Three-point correlation function of giant magnons in the Lunin-Maldacena background

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We compute semiclassical three-point correlation function, or structure constant, of two finite-size (dyonic) giant magnon string states and a light dilaton mode in the Lunin-Maldacena background, which is the γ-deformed, or TsT-transformed $AdS_5 \times S^5_\gamma$, dual to $\mathcal{N} = 1$ super Yang-Mills theory in four dimensions. We also prove that an important relation between the structure constant and the conformal dimension, checked for the $\mathcal{N} = 4$ super Yang-Mills case, still holds for the γ-deformed string background.

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I. INTRODUCTION

The AdS/CFT duality correspondence between type IIB string theory on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ super Yang-Mills (SYM) theory [1] has been actively investigated and led to many exciting developments toward understanding nonperturbative structures of the string and gauge theories. In particular, the integrability structure has been discovered in the computations of energies of string states and conformal dimensions of the gauge theory in the planar limit.

In view of the AdS/CFT duality, the general correlation functions of primary operators of the SYM as a conformal field theory (CFT) should be related to those of the corresponding closed-string vertex operators in the string theory side. The conformal dimensions appear as critical exponents of the two-point correlation functions. Three-point correlation functions are important for the conformal bootstrap approaches. As is well known, the correlation functions of any CFT can be determined, in principle, in terms of the basic conformal data $\{\Delta_i, C_{ijk}\}$, where Δ_i are the conformal dimensions defined by the two-point correlation functions

$$\langle \mathcal{O}_i^\dagger(x_1)\mathcal{O}_j(x_2)\rangle = \frac{C_{12}\delta_{ij}}{|x_1 - x_2|^{2\Delta_i}} \tag{1.1}$$

and C_{ijk} are the structure constants in the operator product expansion

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\mathcal{O}_k(x_3)\rangle = \frac{C_{ijk}}{|x_1 - x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_1 - x_3|^{\Delta_1+\Delta_3-\Delta_2}|x_2 - x_3|^{\Delta_2+\Delta_3-\Delta_1}}. \tag{1.2}$$

Thus, the determination of the initial conformal data for a given CFT is the most important step in the conformal bootstrap approach.

Most of recent studies on the three-point functions are focused on the string theory side in the strong coupling limit using the holographic principle. In particular, the three-point functions of two heavy operators and a light dilaton operator can be approximated by a supergravity vertex operator evaluated at the heavy classical string configuration:

$$\langle V_H(x_1)V_H(x_2)V_L(x_3)\rangle = V_L(x_3)_{\text{classical}}.$$

For $|x_1| = |x_2| = 1$, $x_3 = 0$, the correlation function reduces to

$$\langle V_H(x_1)V_H(x_2)V_L(0)\rangle = \frac{C_{123}}{|x_1 - x_2|^{2\Delta_H}}.$$

Then, the normalized structure constants

$$\mathcal{C}_3 = \frac{C_{123}}{C_{12}}$$

can be found from

$$\mathcal{C}_3 = c_\Delta V_L(0)_{\text{classical}}, \tag{1.3}$$

where c_Δ is the normalized constant of the corresponding light vertex operator.

Recently, there has been an impressive progress in the semiclassical calculations of two, three, and four-point functions with two heavy operators [2–22]. An efficient method to compute two-point correlation functions in the strong coupling limit is to evaluate string partition function for a heavy string state propagating in the AdS space between two boundary points based on a path integral method [2,3]. This method has been extended to the three-point functions of two heavy string states and a supergravity mode which corresponds to a marginal deformation of the SYM two-point functions by the Lagrangian [4–6]. Another direction is to compute the structure

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constants using the Bethe eigenstates of the underlying integrable spin chain in the weak coupling limit of the SYM [23].

Most of these works, however, considered a limit of $J \gg \sqrt{\lambda}$ where J stands for the large angular momentum of a string state or the length of the SYM operator. It is important to generalize this to the case of finite J. Several important integrability techniques have been developed for computing conformal dimensions with the finite J. The finite-size effect for the structure constants have been first studied and compared with the same effect for the conformal dimensions in [19]. However, almost all of these achievements are made for the case the maximal $\mathcal{N}=4$ SYM.

An interesting aspect in the AdS/CFT duality is the role of the supersymmetry because extended conformal symmetries are intimately related to the structure constants. To see this, we think it is important to compute the structure constants of another CFT which has less supersymmetry and holographic duality with a string theory. Such correspondence between gauge and string theory models with reduced supersymmetry is provided by an exactly marginal deformation of $\mathcal{N}=4$ super Yang-Mills theory [24] and string theory on a β-deformed $AdS_5\times S^5$ background suggested by Lunin and Maldacena in [25]. When $\beta\equiv\gamma$ is real, the deformed background can be obtained from $AdS_5\times S^5$ by the so-called TsT transformation, which preserves the classical integrability of string theory on $AdS_5\times S^5$ [26].

Motivated by the above, we investigate in this paper semiclassical correlators in the framework of the duality between string theory in $AdS_5\times S^5_\gamma$ and $\mathcal{N}=1$ SYM. We notice that there is a paper devoted to the same topic [22]. where the three-point correlation function of two infinite-size giant magnons and the dilaton has been obtained. Our object is to extend the results to finite-size system, $J\sim\sqrt{\lambda}\gg 1$, for the case of *finite-size* giant magnons. This paper is organized as follows. In Sec. II we solve classical integrable system to express the energies (or conformal dimensions) and structure constants in terms of the finite angular momenta. This is our main result. We check the validity of our results by comparing them with the conformal dimensions of finite-size system in Sec. III. For this purpose, we use an expansion of large but finite value of J. We will conclude with a brief summary in Sec. IV and some mathematical formulae in the Appendix.

II. THREE-POINT CORRELATION FUNCTION

The bosonic part of the Green-Schwarz action for strings on the γ_i-deformed $AdS_5\times S^5_{\gamma_i}$ [27] reduced to $R_t\times S^5_{\gamma_i}$ can be written as (the common radius R of AdS_5 and $S^5_{\gamma_i}$ is set to 1)

$$\begin{aligned} S = -\frac{T}{2}\int d\tau d\sigma\{&\sqrt{-\gamma}\gamma^{ab}[-\partial_a t\partial_b t + \partial_a r_i\partial_b r_i \\ &+ Gr_i^2\partial_a\phi_i\partial_b\phi_i + Gr_1^2r_2^2r_3^2(\tilde{\gamma}_i\partial_a\phi_i)(\tilde{\gamma}_j\partial_b\phi_j)] \\ &- 2G\epsilon^{ab}(\tilde{\gamma}_3 r_1^2r_2^2\partial_a\phi_1\partial_b\phi_2 + \tilde{\gamma}_1 r_2^2r_3^2\partial_a\phi_2\partial_b\phi_3 \\ &+ \tilde{\gamma}_2 r_3^2r_1^2\partial_a\phi_3\partial_b\phi_1)\}, \end{aligned} \tag{2.1}$$

where T is the string tension, γ^{ab} is the worldsheet metric, ϕ_i are the three isometry angles of the deformed $S^5_{\gamma_i}$, and

$$\sum_{i=1}^3 r_i^2 = 1, \qquad G^{-1} = 1 + \tilde{\gamma}_3^2r_1^2r_2^2 + \tilde{\gamma}_1^2r_2^2r_3^2 + \tilde{\gamma}_2^2r_1^2r_3^2. \tag{2.2}$$

The deformation parameters $\tilde{\gamma}_i$ are related to γ_i which appear in the dual gauge theory as follows

$$\tilde{\gamma}_i = 2\pi T\gamma_i = \sqrt{\lambda}\gamma_i.$$

When $\tilde{\gamma}_i=\tilde{\gamma}$ this becomes the supersymmetric background of [25], and the deformation parameter γ enters the $\mathcal{N}=1$ SYM superpotential in the following way

$$W \propto \mathrm{tr}(e^{i\pi\gamma}\Phi_1\Phi_2\Phi_3 - e^{-i\pi\gamma}\Phi_1\Phi_3\Phi_2).$$

This is the case we are going to explore here.

To consider the (dyonic) giant magnon string solution, we restrict ourselves to the subspace $R_t\times S^3_\gamma$, parameterize (see (2.2))

$$r_1 = \sin\theta, \qquad r_2 = \cos\theta,$$

and use the ansatz [28]

$$\begin{aligned} &t(\tau,\sigma) = \kappa\tau, \qquad \theta(\tau,\sigma) = \theta(\xi), \\ &\phi_j(\tau,\sigma) = \omega_j\tau + f_j(\xi), \qquad \xi = \alpha\sigma + \beta\tau, \\ &\kappa, \omega_j, \alpha, \beta = \text{constants}, \qquad j = 1,2. \end{aligned} \tag{2.3}$$

Then the string Lagrangian in conformal gauge, on the γ-deformed three-sphere, can be written as (prime is used for $d/d\xi$)

$$\begin{aligned} \mathcal{L}_\gamma = (\alpha^2-\beta^2)\Big[&\theta'^2 + G\sin^2\theta\Big(f_1' - \frac{\beta\omega_1}{\alpha^2-\beta^2}\Big)^2 \\ &+ G\cos^2\theta\Big(f_2' - \frac{\beta\omega_2}{\alpha^2-\beta^2}\Big)^2 \\ &- \frac{\alpha^2}{(\alpha^2-\beta^2)^2}G(\omega_1^2\sin^2\theta + \omega_2^2\cos^2\theta) \\ &+ 2\alpha\tilde{\gamma}G\sin^2\theta\cos^2\theta\frac{\omega_2 f_1' - \omega_1 f_2'}{\alpha^2-\beta^2}\Big], \end{aligned} \tag{2.4}$$

where

$$G = \frac{1}{1+\tilde{\gamma}^2\sin^2\theta\cos^2\theta}.$$

The equations of motion for $f_{1,2}$ following from (2.4) can be integrated once to give

$$f_1' = \frac{1}{\alpha^2 - \beta^2}\left[\frac{C_1}{\sin^2\theta} + \beta\omega_1 - \tilde{\gamma}(\alpha\omega_2 - \tilde{\gamma}C_1)\cos^2\theta\right],$$
$$f_2' = \frac{1}{\alpha^2 - \beta^2}\left[\frac{C_2}{\cos^2\theta} + \beta\omega_2 + \tilde{\gamma}(\alpha\omega_1 + \tilde{\gamma}C_2)\sin^2\theta\right], \tag{2.5}$$

where $C_{1,2}$ are integration constants.

Replacing (2.5) into the Virasoro constraints one finds the first integral θ' of the equation of motion for θ and a relation among the parameters

$$\theta'^2 = \frac{1}{(\alpha^2-\beta^2)^2}\left[(\alpha^2+\beta^2)\kappa^2 - \frac{C_1^2}{\sin^2\theta} - \frac{C_2^2}{\cos^2\theta} - (\alpha\omega_1+\tilde{\gamma}C_2)^2\sin^2\theta - (\alpha\omega_2 - \tilde{\gamma}C_1)^2\cos^2\theta\right], \tag{2.6}$$

$$\omega_1 C_1 + \omega_2 C_2 + \beta\kappa^2 = 0. \tag{2.7}$$

Now, we introduce the variable

$$\chi = \cos^2\theta,$$

and the parameters

$$v = -\frac{\beta}{\alpha}, \qquad u = \frac{\Omega_2}{\Omega_1}, \qquad W = \left(\frac{\kappa}{\Omega_1}\right)^2,$$
$$K = \frac{C_2}{\alpha\Omega_1}, \qquad \Omega_1 = \omega_1\left(1 + \tilde{\gamma}\frac{C_2}{\alpha\omega_1}\right),$$
$$\Omega_2 = \omega_2\left(1 - \tilde{\gamma}\frac{C_1}{\alpha\omega_2}\right).$$

By using them and (2.7), the three first integrals can be rewritten as

$$f_1' = \frac{\Omega_1}{\alpha}\frac{1}{1-v^2}\left[\frac{vW - uK}{1-\chi} - v(1-\tilde{\gamma}K) - \tilde{\gamma}u\chi\right],$$
$$f_2' = \frac{\Omega_1}{\alpha}\frac{1}{1-v^2}\left[\frac{K}{\chi} - uv(1-\tilde{\gamma}K) - \tilde{\gamma}v^2W + \tilde{\gamma}(1-\chi)\right],$$
$$\theta' = \frac{\Omega_1}{\alpha}\frac{\sqrt{1-u^2}}{1-v^2}\sqrt{\frac{(\chi_p-\chi)(\chi-\chi_m)(\chi-\chi_n)}{\chi(1-\chi)}}, \tag{2.8}$$

where

$$\chi_p + \chi_m + \chi_n = \frac{2-(1+v^2)W - u^2}{1-u^2},$$
$$\chi_p\chi_m + \chi_p\chi_n + \chi_m\chi_n = \frac{1-(1+v^2)W + (vW-uK)^2 - K^2}{1-u^2},$$
$$\chi_p\chi_m\chi_n = -\frac{K^2}{1-u^2}. \tag{2.9}$$

We are interested in the case of finite-size giant magnons, which corresponds to

$$0 < \chi_m < \chi < \chi_p < 1, \qquad \chi_n < 0.$$

Replacing (2.8) and (2.9) in (2.4), we find the final form of the Lagrangian to be (we set $\alpha = \Omega_1 = 1$ for simplicity)

$$\mathcal{L}_f = -\frac{1}{1-v^2}[2 - (1+v^2)W - 2\tilde{\gamma}K - 2(1 - \tilde{\gamma}K - u(u - \tilde{\gamma}uK + \tilde{\gamma}vW))\chi].$$

To obtain the finite-size effect on the three-point correlator, we use (1.3) and the explicit expression for the dilaton vertex[1]

$$V^d = (Y_4 + Y_5)^{-4}[z^{-2}(\partial_+ x_m\partial_- x^m + \partial_+ z\partial_- z) + \partial_+ X_k\partial_- X_k], \tag{2.10}$$

where

$$Y_4 = \frac{1}{2z}(x^m x_m + z^2 - 1), \qquad Y_5 = \frac{1}{2z}(x^m x_m + z^2 + 1).$$

Here, x_m, z are coordinates on AdS_5, while X_k are the coordinates on S^5. For giant magnons, this leads to [16,19] ($i\tau = \tau_e$)

$$\mathcal{C}_3^{\tilde{\gamma}} = c_\Delta^d\int_{-\infty}^{\infty}\frac{d\tau_e}{\cosh^4(\kappa\tau_e)}\int_{-L}^{L}d\sigma(\kappa^2 + \mathcal{L}_f). \tag{2.11}$$

Performing the integrations in (2.11), one finds

$$\mathcal{C}_3^{\tilde{\gamma}} = \frac{16}{3}c_\Delta^d\frac{1}{\sqrt{(1-u^2)W(\chi_p-\chi_n)}} \times [((1-u^2)(1-\tilde{\gamma}K) - \tilde{\gamma}uvW)\sqrt{\chi_p-\chi_n}\mathbf{E}(1-\epsilon) + ((W(1-\tilde{\gamma}uv\chi_n) - (1-\tilde{\gamma}K) \times (1-(1-u^2)\chi_n))\mathbf{K}(1-\epsilon))], \tag{2.12}$$

where $\mathbf{K}(1-\epsilon)$ and $\mathbf{E}(1-\epsilon)$ are the complete elliptic integrals of first and second kind, and the following notation has been introduced

$$\epsilon = \frac{\chi_m - \chi_n}{\chi_p - \chi_n}. \tag{2.13}$$

This is our *exact* result for the normalized coefficient $\mathcal{C}_3^{\tilde{\gamma}}$ in the three-point correlation function, corresponding to the case when the heavy vertex operators are *finite-size* dyonic giant magnons living on the γ-deformed three-sphere.

For further purposes, let us also write down the exact expressions for the conserved charges and the angular differences

$$\mathcal{E} \equiv \frac{2\pi E}{\sqrt{\lambda}} = 2\frac{(1-v^2)\sqrt{W}}{\sqrt{1-u^2}}\frac{\mathbf{K}(1-\epsilon)}{\sqrt{\chi_p-\chi_n}}, \tag{2.14}$$

[1] Actually, this vertex corresponds to dilaton with zero Kaluza-Klein momentum: $j = 0$. For the general case, $j \neq 0$, see [6,16].

$$\mathcal{J}_1 \equiv \frac{2\pi J_1}{\sqrt{\lambda}} = \frac{2}{\sqrt{1-u^2}}\Big[\frac{1-\chi_n - v(vW-uK)}{\sqrt{\chi_p-\chi_n}}\mathbf{K}(1-\epsilon) - \sqrt{\chi_p-\chi_n}\mathbf{E}(1-\epsilon)\Big], \tag{2.15}$$

$$\mathcal{J}_2 \equiv \frac{2\pi J_2}{\sqrt{\lambda}} = \frac{2}{\sqrt{1-u^2}}\Big[\frac{u\chi_n - vK}{\sqrt{\chi_p-\chi_n}}\mathbf{K}(1-\epsilon) + u\sqrt{\chi_p-\chi_n}\mathbf{E}(1-\epsilon)\Big], \tag{2.16}$$

$$p_1 \equiv \Delta\phi_1 = \phi_1(L)-\phi_1(-L) = \frac{2}{\sqrt{1-u^2}}\Big\{\frac{vW-uK}{(1-\chi_p)\sqrt{\chi_p-\chi_n}} \times \Pi\Big(-\frac{\chi_p-\chi_m}{1-\chi_p}\Big|1-\epsilon\Big) - [v(1-\tilde\gamma K)+\tilde\gamma u\chi_n] \times \frac{\mathbf{K}(1-\epsilon)}{\sqrt{\chi_p-\chi_n}} - \tilde\gamma u\sqrt{\chi_p-\chi_n}\mathbf{E}(1-\epsilon)\Big\}, \tag{2.17}$$

$$p_2 \equiv \Delta\phi_2 = \phi_2(L)-\phi_2(-L) = \frac{2}{\sqrt{1-u^2}}\Big\{\frac{K}{\chi_p\sqrt{\chi_p-\chi_n}}\Pi\Big(1-\frac{\chi_m}{\chi_p}\Big|1-\epsilon\Big) - [uv+\tilde\gamma v(vW-uK)-\tilde\gamma(1-\chi_n)] \times \frac{\mathbf{K}(1-\epsilon)}{\sqrt{\chi_p-\chi_n}} - \tilde\gamma\sqrt{\chi_p-\chi_n}\,\mathbf{E}(1-\epsilon)\Big\}. \tag{2.18}$$

Here, E, $J_{1,2}$ are the string energy and angular momenta, while $\phi_{1,2}$ are the isometry angles on the γ-deformed three-sphere. $\Pi(m|n)$ is the complete elliptic integral of third kind.

III. SMALL ϵ EXPANSIONS

For the case of dilaton operator with zero Kaluza-Klein momentum, the three-point function of the SYM can be easily related to the conformal dimension of the heavy operators. This corresponds to shift ‘t Hooft coupling constant which is the overall coefficient of the Lagrangian [5]. This gives an important relation between the structure constant and the conformal dimension as follows:

$$\mathcal{C}_3^{\tilde\gamma} = \frac{32\pi}{3}c_\Delta^d\sqrt{\lambda}\partial_\lambda\Delta. \tag{3.1}$$

We want to show here that this relation holds also for the case of finite-size giant magnons on $R_t\times S^3_\gamma$ ($J_2=0$), assuming that $\Delta = E - J_1$, and considering the limit $\epsilon\to 0$. To this end, we introduce the expansions[2]

[2]The expansions for the elliptic integrals we use are given in the Appendix.

$$\begin{aligned}
\chi_p &= \chi_{p0} + (\chi_{p1}+\chi_{p2}\log(\epsilon))\epsilon,\\
\chi_m &= \chi_{m0} + (\chi_{m1}+\chi_{m2}\log(\epsilon))\epsilon,\\
\chi_n &= \chi_{n0} + (\chi_{n1}+\chi_{n2}\log(\epsilon))\epsilon,\\
v &= v_0 + (v_1+v_2\log(\epsilon))\epsilon,\\
u &= u_0 + (u_1+u_2\log(\epsilon))\epsilon,\\
W &= W_0 + (W_1+W_2\log(\epsilon))\epsilon,\\
K &= K_0 + (K_1+K_2\log(\epsilon))\epsilon.
\end{aligned} \tag{3.2}$$

A few comments are in order. To be able to reproduce the dispersion relation for the infinite-size giant magnons, we set

$$\chi_{m0} = \chi_{n0} = K_0 = 0, \qquad W_0 = 1. \tag{3.3}$$

In addition, one can check that if we keep the coefficients χ_{m2}, χ_{n2}, W_2 and K_2 nonzero, the known leading correction to the giant magnon energy-charge relation [29] will be modified by a term proportional to $\mathcal{J}_1^2$. That is why we choose

$$\chi_{m2} = \chi_{n2} = W_2 = K_2 = 0. \tag{3.4}$$

Finally, since we are considering for simplicity giant magnons with one angular momentum, we also set

$$u_0 = 0, \tag{3.5}$$

because the leading term in the ϵ-expansion of $\mathcal{J}_2$ is proportional to u_0. By replacing (3.2) in (2.9) and (2.13), and taking into account (3.3), (3.4), and (3.5), we obtain

$$\begin{aligned}
\chi_{p0} &= 1-v_0^2,\\
\chi_{p1} &= \frac{v_0}{1-v_0^2}\Big[v_0\sqrt{(1-v_0^2)^4-4K_1^2(1-v_0^2)} - 2(1-v_0^2)v_1\Big],\\
\chi_{p2} &= -2v_0v_2,\\
\chi_{m1} &= \frac{(1-v_0^2)^2+\sqrt{(1-v_0^2)^4-4K_1^2(1-v_0^2)}}{2(1-v_0^2)},\\
\chi_{n1} &= -\frac{(1-v_0^2)^2-\sqrt{(1-v_0^2)^4-4K_1^2(1-v_0^2)}}{2(1-v_0^2)},\\
W_1 &= -\frac{\sqrt{(1-v_0^2)^4-4K_1^2(1-v_0^2)}}{1-v_0^2}.
\end{aligned} \tag{3.6}$$

The other parameters in (3.2) and (3.6) can be found in the following way. First, we impose the conditions $J_2 = 0$ and p_1 to be independent of ϵ. This leads to four equations with solution

$$v_1 = \frac{v_0\sqrt{(1-v_0^2)^4 - 4K_1^2(1-v_0^2)}(1-\log 16)}{4(1-v_0^2)},$$
$$v_2 = \frac{v_0\sqrt{(1-v_0^2)^4 - 4K_1^2(1-v_0^2)}}{4(1-v_0^2)}, \tag{3.7}$$
$$u_1 = \frac{K_1 v_0 \log 4}{1-v_0^2}, \qquad u_2 = -\frac{K_1 v_0}{2(1-v_0^2)},$$

where

$$v_0 = \cos\frac{p_1}{2}. \tag{3.8}$$

Next, to the leading order, the expansions for $\mathcal{J}_1$ and $p_2 = 2\pi n_2$ $(n_2 \in \mathbb{Z})$[3] give

$$\epsilon = 16\exp\left(-2 - \frac{\mathcal{J}_1}{\sin\frac{p_1}{2}}\right),$$
$$K_1 = \frac{1}{2}\sin^3\frac{p_1}{2}\sin\Phi, \tag{3.9}$$
$$\Phi = 2\pi\left(n_2 - \frac{\tilde{\gamma}}{\sqrt{\lambda}}J_1\right).$$

Now, we consider the limit $\epsilon \to 0$ in the expression (2.12) for the structure constant in the 3-point correlation function, by using (3.2), (3.3), (3.4), (3.5), (3.6), and (3.7), and obtain

$$\begin{aligned}\mathcal{C}_3^{\tilde{\gamma}} \approx &\frac{4}{3}c_\Delta^d\frac{1}{(1-v_0^2)^{3/2}}\Big[4+4v_0^4(1-\tilde{\gamma}K_1(1-\log 4)\epsilon)\\ &-v_0^2\Big(8+\Big(\sqrt{(1-v_0^2)^4-4K_1^2(1-v_0^2)}(1-\log 16)\\ &-8\tilde{\gamma}K_1(1-\log 4)\Big)\epsilon\Big)-\Big(4\tilde{\gamma}K_1(1-\log 4)\\ &-\sqrt{(1-v_0^2)^4-4K_1^2(1-v_0^2)}(1-\log 256)\Big)\epsilon\\ &-\Big(v_0^2\sqrt{(1-v_0^2)^4-4K_1^2(1-v_0^2)}\\ &+2\tilde{\gamma}K_1(1-v_0^2)^2\Big)\epsilon\log\epsilon\\ &+\sqrt{(1-v_0^2)^4-4K_1^2(1-v_0^2)}\epsilon\log(16\epsilon)\Big].\end{aligned} \tag{3.10}$$

According to (3.8) and (3.9), the above expression for $\mathcal{C}_3^{\tilde{\gamma}}$ can be rewritten in terms of p_1, $\mathcal{J}_1$, as

$$\begin{aligned}\mathcal{C}_3^{\tilde{\gamma}} \approx \frac{16}{3}c_\Delta^d\sin\frac{p_1}{2}\Big[&1-4\sin^2\frac{p_1}{2}\Big(\cos\Phi+\mathcal{J}_1\csc\frac{p_1}{2}\cos\Phi\\ &-\tilde{\gamma}\mathcal{J}_1\sin\Phi\Big)e^{-2-(\mathcal{J}_1/\sin(p_1/2))}\Big].\end{aligned} \tag{3.11}$$

[3]This follows from the periodicity condition on ϕ_2.

In order to check if the equality (3.1) holds for the present case, let us now consider the dispersion relation of giant magnons on TsT-transformed $AdS_5 \times S^5$, including the leading finite-size correction, which is known to be [30,31]

$$\begin{aligned}E - J_1 = \frac{\sqrt{\lambda}}{\pi}\sin(p/2)\Big[&1-4\sin^2(p/2)\cos\Phi\\ &\times\exp\Big(-2-\frac{2\pi J_1}{\sqrt{\lambda}\sin(p/2)}\Big)\Big].\end{aligned} \tag{3.12}$$

Taking the λ derivative of (3.12), one finds

$$\begin{aligned}\lambda\partial_\lambda\Delta = \frac{\sqrt{\lambda}}{2\pi}\sin\frac{p}{2}\Big[&1-4\sin^2\frac{p}{2}(\cos\Phi+\mathcal{J}_1\csc\frac{p}{2}\cos\Phi\\ &-\tilde{\gamma}\mathcal{J}_1\sin\Phi)e^{-2-(\mathcal{J}_1/\sin(p/2))}\Big].\end{aligned} \tag{3.13}$$

Identifying $p \equiv p_1$, and comparing (3.11) with (3.13), we see that the equality (3.1) is also valid for the γ-deformed case.

IV. CONCLUDING REMARKS

In this note, we have derived the structure constant in the three-point correlation function of two finite-size (dyonic) giant magnon string states and a light dilaton state in the semiclassical approximation, for the case of γ-deformed (TsT-transformed) $AdS_5 \times S^5$, dual to $\mathcal{N} = 1$ SYM, arising as an exactly marginal deformation of $\mathcal{N} = 4$ SYM.

We have also found that the important relation between the structure constant and the derivative of the conformal dimension with respect to the t'Hooft coupling λ still holds for the γ-deformed case.

It will be interesting to consider correlation functions of other light operators or even all the heavy string states in the future.

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APPENDIX: ELLIPTIC INTEGRALS AND ϵ-EXPANSIONS

The elliptic integrals appearing in the main text are given by

$$\int_{\chi_m}^{\chi_p} \frac{d\chi}{\sqrt{(\chi_p-\chi)(\chi-\chi_m)(\chi-\chi_n)}} = \frac{2}{\sqrt{\chi_p-\chi_n}}\mathbf{K}(1-\epsilon),$$

$$\int_{\chi_m}^{\chi_p} \frac{\chi d\chi}{\sqrt{(\chi_p-\chi)(\chi-\chi_m)(\chi-\chi_n)}} = \frac{2\chi_n}{\sqrt{\chi_p-\chi_n}}\mathbf{K}(1-\epsilon) + 2\sqrt{\chi_p-\chi_n}\mathbf{E}(1-\epsilon),$$

$$\int_{\chi_m}^{\chi_p} \frac{d\chi}{\chi\sqrt{(\chi_p-\chi)(\chi-\chi_m)(\chi-\chi_n)}} = \frac{2}{\chi_p\sqrt{\chi_p-\chi_n}}\Pi\left(1-\frac{\chi_m}{\chi_p}\,\middle|\,1-\epsilon\right),$$

$$\int_{\chi_m}^{\chi_p} \frac{d\chi}{(1-\chi)\sqrt{(\chi_p-\chi)(\chi-\chi_m)(\chi-\chi_n)}} = \frac{2}{(1-\chi_p)\sqrt{\chi_p-\chi_n}}\Pi\left(-\frac{\chi_p-\chi_m}{1-\chi_p}\,\middle|\,1-\epsilon\right),$$

where

$$\epsilon = \frac{\chi_m-\chi_n}{\chi_p-\chi_n}.$$

We use the following expansions for the complete elliptic integrals [32]

$$\mathbf{K}(1-\epsilon) = -\frac{1}{2}\log\left(\frac{\epsilon}{16}\right) - \frac{1}{4}\left(1+\frac{1}{2}\log\left(\frac{\epsilon}{16}\right)\right)\epsilon + \ldots,$$

$$\mathbf{E}(1-\epsilon) = 1 - \frac{1}{4}\left(1+\log\left(\frac{\epsilon}{16}\right)\right)\epsilon + \ldots,$$

$$\Pi(-n|1-\epsilon) = \frac{2\sqrt{n}\arctan(\sqrt{n}) - \log(\frac{\epsilon}{16})}{2(1+n)} - \frac{2-4\sqrt{n}\arctan(\sqrt{n}) + (1-n)\log(\frac{\epsilon}{16})}{8(1+n)^2}\epsilon + \ldots, \qquad n>0.$$

We use also the equality [33]

$$\Pi(\nu|m) = \frac{q_1}{q}\Pi(\nu_1|m) - \frac{m}{q\sqrt{-\nu\nu_1}}\mathbf{K}(m),$$

where

$$q = \sqrt{(1-\nu)\left(1-\frac{m}{\nu}\right)}, \qquad q_1 = \sqrt{(1-\nu_1)\left(1-\frac{m}{\nu_1}\right)}, \qquad \nu = \frac{\nu_1-m}{\nu_1-1}, \qquad \nu_1<0, \qquad m<\nu<1.$$

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