\tau_0 - \tau_{\rm rec})} \longrightarrow C_\ell^{TT} \propto k^3 P_{\mathcal{R}}(k)\big|_{k\approx\ell/(\tau_0-\tau_{\rm rec})} \underbrace{\int \mathrm{d}\ln x\; j_\ell^2(x)}_{\propto \ell(\ell+1)}$$

$$\Longrightarrow \qquad \boxed{\underbrace{\ell(\ell+1)C_\ell^{TT}}_{\text{observed}} \propto \underbrace{\Delta_s^2(k)\big|_{k\approx\ell/(\tau_0-\tau_{\rm rec})}}_{\text{primordial}} \propto \ell^{n_s-1}}$$

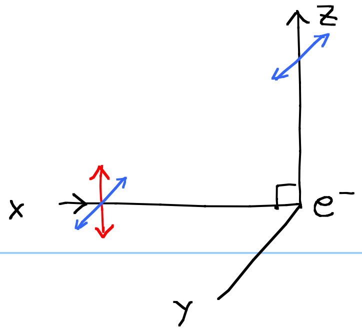
2. Polarization

Compton scattering

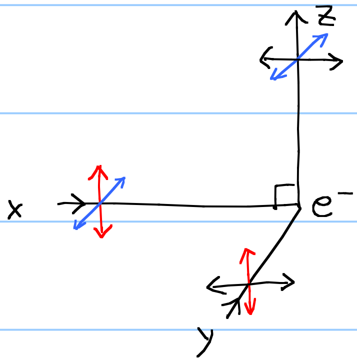


Thomson scattering multiphoton scattering

consider only 90° reflections

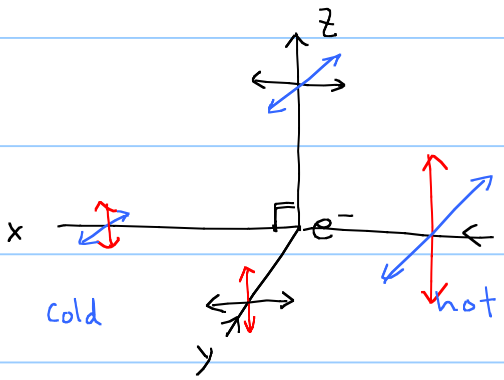


is not allowed



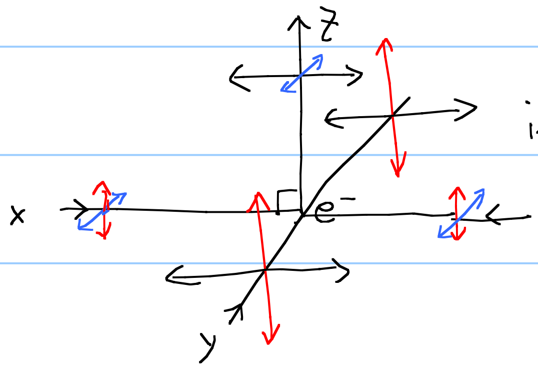
if intensities from x & y are same (uniform)

no polarization!



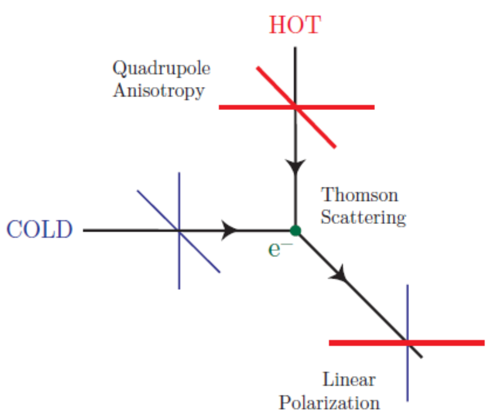
if dipole distribution in x-axis, still

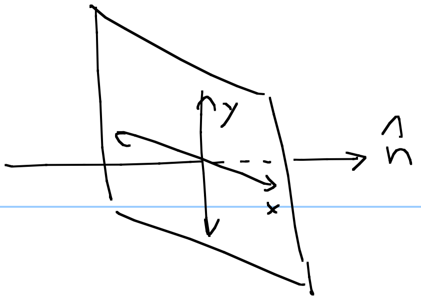
no polarization!

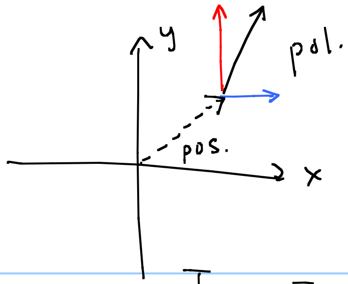


if quadrupole distribution in x-y

polarization!





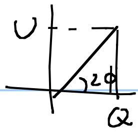


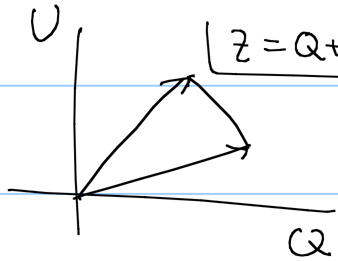
stokes parameters

$$I_{ij} = \begin{pmatrix} T+Q & U \\ U & T-Q \end{pmatrix} = \underbrace{T\,\mathbb{1}}_{\text{only total intensity}} + \begin{pmatrix} Q & U \\ U & -Q \end{pmatrix}$$

x axis

$$\begin{pmatrix} Q & U \\ U & -Q \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} Q \\ U \end{pmatrix}$$





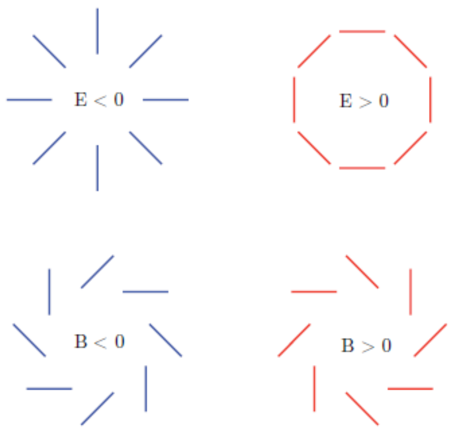
$z = Q + iU$

$\underline{\ell = 2}$

$$(Q \pm iU)(\hat{n}) = \sum_{\ell m} a^{\pm}_{\ell m} Y_{\ell m}(\hat{n})$$

$$Q(\hat{n}) = \sum_{\ell m} \underbrace{\frac{1}{2}(a^{+}_{\ell m} + a^{-}_{\ell m})}_{-a_{E,\ell m}} Y_{\ell m}(\hat{n}) \equiv -E(\hat{n})$$

$$U(\hat{n}) = \sum_{\ell m} \underbrace{\frac{1}{2i}(a^{+}_{\ell m} - a^{-}_{\ell m})}_{-a_{B,\ell m}} Y_{\ell m}(\hat{n}) \equiv -B(\hat{n})$$



Define

$$C_\ell^{XY} \equiv \frac{1}{2\ell+1}\sum_m \langle a^*_{X,\ell m} a_{Y,\ell m} \rangle\,, \qquad X, Y = T, E, B\,.$$

① $\mathcal{R} \longrightarrow$ only E

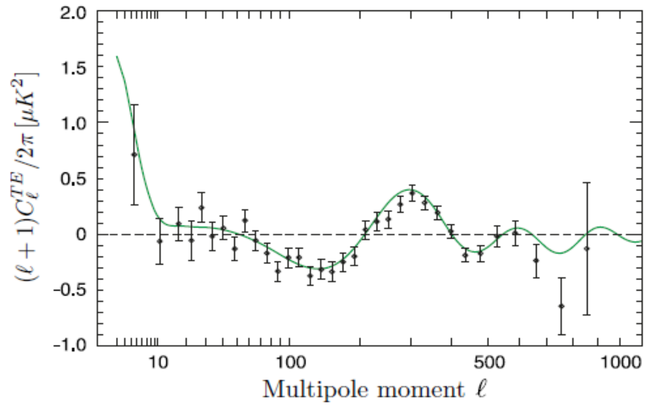
② $h \longrightarrow$ E & B

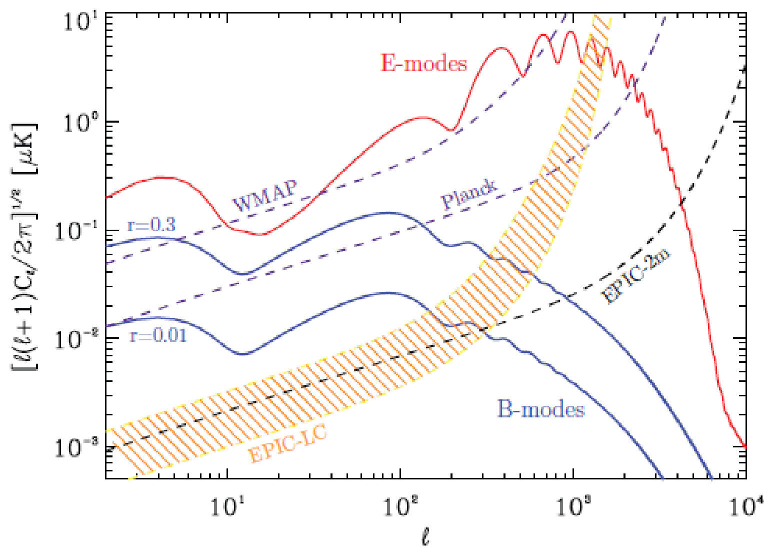
$\Rightarrow$

$$C_\ell^{EE} \approx (4\pi)^2 \int k^2 \mathrm{d}k \overbrace{P_{\mathcal{R}}(k)}^{\text{Inflation}} \Delta_{E\ell}^2(k)\,,$$

$$C_\ell^{TE} \approx (4\pi)^2 \int k^2 \mathrm{d}k \underbrace{P_{\mathcal{R}}(k)}_{\text{Inflation}} \Delta_{T\ell}(k)\Delta_{E\ell}(k)$$

$$C_\ell^{BB} = (4\pi)^2 \int k^2 \mathrm{d}k \underbrace{P_h(k)}_{\text{Inflation}} \Delta_{B\ell}^2(k)$$





② LSS

$$\boxed{P_\delta(k,\tau) = \frac{4}{25}\left(\frac{k}{aH}\right)^4 T_\delta^2(k,\tau) P_{\mathcal{R}}(k)}$$

After reentry : radiation dominates → pressure prevents $\delta_{\text{matter}} \approx 0$
matter dominates → $p=0$ → $\delta \sim a$

$$T_\delta(k) \approx \begin{cases} 1 & k < k_{\text{eq}} \\ (k_{\text{eq}}/k)^2 & k > k_{\text{eq}} \end{cases}$$

← still super → no evolution

$\left(aH = k_{eq}\big|_{t=t_{eq}}\right)$

fitting function

$$T_\delta(q) = \frac{\ln(1+2.34q)}{2.34q}\left[1 + 3.89q + (1.61q)^2 + (5.46q)^3 + (6.71q)^4\right]^{-1/4}$$

, $\delta \propto k$

Current Evidence for Inflation (slow roll)

1. Flatness $\quad \Omega_{total} \cong 1 \pm 0.02$

2. Coherence of CMB radiation

all Fourier mode has the same phase!

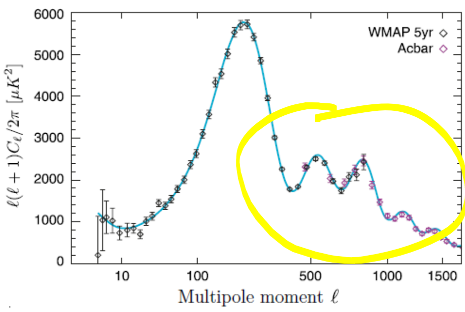
(otherwise, $\langle a_{\ell m}^{*} a_{\ell m} \rangle_{\text{phase average}} \equiv 0$)

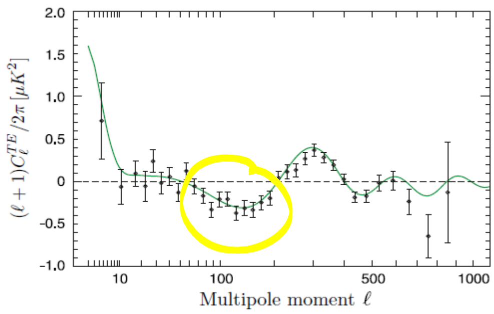
Why? ① maybe, some "physics" after reentry to recomb. lead to it.

this may explain large ℓ mode coherence

but not low ℓ ($\ell < 200$) which can not

reenter into horizon





T & E modes were out of phase

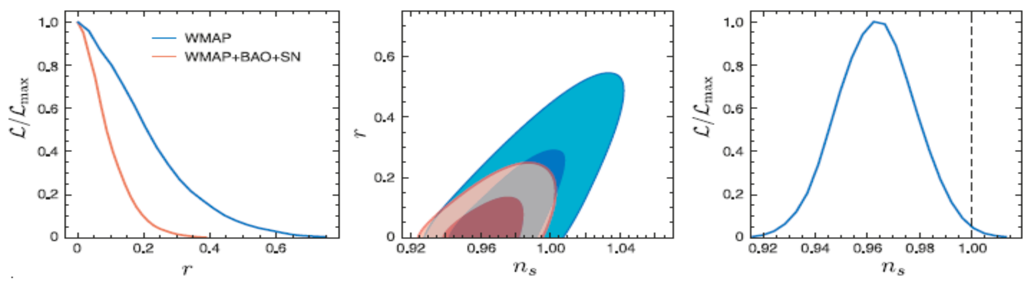
② Only subhorizon befor exit can explain

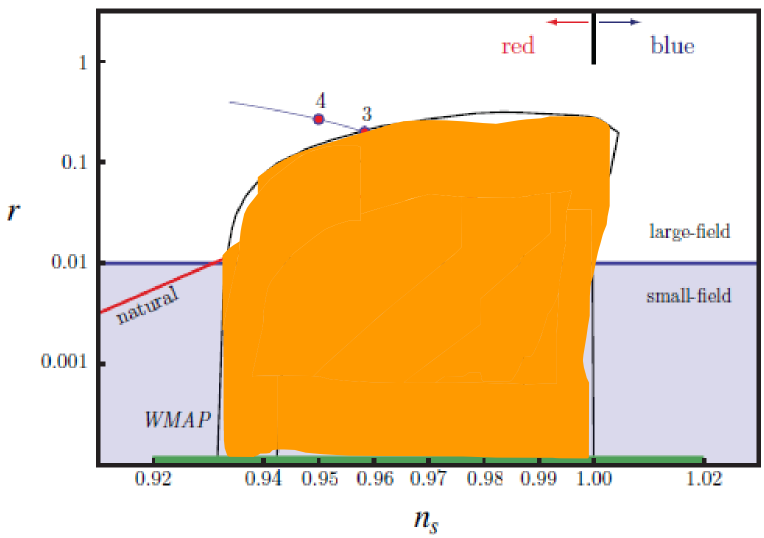
$\Rightarrow$ inflation physics

3. Scale invariance : $n_s \approx 1$

4. Gaussianity : $f_{NL} \approx 0 \quad (-4 < f_{NL} < 80)$

5. Adiabaticity : $\delta\left(\frac{n_m}{n_r}\right) = 0$





POSSIBLE FUTURE OBSERVATIONS & IMPLICATIONS

① CMP pole $\longrightarrow$ C_ℓ^{BB} ? " tensor perturb"

② scale dependence

- slow roll : $\alpha_s \equiv \dfrac{d n_s}{d \ln k} \sim \mathcal{O}(\varepsilon^2) \ll 1$

  if NOT: slow roll inflation is over

- Similarly, n_t & $r = -8 n_t$ relation

  if NOT: slow roll inflation is over

③ Non Gaussianity

$$R = R_g + \frac{3}{5} f_{NL} R_g^2(\vec{x})$$

↳ Fourier $\quad \tilde{R}(\vec{k}) = \tilde{R}_g + \frac{3}{5} f_{NL} \widetilde{R_g^2}$

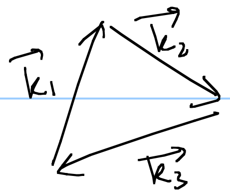
$$\widetilde{R_g^2}(\vec{k}) = \frac{1}{(2\pi)^3} \int d\vec{k}' \, \tilde{R}_g(\vec{k}') \, \tilde{R}_g(\vec{k}-\vec{k}')$$

$$\langle \tilde{R}(\vec{k}_1) \, \tilde{R}(\vec{k}_2) \, \tilde{R}(\vec{k}_3) \rangle$$

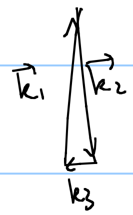
$$= \cancel{\langle \tilde{R}_g(\vec{k}_1) \, \tilde{R}_g(\vec{k}_2) \, \tilde{R}_g(\vec{k}_3) \rangle} + \frac{3}{5} f_{NL} \langle \tilde{R}_g(k_1) \tilde{R}_g(k_2) \int dk' \, \tilde{R}_g(k') \, \tilde{R}_g(k_3 - k') \rangle$$

$$+ (3 \to 1) + (3 \to 2)$$

$$= \frac{6}{5} f_{NL} \underbrace{\delta(k_1 + k_2 + k_3)} \left[P_R(k_1) P_R(k_2) + P_R(k_2) P_R(k_3) + P_R(k_1) P_R(k_3) \right]$$

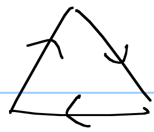


(1) local shape:



- multi-field
- curvaton : iso curvature field converted to curvature pert.
- Ekpyrotic

(2) equilateral shape:



- single field with DBI

④ Non-adiabaticity

multi-field : relative density fluctuation matters are possible

CMB limits this already very small