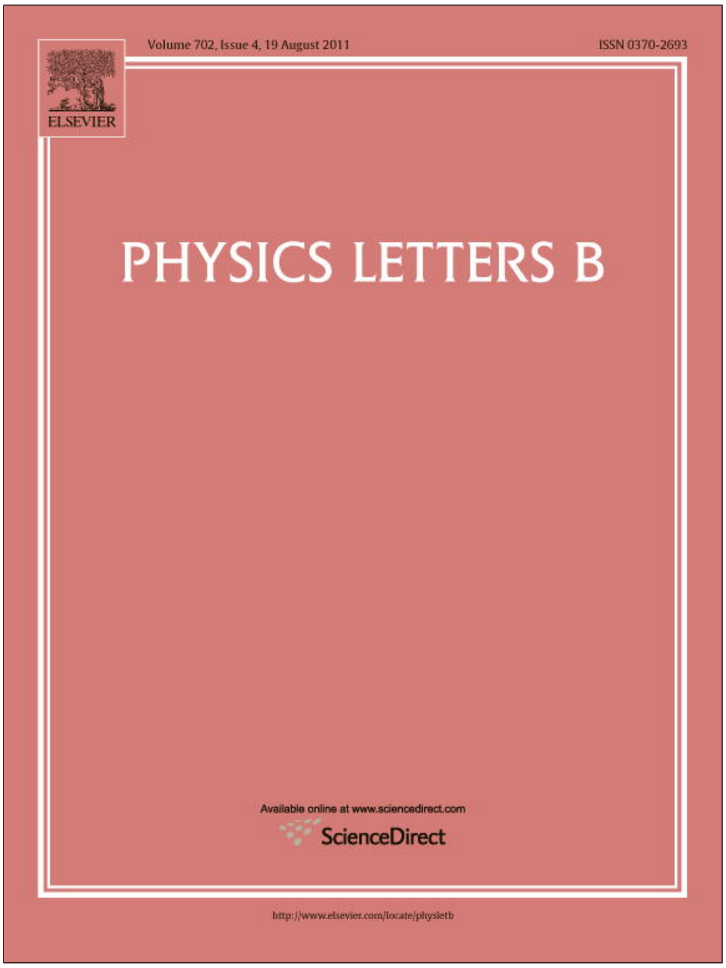
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Three-point correlation functions of giant magnons with finite size

Changrim Ahn *, Plamen Bozhilov [1]

Institute for the Early Universe and Department of Physics, Ewha Womans University, DaeHyun 11-1, Seoul 120-750, South Korea

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ABSTRACT

We compute holographic three-point correlation functions or structure constants of a zero-momentum dilaton operator and two (dyonic) giant magnon string states with a finite-size length in the semiclassical approximation. We show that the semiclassical structure constants match exactly with the three-point functions between two $su(2)$ magnon single trace operators with finite size and the Lagrangian in the large 't Hooft coupling constant limit. A special limit $J \gg \sqrt{\lambda}$ of our result is compared with the relevant result based on the Lüscher corrections.

1. Introduction

Correlation functions of conformal field theories (CFTs) can be in principle determined in terms of basic conformal data $\{\Delta_i, C_{ijk}\}$, where Δ_i's are conformal dimensions defined by two-point correlation functions

$$\left\langle \mathcal{O}_i^\dagger(x_1)\mathcal{O}_j(x_2)\right\rangle = \frac{\delta_{ij}}{|x_1 - x_2|^{2\Delta_i}} \tag{1.1}$$

and C_{ijk}'s are structure constants by three-point correlation functions

$$\left\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\mathcal{O}_k(x_3)\right\rangle = \frac{C_{ijk}}{|x_1 - x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_1 - x_3|^{\Delta_1+\Delta_3-\Delta_2}|x_2 - x_3|^{\Delta_2+\Delta_3-\Delta_1}}. \tag{1.2}$$

Hence a complete determination of conformal data for a given CFT is a most important step in the conformal bootstrap procedure. While this is well-established in two dimensions where the conformal symmetry is infinite-dimensional [1], it is extremely difficult to extend the procedure to higher space–time dimensions.

In four dimensions, the AdS/CFT correspondence between the $\mathcal{N} = 4$ super-Yang–Mills theory (SYM) and type IIB string theory moving on $AdS_5 \times S^5$ target space has provided a most promising framework [2]. A lot of impressive progresses have been made in this field based on the integrability discovered in the planar limit of the SYM. In particular, the thermodynamic Bethe ansatz approach based on non-perturbative world-sheet S-matrix has been formulated to provide the conformal dimensions of SYM operators with arbitrary number of elementary fields for generic value of 't Hooft coupling constant λ (see for a recent review [3]). In strong coupling limit $\lambda \gg 1$ the AdS/CFT correspondence relates the conformal dimensions to energy of certain classical string configurations which can be computed by either solving the superstring sigma model directly such as the algebraic curve method [4] or Neumann–Rosochatius reduction [5].

There have been many interesting progresses on three-point correlation functions in the AdS/CFT context. Three-point functions for chiral primary operators have been computed first in the AdS_5 supergravity approximation where explicit dependence on the coupling constant λ is not apparent [6]. It is only recently that several interesting developments have been made to consider general heavy string states. An efficient method to compute two-point correlation functions in the strong coupling limit is to evaluate string partition function for a heavy string state propagating in the AdS space between two boundary points based on a path integral method [7–9]. This method has been extended to the three-point functions of two heavy string states and a supergravity mode which corresponds to a marginal deformation of the SYM two-point functions by the Lagrangian [10–12]. With these formulation, many interesting checks of three-point functions of two heavy mode and one light mode have been performed [13–23]. Another direction is to compute the structure constants using the Bethe eigenstates of the underlying integrable spin chain in the weak coupling limit of the SYM [24,25].

In this Letter we apply the semiclassical formulation of the three-point correlation functions of a "zero-momentum" dilaton operator which is the Lagrangian as a light operator along with two heavy (dyonic) giant magnon string states. Differently from previous cases with an infinite length of the SYM operator $J \to \infty$ [11,23], we consider finite-size systems $J \sim \sqrt{\lambda} \gg 1$. We show that the semiclassical formulation of the three-point functions is still

* Corresponding author.
E-mail addresses: ahn@ewha.ac.kr (C. Ahn), bozhilov@inrne.bas.bg (P. Bozhilov).

[1] On leave from Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Bulgaria.

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valid for this more general situation. A special limit of our result is $J \gg \sqrt{\lambda}$ where the finite-size corrections to both conformal dimensions and energies of string states have been computed from the Lüscher corrections.

The three-point functions of two heavy operators and a light operator can be approximated by a supergravity vertex operator evaluated at the heavy classical string configuration:

$$\langle V_{H_1}(x_1)V_{H_2}(x_2)V_L(x_3)\rangle = V_L(x_3)_{\text{classical}}.$$

For $|x_1| = |x_2| = 1$, $x_3 = 0$, the correlation function reduces to

$$\langle V_{H_1}(x_1)V_{H_2}(x_2)V_L(0)\rangle = \frac{C_{123}}{|x_1 - x_2|^{2\Delta_{H_1}}}.$$

Then, the normalized structure constants

$$\mathcal{C}_3 = \frac{C_{123}}{C_{12}}$$

can be found from [23]

$$\mathcal{C}_3 = c_\Delta V_L(0)_{\text{classical}}. \tag{1.3}$$

Here, c_Δ is the normalized constant of the corresponding light vertex operator.

We restrict our consideration to the zero-momentum dilaton operator, namely the Lagrangian whose vertex operator is given by

$$V^d = (Y_4 + Y_5)^{-4}\left[z^{-2}\left(\partial_+ x_m \partial_- x^m + \partial_+ z \partial_- z\right) + \partial_+ X_k \partial_- X_k\right], \tag{1.4}$$

where

$$Y_4 = \frac{1}{2z}\left(x^m x_m + z^2 - 1\right), \qquad Y_5 = \frac{1}{2z}\left(x^m x_m + z^2 + 1\right),$$

and x_m, z are coordinates on AdS_5, while X_k are the coordinates on S^5.

2. Giant magnons with finite size

The finite-size giant magnon solution [26–28], in the notations of [29] can be represented as ($i\tau = \tau_e$)

$$x_{0e} = \tanh(\kappa\tau_e), \qquad x_i = 0, \qquad z = \frac{1}{\cosh(\kappa\tau_e)},$$

$$\cos\theta = \sqrt{1 - v^2\kappa^2}\, dn\left(\frac{\sqrt{1 - v^2\kappa^2}}{1 - v^2}(\sigma - v\tau)\Big|1 - \epsilon\right),$$

$$\phi = \frac{\tau - v\sigma}{1 - v^2} + \frac{1}{v\sqrt{1 - v^2\kappa^2}} \times \Pi\left(-\frac{1 - v^2\kappa^2}{v^2\kappa^2}(1 - \epsilon),\; am\left(\frac{\sqrt{1 - v^2\kappa^2}}{1 - v^2}(\sigma - v\tau)\right)\Big|1 - \epsilon\right), \tag{2.1}$$

where $dn(x|y)$ is one of the Jacobi elliptic functions, $am(x)$ is the Jacobi amplitude, and $\Pi(x, y|z)$ is the incomplete elliptic integral of third kind, and

$$\epsilon = \frac{1 - \kappa^2}{1 - v^2\kappa^2}.$$

To find the finite-size effect on the three-point correlator, we will use (1.3) and (1.4), which computed on (2.1) gives

$$\mathcal{C}_3^d = c_\Delta^d \int_{-\infty}^{\infty} \frac{d\tau_e}{\cosh^4(\kappa\tau_e)} \int_{-L}^{L} d\sigma\, \left[\kappa^2 + \partial_+ X_k \partial_- X_k\right], \tag{2.2}$$

where

$$\partial_+ X_k \partial_- X_k = -\frac{1}{(1 - v^2)\sin^2\theta}\left\{2 - \left(1 + v^2\right)\kappa^2 - \cos^2\theta\left[4 - \left(1 + v^2\right)\kappa^2 - 2\cos^2\theta\right]\right\}.$$

The integration variable σ can be changed to $\xi = \sigma - v\tau$ and to θ,

$$\int_{-L}^{L} d\sigma \cdots = \int_{-L-v\tau}^{L-v\tau} d\xi \cdots = 2\int_{\theta_{\min}}^{\theta_{\max}} \frac{d\theta}{\theta'(\xi)} \cdots \tag{2.3}$$

using the periodic property of $\theta(\xi)$ (periodicity is $2\mathbf{K}(1 - \epsilon)$) and the integral becomes independent of τ. We would like to emphasize that, as explained in [26], unlike the infinite size giant magnon [30], the finite-size giant magnon is *non-rigid*. When $L \to \infty$, the string becomes rigid and the end points touch the equator.

Performing the integrations in (2.2), one finds

$$\mathcal{C}_3^d = \frac{16}{3} c_\Delta^d \sqrt{\frac{1 - v^2}{1 - \epsilon}}\left[\mathbf{E}(1 - \epsilon) - \epsilon\mathbf{K}(1 - \epsilon)\right], \tag{2.4}$$

where $\mathbf{K}(1 - \epsilon)$ and $\mathbf{E}(1 - \epsilon)$ are the complete elliptic integrals of first and second kind. Let us point out that the parameter L in (2.4) is given by

$$L = \frac{1 - v^2}{\sqrt{1 - v^2\kappa^2}}\mathbf{K}(1 - \epsilon).$$

This is our *exact* result for the normalized coefficient $\mathcal{C}_3^d$ in the three-point correlation function, corresponding to the case when the heavy vertex operators are *finite-size* giant magnons, and the light vertex is taken to be the zero-momentum dilaton operator.

For the case of this dilaton operator, the three-point function of the SYM can be easily related to the conformal dimension of the heavy operators. This corresponds to shift 't Hooft coupling constant which is the overall coefficient of the Lagrangian [11]. This gives an important relation between the structure constant and the conformal dimension as follows:

$$\mathcal{C}_3^d = \frac{32\pi}{3} c_\Delta^d \sqrt{\lambda}\, \partial_\lambda \Delta. \tag{2.5}$$

We want to show that this relation is correct for the case of the giant magnons with arbitrary finite size.

In the context of the AdS/CFT correspondence, it is now well-established that the conformal dimension of a single trace operator with one magnon state is the same as $E - J$ in the strong coupling limit. For an exact relation from the gauge theory side, one should solve the thermodynamic Bethe equations [31]. Although this is very complicated and analytic solutions are still not available, it has been shown that finite-size corrections to the conformal dimensions of the SYM (dyonic) giant magnon operators computed by the Lüscher formula for $J \gg \sqrt{\lambda}$ match exactly with $E - J$ of corresponding string state configurations [32,33]. Based on these results, we can assume that the conformal dimensions Δ of the SYM operators are the same as $E - J$ of corresponding string states.

The *exact* classical expression for finite-size giant magnon energy-charge relation is given by [29]

$$E - J \equiv \Delta = \frac{\sqrt{\lambda}}{\pi}\sqrt{\frac{1 - v^2}{1 - v^2\epsilon}}\Big[\mathbf{E}(1 - \epsilon) - \left(1 - \sqrt{\left(1 - v^2\epsilon\right)(1 - \epsilon)}\right)\mathbf{K}(1 - \epsilon)\Big]. \tag{2.6}$$

The corresponding expressions for J and p are

$$J=\frac{\sqrt{\lambda}}{\pi}\sqrt{\frac{1-v^2}{1-v^2\epsilon}}\big[\mathbf{K}(1-\epsilon)-\mathbf{E}(1-\epsilon)\big],$$

$$p=2v\sqrt{\frac{1-v^2\epsilon}{1-v^2}}\left[\frac{1}{v^2}\Pi\left(1-\frac{1}{v^2}\Big|1-\epsilon\right)-\mathbf{K}(1-\epsilon)\right],$$

where J is the angular momentum of the string, and p is the magnon momentum. One can obtain $E-J$ in terms of J and p by eliminating v, ϵ from these expressions.

To take λ-derivative on Δ, we need to know λ dependence of v and ϵ. Our strategy is to find $v'(\lambda)$ and $\epsilon'(\lambda)$ from the conditions that J and p are independent variables of λ, namely,

$$\frac{dJ}{d\lambda}=\frac{dp}{d\lambda}=0. \tag{2.7}$$

Solving these conditions, we find the derivatives of the functions $v(\lambda)$ and $\epsilon(\lambda)$

$$\frac{dv}{d\lambda}=-\frac{v(1-v^2)\epsilon[\mathbf{E}(1-\epsilon)-\mathbf{K}(1-\epsilon)]^2}{2\lambda(1-\epsilon)[\mathbf{E}(1-\epsilon)^2-v^2\epsilon\mathbf{K}(1-\epsilon)^2]},$$
$$\frac{d\epsilon}{d\lambda}=-\frac{\epsilon[\mathbf{E}(1-\epsilon)-\mathbf{K}(1-\epsilon)][\mathbf{E}(1-\epsilon)-v^2\epsilon\mathbf{K}(1-\epsilon)]}{\lambda[\mathbf{E}(1-\epsilon)^2-v^2\epsilon\mathbf{K}(1-\epsilon)^2]}. \tag{2.8}$$

Replacing (2.8) into the derivative of (2.6), one finds

$$\lambda\partial_\lambda\Delta=\frac{\sqrt{\lambda}}{2\pi}\sqrt{\frac{1-v^2}{1-\epsilon}}\big[\mathbf{E}(1-\epsilon)-\epsilon\mathbf{K}(1-\epsilon)\big]. \tag{2.9}$$

Comparing (2.4) and (2.9), we conclude that the equality (2.5) holds.

Next, we would like to compare (2.4) with the known leading finite-size correction to the giant magnon dispersion relation [26]. To this end, we have to consider the limit $\epsilon\to 0$ in (2.4). Taking into account the behavior of the elliptic integrals in the $\epsilon\to 0$ limit, we can use the ansatz

$$v(\epsilon)=v_0+v_1\epsilon+v_2\epsilon\log(\epsilon). \tag{2.10}$$

Actually, all parameters in (2.10) are already known and are given by (see for instance [29])

$$v_0=\cos(p/2),\qquad v_1=\frac{1}{4}\sin^2(p/2)\cos(p/2)\big(1-\log(16)\big),$$
$$v_2=\frac{1}{4}\sin^2(p/2)\cos(p/2),$$
$$\epsilon=16\exp\left(-\frac{2\pi J}{\sqrt{\lambda}\sin(p/2)}-2\right). \tag{2.11}$$

Expanding (2.4) in ϵ and using (2.10), (2.11), we obtain

$$C_3^d=\frac{16}{3}c_\Delta^d\sin(p/2)\left[1-4\sin(p/2)\left(\sin(p/2)+\frac{2\pi J}{\sqrt{\lambda}}\right)\times\exp\left(-\frac{2\pi J}{\sqrt{\lambda}\sin(p/2)}-2\right)\right]. \tag{2.12}$$

On the other hand, from the giant magnon dispersion relation, including the leading finite-size effect,

$$\Delta=\frac{\sqrt{\lambda}}{\pi}\sin(p/2)\left[1-4\sin^2(p/2)\exp\left(-\frac{2\pi J}{\sqrt{\lambda}\sin(p/2)}-2\right)\right],$$

one finds

$$\lambda\partial_\lambda\Delta=\frac{\sqrt{\lambda}}{2\pi}\sin(p/2)\left[1-4\sin(p/2)\left(\sin(p/2)+\frac{2\pi J}{\sqrt{\lambda}}\right)\times\exp\left(-\frac{2\pi J}{\sqrt{\lambda}\sin(p/2)}-2\right)\right]. \tag{2.13}$$

This confirms explicitly that the relation (2.5) holds in the small ϵ, i.e. $J\gg\sqrt{\lambda}$ limit.

3. Dyonic giant magnons with finite size

The dyonic finite-size giant magnon solution is given by

$$x_{0e}=\tanh(\kappa\tau_e),\qquad x_i=0,\qquad z=\frac{1}{\cosh(\kappa\tau_e)},$$
$$\cos\theta=z_+dn\left(\frac{\sqrt{1-u^2}}{1-v^2}z_+(\sigma-v\tau)\Big|1-\epsilon\right),$$
$$\phi_1=\frac{\tau-v\sigma}{1-v^2}+\frac{vW}{\sqrt{1-u^2}z_+(1-z_+^2)}\times\Pi\left(-\frac{z_+^2}{1-z_+^2}(1-\epsilon),am\left(\frac{\sqrt{1-u^2}}{1-v^2}z_+(\sigma-v\tau)\right)\Big|1-\epsilon\right),$$
$$\phi_2=u\frac{\tau-v\sigma}{1-v^2}, \tag{3.1}$$

where

$$\epsilon=\frac{z_-^2}{z_+^2},\qquad W=\kappa^2.$$

$z_\pm^2$ can be written as

$$z_\pm^2=\frac{1}{2(1-u^2)}\Big\{q_1+q_2-u^2\pm\sqrt{(q_1-q_2)^2-\big[2(q_1+q_2-2q_1q_2)-u^2\big]u^2}\Big\},$$
$$q_1=1-W,\qquad q_2=1-v^2W.$$

Now, we have to replace into (2.2) the following expression obtained from the above solution

$$\partial_+X_k\partial_-X_k=-\frac{1}{(1-v^2)\sin^2\theta}\Big\{1-v^2W^2+\big(1-u^2\big)z_+^4\epsilon+2\big(1-u^2\big)\cos^4\theta-\cos^2\theta\big[2+z_+^2(1+\epsilon)-u^2\big(1+z_+^2(1+\epsilon)\big)\big]\Big\}.$$

Computing the integrals in (2.2), we find

$$C_3^d=\frac{8}{3}c_\Delta^d\frac{1}{\sqrt{(1-u^2)W\chi_p(1-\chi_p)}}\times\Big\{(1-\chi_p)\big[2\big(1-u^2\big)\chi_p\mathbf{E}(1-\epsilon)-\big(u^2-\big(1-v^2\big)W+\big(1-u^2\big)(1+\epsilon)\chi_p\big)\mathbf{K}(1-\epsilon)\big]-\big(1-v^2W^2-\chi_p-(1-\chi_p)\big(\epsilon\chi_p+u^2(1-\epsilon\chi_p)\big)\big)\times\Pi\left(-\frac{\chi_p}{1-\chi_p}(1-\epsilon)\Big|1-\epsilon\right)\Big\}, \tag{3.2}$$

where we introduced the notations

$$\chi_p=z_+^2,\qquad \chi_m=z_-^2\quad\Rightarrow\quad \epsilon=\frac{\chi_m}{\chi_p}.$$

This is our *exact* result for the normalized coefficient C_3^d in the three-point correlation function, corresponding to the case when the heavy vertex operators are *finite-size* dyonic giant magnons.

To check the relation (2.5), we need to know Δ. As GM case, we claim that this is given by $E-J_1$. The explicit results are given by [34]

$$\mathcal{E} = \frac{2\sqrt{W}(1-v^2)}{\sqrt{1-u^2}\sqrt{\chi_p}}\mathbf{K}(1-\epsilon),$$

$$\mathcal{J}_1 = \frac{2\sqrt{\chi_p}}{\sqrt{1-u^2}}\left[\frac{1-v^2W}{\chi_p}\mathbf{K}(1-\epsilon) - \mathbf{E}(1-\epsilon)\right], \tag{3.3}$$

$$\mathcal{J}_2 = \frac{2u\sqrt{\chi_p}}{\sqrt{1-u^2}}\mathbf{E}(1-\epsilon),$$

$$p = \frac{2v}{\sqrt{1-u^2}\sqrt{\chi_p}} \times \left[\frac{W}{1-\chi_p}\Pi\left(-\frac{\chi_p}{1-\chi_p}(1-\epsilon)\middle|1-\epsilon\right) - \mathbf{K}(1-\epsilon)\right], \tag{3.4}$$

and

$$\mathcal{E} = \frac{2\pi E}{\sqrt{\lambda}}, \qquad \mathcal{J}_{1,2} = \frac{2\pi J_{1,2}}{\sqrt{\lambda}}.$$

In this case, we need to obtain $v'(\lambda), \epsilon'(\lambda), u'(\lambda)$ from the condition that J_1, J_2, p be independent of λ. It turns out that the exact calculations for these are too complicated. Instead, we will just focus on the $\epsilon \to 0$ limit of (3.2) and λ derivative of Δ from the Lüscher formula to check (2.5). To this end, we will use the expansions

$$\chi_p = \chi_{p0} + \big(\chi_{p1} + \chi_{p2}\log(\epsilon)\big)\epsilon, \qquad \chi_m = \chi_{m1}\epsilon,$$
$$v = v_0 + \big(v_1 + v_2\log(\epsilon)\big)\epsilon, \qquad u = u_0 + \big(u_1 + u_2\log(\epsilon)\big)\epsilon,$$
$$W = 1 + W_1\epsilon. \tag{3.5}$$

First note that χ_p and χ_m satisfy the following relations

$$\chi_p + \chi_m = \frac{2-(1+v^2)W-u^2}{1-u^2},$$
$$\chi_p\chi_m = \frac{1-(1+v^2)W - v^2W^2}{1-u^2}. \tag{3.6}$$

Expanding (3.6) and using the definition of ϵ, we arrive at

$$\chi_{p0} = 1 - \frac{v_0^2}{1-u_0^2},$$
$$\chi_{p1} = \frac{v_0}{(1-v_0^2)(1-u_0^2)^2}\Big\{v_0\big[\big(1-v_0^2\big)^2 - 3\big(1-v_0^2\big)u_0^2 + 2u_0^4 - 2\big(1-v_0^2\big)u_0u_1\big] - 2\big(1-v_0^2\big)\big(1-u_0^2\big)v_1\Big\},$$
$$\chi_{p2} = -2v_0\frac{v_2 + (v_0u_2 - u_0v_2)u_0}{(1-u_0^2)^2},$$
$$\chi_{m1} = 1 - \frac{v_0^2}{1-u_0^2},$$
$$W_1 = -\frac{(1-u_0^2-v_0^2)^2}{(1-u_0^2)(1-v_0^2)}. \tag{3.7}$$

The coefficients in the expansions of v and u, we take from [35], where for the case under consideration we have to set $K_1 = \chi_{n1} = 0$, or equivalently $\Phi = 0$. This gives

$$v_0 = \frac{\sin(p)}{\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}}, \qquad u_0 = \frac{\mathcal{J}_2}{\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}},$$
$$v_1 = \frac{v_0(1-v_0^2-u_0^2)}{4(1-u_0^2)(1-v_0^2)}\big[\big(1-v_0^2\big)\big(1-\log(16)\big) - u_0^2\big(5 - v_0^2\big(1+\log(16)\big) - \log(4096)\big)\big],$$
$$v_2 = \frac{v_0(1-v_0^2-u_0^2)}{4(1-u_0^2)(1-v_0^2)}\big[1 - v_0^2 - u_0^2\big(3+v_0^2\big)\big],$$
$$u_1 = \frac{u_0(1-v_0^2-u_0^2)}{4(1-v_0^2)}\big[1-\log(16) - v_0^2\big(1+\log(16)\big)\big],$$
$$u_2 = \frac{u_0(1-v_0^2-u_0^2)}{4(1-v_0^2)}\big(1+v_0^2\big). \tag{3.8}$$

We need also the expression for ϵ. To the leading order, it can be written as [35]

$$\epsilon = 16\exp\left(-\frac{2 - \frac{2v_0^2}{1-u_0^2} + \mathcal{J}_1\sqrt{1-v_0^2-u_0^2}}{1-v_0^2}\right). \tag{3.9}$$

By using (3.7), (3.8) and (3.9) in the ϵ-expansion of (3.2), we derive

$$\mathcal{C}_3^d = \frac{16}{3}c_\Delta^d\Bigg\{\frac{\mathcal{J}_2^2 + 4\sin^2(p/2) - 16\sin^4(p/2)\exp(f)}{2\sqrt{\mathcal{J}_2^2+4\sin^2(p/2)}} + \frac{1}{(\mathcal{J}_2^2+4\sin^2(p/2))(\mathcal{J}_2^2+4\sin^4(p/2))^2} \times\Big[32\exp(f)\Big(2\mathcal{J}_2^2\sqrt{\mathcal{J}_2^2+4\sin^2(p/2)} - 3\mathcal{J}_1 + 2\Big(\mathcal{J}_1\big(2+\mathcal{J}_2^2\big) + \mathcal{J}_2^2\sqrt{\mathcal{J}_2^2+4\sin^2(p/2)}\Big)\cos(p) - \mathcal{J}_1\cos(2p)\Big)\sin^8(p/2)\Big] - \frac{\mathcal{J}_2^2}{2\sqrt{\mathcal{J}_2^2+4\sin^2(p/2)}} - \frac{8\mathcal{J}_2^2\sin^4(p/2)}{(\mathcal{J}_2^2+4\sin^2(p/2))^{3/2}}\exp(f)\Bigg\}, \tag{3.10}$$

where

$$f = -\frac{2(\mathcal{J}_1 + \sqrt{\mathcal{J}_2^2+4\sin^2(p/2)})\sqrt{\mathcal{J}_2^2+4\sin^2(p/2)}\sin^2(p/2)}{\mathcal{J}_2^2+4\sin^4(p/2)}.$$

On the other hand, from the dyonic giant magnon dispersion relation, including the leading finite-size correction,

$$\Delta_{dyonic} = \frac{\sqrt{\lambda}}{2\pi}\left[\sqrt{\mathcal{J}_2^2+4\sin^2(p/2)} - \frac{16\sin^4(p/2)}{\sqrt{\mathcal{J}_2^2+4\sin^2(p/2)}}\exp(f)\right], \tag{3.11}$$

one obtains

$$\lambda\partial_\lambda\Delta_{dyonic} = \frac{\sqrt{\lambda}}{2\pi}\Bigg\{\frac{\mathcal{J}_2^2 + 4\sin^2(p/2) - 16\sin^4(p/2)\exp(f)}{2\sqrt{\mathcal{J}_2^2+4\sin^2(p/2)}} + \frac{1}{(\mathcal{J}_2^2+4\sin^2(p/2))(\mathcal{J}_2^2+4\sin^4(p/2))^2} \times\Big[32\exp(f)\Big(2\mathcal{J}_2^2\sqrt{\mathcal{J}_2^2+4\sin^2(p/2)} - 3\mathcal{J}_1 + 2\Big(\mathcal{J}_1\big(2+\mathcal{J}_2^2\big) + \mathcal{J}_2^2\sqrt{\mathcal{J}_2^2+4\sin^2(p/2)}\Big)\cos(p) - \mathcal{J}_1\cos(2p)\Big)\sin^8(p/2)\Big] - \frac{\mathcal{J}_2^2}{2\sqrt{\mathcal{J}_2^2+4\sin^2(p/2)}} - \frac{8\mathcal{J}_2^2\sin^4(p/2)}{(\mathcal{J}_2^2+4\sin^2(p/2))^{3/2}}\exp(f)\Bigg\}. \tag{3.12}$$

Comparing (3.10) and (3.12), we see that the relation (2.5) is also valid for finite-size dyonic giant magnons, as it should be.

4. Concluding remarks

In this Letter we have considered three-point correlation functions in the strong coupling side of the AdS/CFT correspondence. We have used a formulation for semiclassical structure constants of a zero-momentum dilaton operator and two heavy string states of (dyonic) giant magnons of finite-size and showed that they match with the expected results coming from derivatives of two-point correlation functions w.r.t. 't Hooft coupling constant.

It is still not clear how to overcome the key approximations we and many other related papers have assumed. It should be essential to utilize the integrability to consider correlation functions for arbitrary value of 't Hooft coupling constant. Developments in this line have been recently reported in [24,25]. Another issue is to include other light operators such as generic dilaton operators which is dual to the SYM or even general heavy string states.

Considering the remarkable developments on the two-point functions, the semiclassical analysis has made crucial contributions in figuring out exact integrability structure hidden in the AdS/CFT correspondence. We hope that our semiclassical results for the generic finite-size operators can be a small starting point toward exact formulation of the three-point correlation functions.

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