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Finite-size effect of η-deformed $AdS_5 \times S^5$ at strong coupling

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ABSTRACT

We compute Lüscher corrections for a giant magnon in the η-deformed $(AdS_5 \times S^5)_\eta$ using the $su(2|2)_q$-invariant S-matrix at strong coupling and compare with the finite-size effect of the corresponding string state, derived previously. We find that these two results match and confirm that the $su(2|2)_q$-invariant S-matrix is describing world-sheet excitations of the η-deformed background.

1. Introduction

AdS/CFT duality [1], a correspondence between string theories in AdS background with certain supersymmetric and conformal Yang–Mills theories on the boundary space-time of the AdS space, has been a hot topic for theoretical researches and produced many important quantitative results and applications (for overview see [2]). In these developments, integrability has played a crucial role on both sides of the correspondence. Two-dimensional world-sheet actions for the string theory moving in the background are described by nonlinear sigma models on coset group manifolds which are classically integrable. Aspects of quantum integrable structure of supersymmetric Yang–Mills theories appear in Bethe ansatz equations and related exact integrable machineries which can determine conformal dimensions of the CFTs. Quantum S-matrices of the world-sheet actions provide integrable framework which interpolates from the strong to weak coupling limits.

An important direction of research is to find new AdS/CFT pairs which show novel integrability structures. One such string theory, which has been studied recently, is the type IIB superstring theory in the η-deformed target space $(AdS_5 \times S^5)_\eta$ for a real parameter η [3]. The classical integrability of nonlinear sigma model is provided by solutions of the classical Yang–Baxter equation [4]. (See [5–7] for related issues.) It has been conjectured in [3] that full quantum S-matrix of the deformed sigma model is given by the R-matrix of the q-deformed Hubbard model which has been proposed much earlier in [8]. When q is a complex phase, the dressing phase of the S-matrix and bound-states have been analyzed in [9]. Scattering amplitudes of bosonic excitations for small values of the world-sheet momentum have been computed and shown to agree with the q-deformed S-matrix in the large string tension (strong coupling) limit for real q with explicit relation with η [10]. Based on the exact S-matrix, thermodynamic Bethe ansatz equations for ground states and dressing phase for real q have been studied in [11].

A pertinent issue which should be mentioned is that the deformed sigma model is not a fully consistent string theory at quantum level. It has been found that this η-deformed sigma model does not solve the type IIB supergravity equations of motion [12], but solves, instead, a generalization of them [13]. These generalized ones allow only scale invariance but not full Weyl invariance at one-loop [14]. The Weyl invariance can be restored if the deformation is generalized by some modified solutions of the Yang–Baxter equation [15]. This suggests that one should pay attention while treating the η-deformed theory at quantum level.

In this letter, we provide another evidence for the q-deformed S-matrix to describe the string theory on the η-deformed geometry. For this purpose, we consider finite-size effects of a giant magnon state, a classical string configuration living on a subspace of the $(AdS_5 \times S^5)_\eta$ [16]. These corrections have been computed for the undeformed $AdS_5 \times S^5$ in [17,18] and for the γ-deformed $AdS_5 \times S^5$ in [19,20] from both directions of string solutions and world-sheet S-matrices. For the η-deformed case, this effect has been studied from only string theory side in [21], which will be reviewed in sect. 2. Exact q-deformed S-matrix and related formula will be presented in sect. 3. We present our computation of the Lüscher corrections for a giant magnon based on q-deformed S-matrix in sect. 4 along with a conjecture on the deformed dressing phase in sect. 5. In sect. 6, we conclude with a short summary and comments.

2. Finite-size effect of a giant magnon in $(AdS_5 \times S^5)_\eta$

In this section, we give a brief review on computing the energy of a giant magnon using Neumann–Rosochatius ansatz fol-

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lowing [21]. The giant magnon is defined in the $R_t \times S^3_\eta$ subspace of $(AdS_5 \times S^5)_\eta$, where background metric and B-field are given by

$$g_{tt} = -1, \quad g_{\phi_1\phi_1} = \sin^2\theta, \quad g_{\phi_2\phi_2} = \frac{\cos^2\theta}{1+\tilde{\eta}^2\sin^2\theta},$$
$$g_{\theta\theta} = \frac{1}{1+\tilde{\eta}^2\sin^2\theta}, \quad b_{\phi_2\theta} = -\tilde{\eta}\frac{\sin 2\theta}{1+\tilde{\eta}^2\sin^2\theta}. \tag{2.1}$$

Deformation parameter $\tilde{\eta}$ is related to original parameter η by $\tilde{\eta} = 2\eta/(1-\eta^2)$.

One can solve the giant magnon configuration using an ansatz for the dynamics of the target space coordinates

$$t(\tau,\sigma) = \kappa\tau, \quad \phi_i(\tau,\sigma) = \omega_i\tau + F_i(\xi), \quad \theta(\tau,\sigma) = \theta(\xi),$$
$$\xi = \sigma - v\tau, \quad i = 1,2, \tag{2.2}$$

where τ and σ are the string world-sheet coordinates and the Virasoro constraints. If we restrict further to S^2 by setting the isometry angle ϕ_2 to zero, conserved charges E_s, J_1 corresponding to other isometric coordinates t, ϕ_1 are given by complete elliptic integrals of first and third kinds ($W = \kappa^2/\omega_1^2$):

$$E_s = \frac{2T}{\tilde{\eta}}\frac{(1-v^2)\sqrt{W}}{\sqrt{(\chi_\eta-\chi_m)(\chi_p-\chi_n)}}\mathbf{K}(1-\epsilon),$$
$$J_1 = \frac{2T}{\tilde{\eta}\sqrt{(\chi_\eta-\chi_m)(\chi_p-\chi_n)}}\left[\left(1-v^2W-\chi_\eta\right)\mathbf{K}(1-\epsilon)\right.$$
$$\left. + (\chi_\eta-\chi_p)\,\Pi\left(\frac{\chi_p-\chi_m}{\chi_\eta-\chi_m}, 1-\epsilon\right)\right], \tag{2.3}$$

where the parameters are satisfying

$$\chi_m = \frac{\chi_\eta\chi_p}{\chi_\eta-(1-\epsilon)\chi_p}\epsilon, \quad \epsilon = \frac{\chi_m(\chi_\eta-\chi_p)}{\chi_p(\chi_\eta-\chi_m)},$$
$$\frac{(1-\epsilon)\chi_p^2 - 2\epsilon\chi_p\chi_\eta - \chi_\eta^2}{\chi_\eta-(1-\epsilon)\chi_p} + 3 - (1+v^2)W + \frac{1}{\tilde{\eta}^2} = 0,$$
$$\chi_p\chi_\eta + \frac{\epsilon\chi_p\chi_\eta(\chi_p+\chi_\eta)}{\chi_\eta-(1-\epsilon)\chi_p}$$
$$-\frac{2-(1+v^2)W + \left(3-\left(2+v^2(2-W)\right)W\right)\tilde{\eta}^2}{\tilde{\eta}^2} = 0,$$
$$\frac{\epsilon\chi_p^2\chi_\eta^2}{\chi_\eta-(1-\epsilon)\chi_p} - \frac{(1+\tilde{\eta}^2)(1-W)(1-v^2W)}{\tilde{\eta}^2} = 0.$$

The momentum of a giant magnon, which is related to the deficit angle by $\Delta\phi_1 = p$, satisfies

$$p = \frac{2v}{\tilde{\eta}\sqrt{\chi_p(\chi_\eta-\chi_m)}}\left\{-v\,\mathbf{K}(1-\epsilon) + \frac{W}{(\chi_\eta-1)(1-\chi_p)}\right.$$
$$\times\left[(\chi_\eta-\chi_p)\,\Pi\left(-\frac{(\chi_\eta-1)(\chi_p-\chi_m)}{(\chi_\eta-\chi_m)(1-\chi_p)}, 1-\epsilon\right)\right.$$
$$\left.\left. - (1-\chi_p)\,\mathbf{K}(1-\epsilon)\right]\right\}. \tag{2.4}$$

Eqs. (2.3) and (2.4) generate the dispersion relation of a giant magnon at finite J_1.

In the limit of $J_1 \gg g \gg 1$ one can solve the parameter relations in terms of small ϵ-expansions to determine the energy and angular momentum

$$E_s - J_1 = \frac{2g\sqrt{1+\tilde{\eta}^2}}{\tilde{\eta}}\,\text{arcsinh}\left(\tilde{\eta}\sin\frac{p}{2}\right)$$
$$-\frac{8g(1+\tilde{\eta}^2)^{3/2}\sin^3\frac{p}{2}}{\sqrt{1+\tilde{\eta}^2\sin^2\frac{p}{2}}}$$
$$\times\exp\left(-\frac{J_1}{g}\sqrt{\frac{1+\tilde{\eta}^2\sin^2\frac{p}{2}}{(1+\tilde{\eta}^2)\sin^2\frac{p}{2}}}\right). \tag{2.5}$$

The first term is the energy dispersion relation of a giant magnon in the infinite volume and the second one is the small finite-size (or finite J_1) correction. In next sections, we are going to reproduce this result from the $su(2|2)_q$ S-matrix.

3. q-Deformed S-matrix

The quantum-deformed S-matrix can be written as a graded tensor product of $su(2|2)_q$-invariant matrix as follows:

$$\mathcal{S}(p_1,p_2) = S_{su(2)}\,\mathbf{S}\hat{\otimes}\dot{\mathbf{S}}. \tag{3.6}$$

The overall scalar factor $S_{su(2)}$ is given by [10][1]

$$S_{su(2)}(p_1,p_2) = \frac{1}{\sigma^2(p_1,p_2)}\frac{x_1^+ + \xi}{x_1^- + \xi}\frac{x_2^- + \xi}{x_2^+ + \xi}\frac{x_1^- - x_2^+}{x_1^+ - x_2^-}\frac{1-\frac{1}{x_1^- x_2^+}}{1-\frac{1}{x_1^+ x_2^-}}, \tag{3.7}$$

with q-deformed dressing phase σ. The $su(2|2)_q$-invariant S-matrix has 16×16 elements, $S_{ij}^{i'j'}$, $i,j,i',j' = 1,\ldots,4$. For Lüscher correction, the matrix elements we need are

$$S_{11}^{11} = a_1, \quad S_{12}^{12} = \frac{q}{2}a_1 + \frac{1}{2}a_2, \quad S_{13}^{13} = S_{14}^{14} = a_5 \tag{3.8}$$

$$a_1 = 1, \quad a_2 = -q + \frac{2}{q}\frac{x_1^-(1-x_2^-x_1^+)(x_1^+-x_2^+)}{x_1^+(1-x_2^-x_1^-)(x_1^- - x_2^+)},$$
$$a_5 = \frac{x_1^+ - x_2^+}{\sqrt{q}U_1V_1(x_1^- - x_2^+)}. \tag{3.9}$$

The parameters $x^\pm$ satisfy a shortening relation

$$\frac{1}{q}\left(x^+ + \frac{1}{x^+}\right) - q\left(x^- + \frac{1}{x^-}\right) = \left(q - \frac{1}{q}\right)\left(\xi + \frac{1}{\xi}\right), \tag{3.10}$$

and relate to energy $\mathcal{E}$ and momentum p by

$$V^2 = q\frac{x^+}{x^-}\frac{x^- + \xi}{x^+ + \xi} \equiv q^{\mathcal{E}}, \quad U^2 = \frac{1}{q}\frac{x^+ + \xi}{x^- + \xi} \equiv e^{ip}. \tag{3.11}$$

The constant ξ is related to the string tension g and deformation parameter q by

$$\xi = -\frac{i}{2}\frac{g(q-q^{-1})}{\sqrt{1-\frac{g^2}{4}(q-q^{-1})^2}}. \tag{3.12}$$

It is claimed that the quantum group parameter q is related to $\tilde{\eta}$ by

$$q = e^{-\nu/g} \quad \text{with} \quad \nu = \frac{\tilde{\eta}}{\sqrt{1+\tilde{\eta}^2}}. \tag{3.13}$$

General energy–momentum dispersion relation follows from this

[1] Several different candidates have been proposed in [9]. We have checked that only this one is consistent with the finite-size correction.

$$\mathcal{E}(p) = \frac{2g}{\nu} \operatorname{arcsinh}\left(\frac{\xi}{i}\sqrt{\frac{1}{4g^2\cosh^2\frac{\nu}{2g}} + \sin^2\frac{p}{2}}\right). \tag{3.14}$$

At strong coupling limit $g \gg 1$, Eqs. (3.12) and (3.13) lead to

$$\xi = i\tilde{\eta} + \mathcal{O}(g^{-1}). \tag{3.15}$$

From Eqs. (3.10) and (3.11), one can expand the parameters

$$x^{\pm}(p) = x_0^{\pm}(p) + \frac{1}{g}\, x_1^{\pm}(p) + \mathcal{O}(g^{-2}), \tag{3.16}$$

where

$$x_0^{\pm}(p) = e^{\pm ip/2}\left(\sqrt{1+\tilde{\eta}^2\sin^2\frac{p}{2}} \mp \tilde{\eta}\sin\frac{p}{2}\right), \tag{3.17}$$

$$x_1^{\pm}(p) = \frac{\left(x_0^{\pm}(p)+i\tilde{\eta}\right)\left(\tilde{\eta}\, x_0^{\pm}(p)-i\right)}{\sqrt{1+\tilde{\eta}^2}\left(x_0^{-}(p)-x_0^{+}(p)\right)}. \tag{3.18}$$

Also the dispersion relation in Eq. (3.14) becomes

$$\mathcal{E}_0(p) = \frac{2g\sqrt{1+\tilde{\eta}^2}}{\tilde{\eta}} \operatorname{arcsinh}\left(\tilde{\eta}\sin\frac{p}{2}\right), \tag{3.19}$$

which is consistent with that of giant magnon string state given in the first term of Eq. (2.5).

4. Lüscher corrections

Leading finite-size corrections in the strong coupling limit are the μ-term Lüscher corrections which arise from residues of S-matrix in the contour integrals of the F-term formula. Explicit μ-term Lüscher formula for one $su(2)$ giant magnon state with $su(2|2)$ index $(1\dot{1})$ is given by [22,18],

$$\delta E_{\mu} = -i\left(1-\frac{\mathcal{E}'(p)}{\mathcal{E}'(\tilde{q}_{\star})}\right) e^{-i\tilde{q}_{\star}J_1} \sum_{j,\dot{j},j',\dot{j}'} \operatorname*{Res}_{q=\tilde{q}}\left[\mathcal{S}_{(1\dot{1})(j\dot{j})}^{(1\dot{1})(j'\dot{j}')}(p,q_{\star}(q))\right], \tag{4.20}$$

where $\tilde{q}$ is the location of the S-matrix poles. The physical giant magnon has momentum p and energy given by (3.19), while the momentum $q_{\star}$ of the virtual particle satisfies the following on-shell relation

$$q^2 + \mathcal{E}^2(q_{\star}) = 0. \tag{4.21}$$

We also use a short notation $\tilde{q}_{\star} = q_{\star}(\tilde{q})$.

We start with locating the poles of the S-matrix. The overall scalar factor $S_{su(2)}(p,q_{\star})$ in (3.7) has both s-channel pole at $x^-(\tilde{q}_{\star}) = x^+(p)$ and t-channel pole at $x^-(\tilde{q}_{\star}) = 1/x^+(p)$. We have checked that the t-channel gives exactly same results as the s-channel. We will present a detailed computation for the s-channel here and multiply by a factor 2 at the end.

Substituting $x^+(p)$ for $x^-(\tilde{q}_{\star})$ in Eq. (3.10), we can compute $x^+(\tilde{q}_{\star})$

$$x^+(\tilde{q}_{\star}) = x_0^+(p) + \frac{3}{g}\,\frac{\left(x_0^{\pm}(p)+i\tilde{\eta}\right)\left(\tilde{\eta}\, x_0^{\pm}(p)-i\right)}{\sqrt{1+\tilde{\eta}^2}\left(x_0^{-}(p)-x_0^{+}(p)\right)} + \mathcal{O}(g^{-2}). \tag{4.22}$$

From Eq. (3.11), we can obtain

$$\begin{aligned} e^{i\tilde{q}_{\star}} &= \frac{1}{q}\,\frac{x^+(\tilde{q}_{\star})+\xi}{x^-(\tilde{q}_{\star})+\xi} \\ &= 1 + \frac{1}{g}\,\frac{\tilde{\eta}\left(x_0^+(p)+x_0^-(p)\right)-2i}{\sqrt{1+\tilde{\eta}^2}\left(x_0^-(p)-x_0^+(p)\right)} + \mathcal{O}(g^{-2}). \end{aligned} \tag{4.23}$$

Using Eq. (3.17), we obtain $\tilde{q}_{\star}$ as follows:

$$i\tilde{q}_{\star} = \frac{1}{g}\,\frac{\sqrt{1+\tilde{\eta}^2\sin^2\frac{p}{2}}}{\sqrt{1+\tilde{\eta}^2}\,\sin\frac{p}{2}} + \mathcal{O}(g^{-2}). \tag{4.24}$$

This leads to the exponentially suppressing factor in the Lüscher formula

$$e^{-i\tilde{q}_{\star}J_1} = \exp\left(-\frac{J_1}{g}\,\frac{\sqrt{1+\tilde{\eta}^2\sin^2\frac{p}{2}}}{\sqrt{1+\tilde{\eta}^2}\,\sin\frac{p}{2}}\right), \tag{4.25}$$

which matches with the string computations shown in (2.5).

The next factor to consider in the Lüscher formula (4.20) is the energy dispersion. Since $\tilde{q}_{\star} \sim \mathcal{O}(g^{-1})$, one should use exact relation (3.14) instead of (3.19) before taking the large g limit along with (4.24). A straightforward computation yields

$$\left(1-\frac{\mathcal{E}'(p)}{\mathcal{E}'(\tilde{q}_{\star})}\right) = \frac{\left(1+\tilde{\eta}^2\right)\sin^2\frac{p}{2}}{1+\tilde{\eta}^2\sin^2\frac{p}{2}}. \tag{4.26}$$

Now we move on to the residue of the S-matrix, which comes from the scalar factor (3.7)

$$\begin{aligned} &\operatorname*{Res}_{q=\tilde{q}}\left[S_{su(2)}(p,q_{\star})\right] \\ &= \frac{2e^{3ip/2}\left[1+ie^{ip/2}\tilde{\eta}\left(\sqrt{1+\tilde{\eta}^2\sin^2\frac{p}{2}}-\tilde{\eta}\sin\frac{p}{2}\right)\right]}{g\sqrt{1+\tilde{\eta}^2}\sin\frac{p}{2}\cdot\sigma^2(p,\tilde{q}_{\star})\cdot x^{-\prime}(\tilde{q}_{\star})}. \end{aligned} \tag{4.27}$$

The last factor can be computed by a trick used in [18]

$$\begin{aligned} &\left.\frac{dx^-(q_{\star})}{dq}\right|_{q=\tilde{q}} \\ &= \frac{dx^+(p)/dp}{dq/dp} \\ &= \frac{-ie^{ip/2}\left[1+ie^{ip/2}\tilde{\eta}\left(\sqrt{1+\tilde{\eta}^2\sin^2\frac{p}{2}}-\tilde{\eta}\sin\frac{p}{2}\right)\right]\sin^2\frac{p}{2}}{\sqrt{1+\tilde{\eta}^2\sin^2\frac{p}{2}}}, \end{aligned} \tag{4.28}$$

where we used (4.21) for dq/dp. Combining these, we get

$$\operatorname*{Res}_{q=\tilde{q}}\left[S_{su(2)}(p,q_{\star})\right] = \frac{2ie^{ip}\sqrt{1+\tilde{\eta}^2\sin^2\frac{p}{2}}}{g\sqrt{1+\tilde{\eta}^2}\sin^3\frac{p}{2}\cdot\sigma^2(p,\tilde{q}_{\star})}. \tag{4.29}$$

The contribution from each matrix element in Eq. (3.8) leads to

$$\left(\frac{1+q}{2}\,a_1 + \frac{1}{2}a_2 + 2a_5\right)^2 \tag{4.30}$$

and becomes 1 in the leading order from (3.9).

5. *q*-Deformed dressing phase

The dressing phase has been proposed first in terms of q-deformed Gamma function for q a complex phase, [9]

$$\begin{aligned} \sigma^2(p_1,p_2) &= \exp i\left[\chi(x_1^+,x_2^+) - \chi(x_1^+,x_2^-) - \chi(x_1^-,x_2^+) \right.\\ &\qquad \left. + \chi(x_1^-,x_2^-)\right], \\ \chi(x_1,x_2) &= i\oint_{|z|=1}\frac{dz}{2\pi i}\,\frac{1}{z-x_1}\oint_{|w|=1}\frac{dw}{2\pi i}\,\frac{1}{w-x_2} \end{aligned} \tag{5.31}$$

$$\times \log \frac{\Gamma_{q^2}\left[1+\frac{i}{2a}(u(z)-u(w))\right]}{\Gamma_{q^2}\left[1-\frac{i}{2a}(u(z)-u(w))\right]}, \tag{5.32}$$

where $a=\nu/g$ for $g \gg 1$ and $u(z)$ is defined by

$$X(z,w) \equiv u(z)-u(w) = \frac{i}{2\nu}\log\left(\frac{z+\frac{1}{z}+\xi+\frac{1}{\xi}}{w+\frac{1}{w}+\xi+\frac{1}{\xi}}\right). \tag{5.33}$$

An integral representation for Γ_{q^2} given in [9] can be analytically continued for real q to get strong coupling limit [10]

$$\log\frac{\Gamma_{q^2}[1+gX]}{\Gamma_{q^2}[1-gX]} \approx g\Bigg\{2X(\log g-1)+X\log(-X^2) - i\frac{2\pi}{\nu}\left[\psi^{-2}\left(1-\frac{i\nu X}{\pi}\right)-\psi^{-2}\left(1+\frac{i\nu X}{\pi}\right)\right]\Bigg\}, \tag{5.34}$$

where ψ^{-2} is the poly-gamma function. The integrals over two unit circles in (5.32) may develop a branch cut for $\nu \geq 1/2$ but can be handled with proper care as pointed out in [11].

For computing $\sigma(p,\tilde{q}_\star)$ at strong coupling, we combine the χ functions with arguments $x^{\pm}(p)$, $x^{\pm}(\tilde{q}_\star)$ given in Eqs. (3.16) and (4.22) to get

$$\log\sigma^2(\tilde{q}_\star,p) = \oint_{|z|=1}\frac{dz}{2\pi i}\oint_{|w|=1}\frac{dw}{2\pi i}\left[\frac{2i\nu x_0^+\left(x_0^++x_0^-+\xi+\frac{1}{\xi}\right)}{g(z-x_0^+)(z-x_0^-)(w-x_0^+)^2}\right] \times g\Bigg\{X\log\frac{g^2}{e^2}+X\log(-X^2) - \frac{2\pi i}{\nu}\left[\psi^{-2}\left(1-\frac{i\nu X}{\pi}\right)-\psi^{-2}\left(1+\frac{i\nu X}{\pi}\right)\right]\Bigg\}, \tag{5.35}$$

with a short notation $x_0^{\pm}=x_0^{\pm}(p)$ given in (3.17). Due to complicated branch cuts appearing in the contour integrals, we could not evaluate this integral analytically. However, we have found numerically that the integration depends on $\tilde{\eta}$ very insensitively within available accuracy. This leads to our conjecture that the dressing phase with given arguments in the strong coupling limit is

$$\sigma^2(\tilde{q}_\star,p) = -2g^2\, e^{-ip}\,\sin^4\frac{p}{2}, \tag{5.36}$$

which is the result for the undeformed case, computed from the AFS phase [23] in [18].

Combining (4.25), (4.26), (4.29) and (5.36) along with a factor $-i$ in (4.20) and 2 for the t-channel contribution, we get the final μ-term Lüscher correction

$$\delta E_\mu = -\frac{8g(1+\tilde{\eta}^2)^{1/2}\sin^3\frac{p}{2}}{\sqrt{1+\tilde{\eta}^2\sin^2\frac{p}{2}}}\exp\left(-\frac{J_1}{g}\sqrt{\frac{1+\tilde{\eta}^2\sin^2\frac{p}{2}}{(1+\tilde{\eta}^2)\sin^2\frac{p}{2}}}\right). \tag{5.37}$$

6. Conclusion

Compared with finite-size giant magnon computation (2.5), the strong coupling Lüscher correction matches quite well except $1+\tilde{\eta}^2$ in the overall factor. We think this factor should be modified in the string theory computation. Apart from this minor discrepancy, both coefficient and exponent of the exponential factor show correct dependence on the momentum and deformation parameter. Our check is valid for the $su(2)$ sector with generic value of p and supports that the q-deformed S-matrix should describe the string theory in the η-deformed AdS background. It will be interesting to further elaborate the q-deformed dressing phase to check (5.36) both numerically and analytically. Another interesting but less studied domain is the weak coupling limit of the S-matrix, which could be related to certain q-deformed spin-chain.

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References

[1] J. Maldacena, The large N limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. 2 (1998) 231, arXiv:hep-th/9711200;
S. Gubser, I. Klebanov, A. Polyakov, Gauge theory correlators from non-critical string theory, Phys. Lett. B 428 (1998) 105, arXiv:hep-th/9802109;
E. Witten, Anti-de Sitter space and holography, Adv. Theor. Math. Phys. 2 (1998) 253, arXiv:hep-th/9802150.

[2] N. Beisert, C. Ahn, L. Alday, Z. Bajnok, J. Drummond, L. Freyhult, N. Gromov, R. Janik, V. Kazakov, T. Klose, G. Korchemsky, C. Kristjansen, M. Magro, T. McLoughlin, J. Minahan, R. Nepomechie, A. Rej, R. Roiban, S. Schafer-Nameki, C. Sieg, M. Staudacher, A. Torrielli, A. Tseytlin, P. Vieira, D. Volin, K. Zoubos, Review of AdS/CFT integrability: an overview, Lett. Math. Phys. 99 (2012) 3, arXiv:1012.3982v5 [hep-th].

[3] F. Delduc, M. Magro, B. Vicedo, An integrable deformation of the $AdS_5 \times S^5$ superstring action, Phys. Rev. Lett. 112 (2014) 051601, arXiv:1309.5850 [hep-th];
On classical q-deformations of integrable sigma-models, J. High Energy Phys. 1311 (2013) 192, arXiv:1308.3581 [hep-th].

[4] C. Klimcik, Yang–Baxter σ-models and dS/AdS T duality, J. High Energy Phys. 0212 (2002) 051, arXiv:hep-th/0210095;
On integrability of the Yang–Baxter σ-model, J. Math. Phys. 50 (2009) 043508, arXiv:0802.3518 [hep-th].

[5] I. Kawaguchi, T. Matsumoto, K. Yoshida, The classical origin of quantum affine algebra in squashed sigma models, J. High Energy Phys. 1204 (2012) 115, arXiv:1201.3058 [hep-th];
On the classical equivalence of monodromy matrices in squashed sigma model, J. High Energy Phys. 1206 (2012) 082, arXiv:1203.3400 [hep-th].

[6] K. Sfetsos, Integrable interpolations: from exact CFTs to non-Abelian T-duals, Nucl. Phys. B 880 (2014) 225, arXiv:1312.4560 [hep-th].

[7] B. Hoare, R. Roiban, A.A. Tseytlin, On deformations of $AdS_n \times S^n$ supercosets, J. High Energy Phys. 1406 (2014) 002, arXiv:1403.5517 [hep-th].

[8] N. Beisert, P. Koroteev, Quantum deformations of the one-dimensional Hubbard model, J. Phys. A 41 (2008) 255204.

[9] B. Hoare, T.J. Hollowood, J.L. Miramontes, q-Deformation of the $AdS_5 \times S^5$ superstring S-matrix and its relativistic limit, J. High Energy Phys. 1203 (2012) 015, arXiv:1112.4485 [hep-th].

[10] G. Arutyunov, R. Borsato, S. Frolov, S-matrix for strings on η-deformed $AdS_5 \times S^5$, J. High Energy Phys. 1404 (2014) 002, arXiv:1312.3542 [hep-th].

[11] G. Arutyunov, M. de Leeuw, S. van Tongeren, On the exact spectrum and mirror duality of the $(AdS_5 \times S^5)_\eta$ superstring, Theor. Math. Phys. 182 (2015) 23, arXiv:1403.6104 [hep-th].

[12] G. Arutyunov, R. Borsato, S. Frolov, Puzzles of η-deformed $AdS_5 \times S^5$, J. High Energy Phys. 1512 (2015) 049, arXiv:1507.04239 [hep-th].

[13] G. Arutyunov, S. Frolov, B. Hoare, R. Roiban, A.A. Tseytlin, Scale invariance of the η-deformed $AdS_5 \times S^5$ superstring, T-duality and modified type II equations, Nucl. Phys. B 903 (2016) 262, arXiv:1511.05795 [hep-th].

[14] L. Wulff, A.A. Tseytlin, Kappa-symmetry of superstring sigma model and generalized 10d supergravity equations, J. High Energy Phys. 1606 (2016) 174, arXiv:1605.04884 [hep-th].

[15] R. Borsato, L. Wulff, Target space supergeometry of η and λ-deformed strings, J. High Energy Phys. 1610 (2016) 045, arXiv:1608.03570 [hep-th].

[16] D. Hofman, J. Maldacena, Giant magnons, J. Phys. A 39 (2006) 13095, arXiv:hep-th/0604135.

[17] G. Arutyunov, S. Frolov, M. Zamaklar, Finite-size effects from giant magnons, Nucl. Phys. B 778 (2007) 1, arXiv:hep-th/0606126.

[18] R.A. Janik, T. Łukowski, Wrapping interactions at strong coupling: the giant magnon, Phys. Rev. D 76 (2007) 126008, arXiv:0708.2208 [hep-th].

[19] D.V. Bykov, S. Frolov, J. High Energy Phys. 0807 (2008) 071, arXiv:0805.1070.

[20] C. Ahn, D. Bombardelli, M. Kim, Finite-size effects of β-deformed AdS_5/CFT_4 at strong coupling, Phys. Lett. B 710 (2012) 467.
[21] C. Ahn, P. Bozhilov, Finite-size giant magnons on η-deformed $AdS_5 \times S^5$, Phys. Lett. B 737 (2014) 293, arXiv:1406.0628 [hep-th].
[22] M. Lüscher, Volume dependence of the energy spectrum in massive quantum field theories. 1. Stable particle states, Commun. Math. Phys. 104 (1986) 177.
[23] G. Arutyunov, S. Frolov, M. Staudacher, J. High Energy Phys. 0410 (2004) 016, arXiv:hep-th/0406256.