Published for SISSA by Springer

Received: *December 27, 2017*
Accepted: *February 10, 2018*
Published: *February 19, 2018*

Some semiclassical structure constants for $AdS_4 \times CP^3$

JHEP02(2018)110

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Abstract: We compute structure constants in three-point functions of three string states in $AdS_4 \times CP^3$ in the framework of the semiclassical approach. We consider HHL correlation functions where two of the states are "heavy" string states of *finite-size* giant magnons carrying one or two angular momenta and the other one corresponds to such "light" states as dilaton operators with *non-zero* momentum, primary scalar operators, and singlet scalar operators with higher string levels.

Keywords: AdS-CFT Correspondence, Integrable Field Theories

ArXiv ePrint: 1712.04816

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https://doi.org/10.1007/JHEP02(2018)110

Contents

1 Introduction

The AdS/CFT duality between string/M theories on Anti-de Sitter (AdS) background and conformal field theories (CFTs) on its boundary has been a most productive research direction ever since it was proposed [1–3]. As a strong-weak coupling duality, integrability has played crucial roles in non-perturbative computations [4]. It has been applied to compute conformal dimensions of the CFTs and energies of corresponding string states. A natural next challenge is to utilize integrability to compute structure constants which determine three-point functions.

A promising recent progress is so-called hexagon amplitude approach [5]. The structure constants are given by sums of hexagon amplitudes, which can be determined exactly in all orders of 't Hooft coupling constant λ. This approach has proved effective in the weak coupling limit [6–8]. However, technical difficulties such as summing up all intermediate states, finite-size effects, etc. become substantial in the strong coupling limit [9].

For two-heavy and one-light operators in the semiclassical limit $\lambda \gg 1$, the "HHL" three-point functions can be obtained from explicit evaluation of light vertex operator with the heavy string configurations [10–12]. In spite of limited applicability, this method is useful to obtain structure constants when the heavy operators have large but finite $J \gg \sqrt{\lambda}$ values. The resulting HHL functions show exponential corrections $e^{-J/\sqrt{\lambda}}$, which can be related to exact S-matrix, hence the integrability [13].

Type IIA string theory on $AdS_4 \times CP^3$ background is dual to $\mathcal{N} = 6$ super Chern-Simons theory in three space-time dimensions, known as ABJM theory [14]. Classical integrability [15, 16] and giant magnon solutions have been studied in [17]–[20]. The HHL 3-point functions have been computed for various string states in $AdS_4 \times CP^3$ [21]–[25]. In this paper, we will focus on finite-size effects of some normalized structure constants in $AdS_4 \times CP^3$ in semiclassical limit where the heavy string states are *finite-size* giant magnons. We also consider various different light string states, such as dilaton operators with *non-zero* momentum, primary scalar operators, and singlet scalar operators on higher string levels.

The paper is organized as follows. In section 2, we introduce preliminary contents along with various giant magnons on CP^3. We present the HHL functions of two giant magnons with dilaton operator with non-zero momentum in section 3, with primary scalar operators in section 4, and with singlet scalar operators on higher string levels in section 5. We conclude the paper in section 6.

2 Preliminaries

2.1 Structure constants

It is known that correlation functions of CFTs can be determined in principle in terms of the basic conformal data $\{\Delta_i, C_{ijk}\}$, where Δ_i are the conformal dimensions defined by two-point normalized correlation functions

$$\left\langle \mathcal{O}_i^\dagger(x_1)\mathcal{O}_j(x_2)\right\rangle = \frac{\delta_{ij}}{|x_1 - x_2|^{2\Delta_i}}$$

and C_{ijk} are structure constants of three-point correlation functions

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\mathcal{O}_k(x_3)\rangle = \frac{C_{ijk}}{|x_1 - x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_1 - x_3|^{\Delta_1+\Delta_3-\Delta_2}|x_2 - x_3|^{\Delta_2+\Delta_3-\Delta_1}}.$$

The HHL three-point functions of two heavy operators and a light operator can be approximated by a supergravity vertex operator evaluated at the heavy classical string configuration [12]:

$$\langle V_H(x_1)V_H(x_2)V_L(x_3)\rangle = V_L(x_3)_{\text{classical}}.$$

For $|x_1| = |x_2| = 1$, $x_3 = 0$, the correlation function reduces to

$$\langle V_{H_1}(x_1)V_{H_2}(x_2)V_L(0)\rangle = \frac{C_{123}}{|x_1 - x_2|^{2\Delta_H}}.$$

Then, the structure constants can be given by

$$C_{123} = c_\Delta V_L(0)_{\text{classical}}, \tag{2.1}$$

where c_Δ is the normalization constant of the corresponding light vertex operator.

2.2 The string Lagrangian and Virasoro constraints

String theory moving on certain background can be described by the Polyakov action

$$S^P = -T\int d^2\xi\sqrt{-\gamma}\gamma^{mn}G_{mn}, \qquad G_{mn} = g_{MN}\partial_m X^M\partial_n X^N, \tag{2.2}$$
$$\partial_m = \partial/\partial\xi^m, \quad m,n=(0,1), \qquad (\xi^0,\xi^1)=(\tau,\sigma), \quad M,N=(0,1,\ldots,9),$$

where T is the string tension. We choose to work in conformal gauge $\gamma^{mn} = \eta^{mn} = diag(-1,1)$, in which the Lagrangian and the Virasoro constraints take the form

$$\mathcal{L}_s = T(G_{00} - G_{11}), \tag{2.3}$$
$$G_{00} + G_{11} = 0, \tag{2.4}$$
$$G_{01} = 0. \tag{2.5}$$

The background metric g_{MN} for $AdS_4 \times CP^3$ is given by

$$ds^2 = g_{MN}dx^M dx^N = R^2\left(ds^2_{AdS^4} + ds^2_{CP^3}\right),$$

where R is related to the string tension T and the 't Hooft coupling constant ($\alpha' = 1$) by

$$TR^2 = \sqrt{2\lambda}.$$

There are also dilaton and RR-forms, which do not influence the motion of the classical strings. Further on, we set $R = 1$.

The metric of CP^3 space can be written as

$$\begin{aligned} ds^2_{CP^3} &= d\theta^2 + \sin^2\theta\left(\frac{1}{2}\cos\vartheta_1 d\varphi_1 - \frac{1}{2}\cos\vartheta_2 d\varphi_2 + d\varphi_3\right)^2 \\ &+ \cos^2\frac{\theta}{2}\left(d\vartheta_1^2 + \sin^2\vartheta_1 d\varphi_1^2\right) + \sin^2\frac{\theta}{2}\left(d\vartheta_2^2 + \sin^2\vartheta_2 d\varphi_2^2\right), \end{aligned} \tag{2.6}$$

where $\theta \in [0,\pi]$, $\vartheta_1, \vartheta_2 \in [0,\pi]$, $\varphi_1, \varphi_2 \in [0, 2\pi]$, $\varphi_3 \in [0, 4\pi]$. The angular coordinates in (2.6) can be expressed also by the following complex coordinates

$$\begin{aligned} z_1 &= \cos\frac{\theta}{2}\cos\frac{\vartheta_1}{2}\exp\left[\frac{i}{2}(\varphi_3+\varphi_1)\right] = r_1\exp(i\phi_1), \\ z_2 &= \sin\frac{\theta}{2}\cos\frac{\vartheta_2}{2}\exp\left[-\frac{i}{2}(\varphi_3-\varphi_2)\right] = r_2\exp(i\phi_2), \\ z_3 &= \cos\frac{\theta}{2}\sin\frac{\vartheta_1}{2}\exp\left[\frac{i}{2}(\varphi_3-\varphi_1)\right] = r_3\exp(i\phi_3), \\ z_4 &= \sin\frac{\theta}{2}\sin\frac{\vartheta_2}{2}\exp\left[-\frac{i}{2}(\varphi_3+\varphi_2)\right] = r_4\exp(i\phi_4), \end{aligned} \tag{2.7}$$
$$\sum_{a=1}^{4} r_a^2 = 1, \qquad \sum_{a=1}^{4} r_a^2\partial_m\phi_a = 0.$$

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2.3 Giant magnons

The giant magnon solutions in CP^3 can be found by the Neumann-Rosochatius (NR) integrable system with the following ansatz for the string embedding [18]

$$\begin{aligned} &t(\tau,\sigma)=\kappa\tau, \qquad r_a(\tau,\sigma)=r_a(\xi), \qquad \phi_a(\tau,\sigma)=\omega_a\tau+f_a(\xi), \\ &\xi=\sigma-v\tau, \qquad \kappa,\omega_a,v=\text{constants}. \end{aligned} \tag{2.8}$$

2.3.1 CP^1 giant magnon

Let us start with the giant magnon living in the $R_t\times CP^1$ subspace. Such subspace can be obtained by setting $\theta=\vartheta_2=\varphi_2=\varphi_3=0$. What remains is[1]

$$ds^2=-dt^2+d\vartheta_1^2+\sin^2\vartheta_1 d\varphi_1^2. \tag{2.9}$$

Then (2.7) becomes

$$\begin{aligned} z_1 &= \cos\frac{\vartheta_1}{2}\exp\left(\frac{i}{2}\varphi_1\right)=r_1\exp(i\phi_1), \\ z_2 &= 0, \\ z_3 &= \sin\frac{\vartheta_1}{2}\exp\left(-\frac{i}{2}\varphi_1\right)=r_3\exp(-i\phi_1), \\ z_4 &= 0. \end{aligned} \tag{2.10}$$

For this case, the induced metric on the string wordsheet is

$$\begin{aligned} G_{00} &= -(\partial_0 t)^2+(\partial_0\vartheta_1)^2+\sin^2\vartheta_1(\partial_0\varphi_1)^2, \\ G_{11} &= -(\partial_1 t)^2+(\partial_1\vartheta_1)^2+\sin^2\vartheta_1(\partial_1\varphi_1)^2, \\ G_{01} &= -\partial_0 t\partial_1 t+\partial_0\vartheta_1\partial_1\vartheta_1+\sin^2\vartheta_1\partial_0\varphi_1\partial_1\varphi_1. \end{aligned}$$

By using (2.8) and (2.9) in (2.3), one can write down the string Lagrangian in the following form (prime is used for $d/d\xi$)

$$\mathcal{L}_s=-T(1-v^2)\left[(\vartheta_1')^2+\sin^2\vartheta_1\left(\left(f_1'+\frac{v\omega_1}{1-v^2}\right)^2-\frac{\omega_1^2}{(1-v^2)^2}\right)\right], \tag{2.11}$$

from which the first integral for f_1 becomes

$$f_1'=\frac{1}{1-v^2}\left(\frac{C_1}{\sin^2\vartheta_1}-v\omega_1\right) \tag{2.12}$$

with an integration constant C_1.

The first Virasoro constraint (2.4) along with (2.12) becomes

$$(\vartheta_1')^2=\frac{1}{(1-v^2)^2}\left[(1+v^2)\kappa^2-\frac{C_1^2}{\sin^2\vartheta_1}-\omega_1^2\sin^2\vartheta_1\right], \tag{2.13}$$

while the second constraint (2.5) determines the constant $C_1=\frac{v\kappa^2}{\omega_1}$. We will further restrict $\omega_1=1$ since we can choose an appropriate unit of τ. In terms of $\chi\equiv\cos^2\vartheta_1$, (2.13) can be written as

$$\chi'=\frac{2}{1-v^2}\sqrt{\chi(\chi_p-\chi)(\chi-\chi_m)}, \qquad \text{with} \quad \chi_p=1-v^2\kappa^2, \quad \chi_m=1-\kappa^2. \tag{2.14}$$

[1] This choice of $CP^1=S^2$ subspace corresponds to "A"-type giant magnon in the ABJM theory [28]. The "B"-type magnon lives in another CP^1 subspace obtained by $\theta=\pi$. Here we consider only A-type.

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2.3.2 RP^3 giant magnon

The RP^3 giant magnon lives in the $R_t \times RP^3$ subspace, which can be obtained from (2.6) by setting $\vartheta_1 = \vartheta_2 = \frac{\pi}{2}$, $\varphi_3 = 0$. The resulting metric is

$$ds^2 = -dt^2 + d\theta^2 + \cos^2\frac{\theta}{2} d\varphi_1^2 + \sin^2\frac{\theta}{2} d\varphi_2^2.$$

Correspondingly, the coordinates (2.7) reduce to

$$\begin{aligned} z_1 &= \frac{1}{\sqrt{2}} \cos\frac{\theta}{2} \exp\left(\frac{i}{2}\varphi_1\right) = r_1 \exp(i\phi_1), \qquad (2.15)\\ z_2 &= \frac{1}{\sqrt{2}} \sin\frac{\theta}{2} \exp\left(\frac{i}{2}\varphi_2\right) = r_2 \exp(i\phi_2),\\ z_3 &= \frac{1}{\sqrt{2}} \cos\frac{\theta}{2} \exp\left(-\frac{i}{2}\varphi_1\right) = r_1 \exp(-i\phi_1),\\ z_4 &= \frac{1}{\sqrt{2}} \sin\frac{\theta}{2} \exp\left(-\frac{i}{2}\varphi_2\right) = r_2 \exp(-i\phi_2). \end{aligned}$$

For the case at hand, the metric induced on the string worldsheet is given by

$$\begin{aligned} G_{00} &= -(\partial_0 t)^2 + (\partial_0\theta)^2 + \cos^2\frac{\theta}{2}(\partial_0\varphi_1)^2 + \sin^2\frac{\theta}{2}(\partial_0\varphi_2)^2,\\ G_{11} &= -(\partial_1 t)^2 + (\partial_1\theta)^2 + \cos^2\frac{\theta}{2}(\partial_1\varphi_1)^2 + \sin^2\frac{\theta}{2}(\partial_1\varphi_2)^2,\\ G_{01} &= -\partial_0 t\partial_1 t + \partial_0\theta\partial_1\theta + \cos^2\frac{\theta}{2}\partial_0\varphi_1\partial_1\varphi_1 + \sin^2\frac{\theta}{2}\partial_0\varphi_2\partial_1\varphi_2. \end{aligned}$$

The string Lagrangian in this case becomes

$$\mathcal{L}_s = -T\left[(1-v^2)(\theta')^2 - \cos^2\frac{\theta}{2}\left((vf_1' - \omega_1)^2 - f_1'\right)^2 - \sin^2\frac{\theta}{2}\left((vf_2' - \omega_2)^2 - f_2'\right)^2\right], \qquad (2.16)$$

from which the first integrals for f_1 and f_2 become

$$f_1' = \frac{1}{1-v^2}\left(\frac{C_1}{\cos^2\frac{\theta}{2}} - v\omega_1\right), \qquad f_2' = \frac{1}{1-v^2}\left(\frac{C_2}{\sin^2\frac{\theta}{2}} - v\omega_2\right) \qquad (2.17)$$

with integration constants C_1, C_2. Since the RP^3 giant magnon should be well-defined at $\theta = \pi$, we impose an extra condition $C_1 = 0$.

The two Virasoro constraints (2.4) and (2.5) are combined along with (2.17) to give a parametric relation

$$v\kappa^2 = C_2\omega_2, \qquad (2.18)$$

and the first integral for θ

$$(\theta')^2 = \frac{1}{(1-v^2)^2}\left[(1+v^2)\kappa^2 - \omega_1^2 - \frac{C_2^2}{\sin^2\frac{\theta}{2}} + (\omega_1^2 - \omega_2^2)\sin^2\frac{\theta}{2}\right]. \qquad (2.19)$$

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This time we fix $\omega_2 \equiv 1$ and define

$$\chi \equiv \cos^2 \frac{\theta}{2}, \qquad u \equiv \frac{\omega_1}{\omega_2}, \tag{2.20}$$

to rewrite this equation as

$$\chi' = \frac{\sqrt{1-u^2}}{1-v^2} \sqrt{\chi(\chi_p - \chi)(\chi - \chi_m)}, \tag{2.21}$$

where χ_p and χ_m satisfy the equalities:

$$\chi_p + \chi_m = \frac{2 - (1+v^2)\kappa^2 - u^2}{1-u^2}, \qquad \chi_p \chi_m = \frac{(1-\kappa^2)(1-v^2\kappa^2)}{1-u^2}. \tag{2.22}$$

3 HHL of dilaton operator with non-zero momentum

The vertex for the dilaton operator with *non-zero* momentum j, originally defined for $AdS_5 \times S^5$ in [12], is modified in the $AdS_4 \times CP^3$ case to

$$\begin{aligned} V^d_{ab}(j) &= \left(\frac{x^m x_m + z^2}{z}\right)^{-\Delta_d} (z_a z_b)^j \left[z^{-2}\left(\partial_+ x_m \partial_- x^m + \partial_+ z \partial_- z\right) + \partial_+ X_k \partial_- X^k\right] \\ x^m x_m &= -x_0^2 + x_i x_i, \qquad i = 1, 2, \end{aligned} \tag{3.1}$$

where we denote the scaling dimension $\Delta_d = 4 + j$ and (x^m, z) as the Poincare coordinates on AdS_4. The coordinates on CP^3 are represented by angular coordinates X_k or equivalently by the complex coordinates z_a defined in (2.7). The choice of the indices (a, b) determines the direction of the momentum in the CP^3 space.

The AdS part of the giant magnon solution is given by (after Euclidean rotation, $i\tau = \tau_e$, where τ is the worldsheet time)

$$x_{0e} = \tanh(\kappa\tau_e), \qquad x_i = 0, \qquad z = \frac{1}{\cosh(\kappa\tau_e)}. \tag{3.2}$$

Replacing (3.2) into (3.1), one finds

$$V^d_{ab}(j) = (\cosh \kappa\tau_e)^{-\Delta_d} (z_a z_b)^j \left(\kappa^2 + \partial_+ X_k \partial_- X^k\right). \tag{3.3}$$

3.1 With the $R_t \times CP^1$ giant magnons

From (2.10) it is clear that non-vanishing HHL is possible only with the vertex $V^d_{13}(j)$. Evaluating the Lagrangian on the giant magnon state,

$$\partial_+ X_k \partial_- X^k = \frac{2}{1-v^2}\left[\chi - 1 + \frac{1}{2}(1+v^2)\kappa^2\right], \tag{3.4}$$

one finds

$$V^d_{13}(j) = \frac{(1-\chi)^{j/2}\left(\kappa^2 + \chi - 1\right)}{2^{j-1}(1-v^2)\left(\cosh \kappa\tau_e\right)^{\Delta_d}}. \tag{3.5}$$

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The normalized structure constant can be obtained by integrating the vertex over the string worldsheet.

$$\begin{aligned}\mathcal{C}_{13}^{CP^1,d}(j) &= c_\Delta^d \int_{-\infty}^{\infty} d\tau_e \int_{-L}^{L} d\sigma V_{13}^d(j) \qquad (3.6)\\ &= \frac{c_\Delta^d \sqrt{\pi}}{2^{j-1}(1-v^2)\kappa} \frac{\Gamma\left(\frac{\Delta_d}{2}\right)}{\Gamma\left(\frac{\Delta_d+1}{2}\right)} \int_{\chi_m}^{\chi_p} \frac{d\chi}{\chi'} (1-\chi)^{j/2}\left(\kappa^2+\chi-1\right),\end{aligned}$$

where we replaced the integration over σ with integration over χ in the following way

$$\int_{-L}^{L} d\sigma = 2\int_{\chi_m}^{\chi_p} \frac{d\chi}{\chi'},$$

using eq. (2.14) for χ'. The parameter L is introduced here in order to take into account the giant magnons in the *finite-size* worldsheet volume.

The final expression of the integral in (3.6) becomes

$$\begin{aligned}\mathcal{C}_{13}^{CP^1,d}(j) &= c_\Delta^d \frac{\Gamma\left(\frac{\Delta_d}{2}\right)}{\Gamma\left(\frac{\Delta_d+1}{2}\right)} \frac{\pi^{3/2}}{2^j\kappa} \chi_p^{-1/2}(1-\chi_p)^{j/2} \times \qquad (3.7)\\ &\times \left[\chi_p F_1\left(\frac{1}{2};-\frac{1}{2},-\frac{j}{2};1;1-\epsilon,\frac{\chi_p(\epsilon-1)}{1-\chi_p}\right) - (1-\kappa^2)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{j}{2};1;1-\epsilon,\frac{\chi_p(\epsilon-1)}{1-\chi_p}\right)\right],\end{aligned}$$

where $F_1(a;b_1,b_2;c;z_1,z_2)$ is a hypergeometric function of two variables (*Appell* F_1) and

$$\epsilon = \frac{\chi_m}{\chi_p}. \qquad (3.8)$$

We used an integral representation for the F_1 in (3.7) [32]

$$F_1(a;b_1,b_2;c;z_1,z_2) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(c-a)} \int_0^1 x^{a-1}(1-x)^{c-a-1}(1-z_1x)^{-b_1}(1-z_2x)^{-b_2}dx.$$

The structure constants are given by the parameters κ, v which are eventually related to the conserved angular momentum J_1 and the worldsheet momentum p (see (2.14), (3.8)) by

$$J_1 = 2T\sqrt{\frac{1-v^2}{1-v^2\epsilon}}\left[\mathbf{K}(1-\epsilon)-\mathbf{E}(1-\epsilon)\right], \qquad (3.9)$$

$$p = 2v\sqrt{\frac{1-v^2\epsilon}{1-v^2}}\left[\frac{1}{v^2}\mathbf{\Pi}\left(1-\frac{1}{v^2}|1-\epsilon\right)-\mathbf{K}(1-\epsilon)\right]. \qquad (3.10)$$

Here $\mathbf{K}$, $\mathbf{E}$ and $\mathbf{\Pi}$ are the complete elliptic integrals of the first, second, and third kinds, respectively.

3.1.1 Leading finite-size corrections

Since J_1 and p define the heavy operators of the dual gauge theory, it is important to express the semiclassical structure constants $\mathcal{C}_{13}^{CP^1,d}(j)$ in terms of them. For given J_1 and

p, one can solve numerically (3.9) and (3.10) to find corresponding values of κ, v, which can be used to evaluate $\mathcal{C}_{13}^{CP^1,d}(j)$ from (3.6).

Explicit computations are possible for the case where J_1 is large but finite $J_1 \gg T$, equivalently, $\epsilon \ll 1$. We start by rewriting (2.14) in the following form

$$(1+\epsilon)\chi_p = 2 - (1+v^2)\kappa^2, \qquad \epsilon\chi_p^2 = (1-\kappa^2)(1-v^2\kappa^2). \tag{3.11}$$

Next, we use the small ϵ-expansions

$$\begin{aligned} \chi_p &= \chi_{p0} + (\chi_{p1} + \chi_{p2}\log\epsilon)\epsilon, \\ v &= v_0 + (v_1 + v_2\log\epsilon)\epsilon, \\ \kappa^2 &= 1 + W_1\epsilon. \end{aligned} \tag{3.12}$$

Replacing (3.12) into (3.11), one finds relations

$$\chi_{p0} = 1 - v_0^2, \quad \chi_{p1} = v_0(v_0 - v_0^3 - 2v_1), \quad \chi_{p2} = -2v_0v_2, \quad W_1 = -1 + v_0^2. \tag{3.13}$$

The expressions for v_0, v_1, v_2 in terms of the worldsheet momentum p can be found from (3.9) and (3.10)

$$\begin{aligned} v_0 &= \cos\frac{p}{2}, \\ v_1 &= \frac{1}{4}\sin^2\frac{p}{2}\cos\frac{p}{2}(1-\log 16), \\ v_2 &= \frac{1}{4}\sin^2\frac{p}{2}\cos\frac{p}{2}, \end{aligned} \tag{3.14}$$

along with the expansion parameter in terms of J_1 and p

$$\epsilon = 16\exp\left(-\frac{J_1}{T\sin\frac{p}{2}} - 2\right). \tag{3.15}$$

The case of $j = 0$. Let us begin with the simplest case $j = 0$, i.e. dilaton with zero momentum, which is just the Lagrangian. The $\mathcal{C}_{13}^{CP^1,d}(j)$ in (3.7) simplifies to

$$\mathcal{C}_{13}^{CP^1,d}(0) = \frac{8\sqrt{\pi}c_\Delta^d}{3\kappa\sqrt{\chi_p}}\left[\chi_p\mathbf{E}(1-\epsilon) - (1-\kappa^2)\mathbf{K}(1-\epsilon)\right]. \tag{3.16}$$

Replacing (3.14), (3.15) into the ϵ-expantion of (3.16), we obtain (for dilaton with zero momentum $\Delta_d = 4$)

$$\mathcal{C}_{13}^{CP^1,d}(0) \approx \frac{8c_\Delta^d}{3}\sin\frac{p}{2}\left[1 - 4\sin\frac{p}{2}\left(\sin\frac{p}{2} + \frac{J_1}{T}\right)\exp\left(-\frac{J_1}{T\sin\frac{p}{2}} - 2\right)\right]. \tag{3.17}$$

This is exactly same result as [30].

The case of j = even integer. In this case, the third arguments $-j/2$ of the *Appell* F_1 functions in (3.7) are negative integers. With the help of Mathematica, we have found that these functions can be expressed in terms of the elliptic integrals, $\mathbf{K}$ and $\mathbf{E}$. (The $j=0$ result analysed above belongs to this case too.) We list explicit functional relations for a few simple cases in the appendix.

Using small ϵ-expansion of the elliptic integrals, one can find the leading finite-size corrections of the HHL for any even j in principle. Here, we present explicit results for $j=2$ ($\Delta_d=6$) as an example:

$$\begin{aligned}\mathcal{C}_{13}^{CP^1,d}(2) &= \frac{\sqrt{\pi}c_\Delta^d\Gamma\left(\frac{\Delta_d}{2}\right)}{6\kappa\sqrt{\chi_p}\Gamma\left(\frac{\Delta_d+1}{2}\right)}\left[\mathbf{K}(1-\epsilon)\left(3\kappa^2+\chi_p^2\epsilon-3\right)-\chi_p\mathbf{E}(1-\epsilon)(3\kappa^2+2(\chi_p+\chi_p\epsilon-3))\right]\\ &\approx \frac{8c_d^\Delta}{45}\sin\frac{p}{2}\left[2+\cos p\right.\\ &\qquad \left.+\left(2\cos p+7\cos 2p-9-\frac{18J_1}{T}\left(\sin\frac{p}{2}+\frac{1}{3}\sin\frac{3p}{2}\right)\right)e^{-\frac{J1}{T\sin\frac{p}{2}}-2}\right].\end{aligned} \tag{3.18}$$

The case of j = odd integer. Since the Appell functions can not be written in terms of the elliptic integrals in this case, we express the F_1 in (3.7) using an infinite sum [31]

$$F_1(a;b_1,b_2;c;z_1,z_2)=\sum_{k=0}^{\infty}\frac{(a)_k(b_2)_k}{(c)_k k!}\,{}_2F_1\left(a+k;b_1;c+k;z_1\right)z_2^k,\quad (a)_k=\frac{\Gamma(a+k)}{\Gamma(a)}. \tag{3.19}$$

We can use small ϵ-expansions for the hypergeometric ${}_2F_1$ functions and resum afterward. The resulting structure constants (3.7) in the leading-order of ϵ are

$$\mathcal{C}_{13}^{CP^1,d}(j)\approx c_\Delta^d\frac{\sqrt{\pi}\Gamma\left(\frac{\Delta_d}{2}\right)}{2^{j-1}\Gamma\left(\frac{\Delta_d+1}{2}\right)}\sin\frac{p}{2}\cos^j\frac{p}{2}\left[{}_2F_1\left(\frac{1}{2};-\frac{j}{2};\frac{3}{2};-\tan^2\frac{p}{2}\right)+\mathcal{C}\,\epsilon\right] \tag{3.20}$$

$$\begin{aligned}\mathcal{C} &= \sec^j\frac{p}{2}-\frac{\pi^{3/2}\csc\frac{\pi j}{2}}{2\Gamma\left(1-\frac{j}{2}\right)\Gamma\left(\frac{1+j}{2}\right)}-\frac{1}{2}\left(\gamma+\log 4-\frac{J_1}{T}\csc\frac{p}{2}\right)\sec^j\frac{p}{2}\\ &\quad -\frac{1}{4}\,{}_2F_1\left(\frac{1}{2};-\frac{j}{2};\frac{3}{2};-\tan^2\frac{p}{2}\right)\sin\frac{p}{2}\left[(1+3j)\sin\frac{p}{2}+(1+j)\frac{J_1}{T}\right]\\ &\quad +\frac{\sqrt{\pi}}{4}\left[2\,{}_2F_1reg^{(0,0,1,0)}\left(\frac{1}{2};-\frac{j}{2};\frac{1}{2};-\tan^2\frac{p}{2}\right)+{}_2F_1reg^{(0,1,0,0)}\left(\frac{1}{2};-\frac{j}{2};\frac{1}{2};-\tan^2\frac{p}{2}\right)\right].\end{aligned} \tag{3.21}$$

Here γ is the Euler's constant and ${}_2F_1\,(a;b;c;z)$ the Gauss hypergeometric function,

$$\begin{aligned}{}_2F_1reg(a,b;c;z) &= \frac{1}{\Gamma(c)}\,{}_2F_1(a,b;c;z),\\ {}_2F_1reg^{(0,1,0,0)}\,(a;b;c;z) &= \frac{\partial}{\partial b}\,{}_2F_1reg\,(a;b;c;z)\,,\\ {}_2F_1reg^{(0,0,1,0)}\,(a;b;c;z) &= \frac{\partial}{\partial c}\,{}_2F_1reg\,(a;b;c;z)\,.\end{aligned}$$

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3.2 With the $R_t \times RP^3$ giant magnons

From (2.15), all combinations (a,b) in the dilaton vertex (3.1) are non-vanishing. However, we will focus on only $V_{13}^d(j)$ and $V_{24}^d(j)$ for which explicit expressions can be obtained. Working in the same way as the CP^1 case (see (3.4)), one derives

$$\partial_+ X_k \partial_- X^k = \frac{2(1-u^2)}{1-v^2}\left[\chi - \frac{1-\frac{1}{2}(1+v^2)\kappa^2}{1-u^2}\right],$$

which can be used to $V_{13}^d(j)$ and $V_{24}^d(j)$:

$$V_{13}^d(j) = \frac{2^{1-j}}{1-v^2}\left(\cosh\kappa\tau_e\right)^{-\Delta_d}\chi^j\left[(1-u^2)\chi-(1-\kappa^2)\right], \tag{3.22}$$

$$V_{24}^d(j) = \frac{2^{1-j}}{1-v^2}\left(\cosh\kappa\tau_e\right)^{-\Delta_d}(1-\chi)^j\left[(1-u^2)\chi-(1-\kappa^2)\right]. \tag{3.23}$$

By integrating $V_{13}^d(j)$ and $V_{24}^d(j)$ over the string worldsheet coordinates, we derive the corresponding structure constants $\mathcal{C}_{13}^{RP^3,d}$ again in terms of the F_1 and ${}_2F_1$:

$$\mathcal{C}_{13}^{RP^3,d}(j) = \frac{c_\Delta^d \pi^{3/2}\sqrt{1-u^2}}{2^{j-1}\kappa}\frac{\Gamma\left(\frac{\Delta_d}{2}\right)}{\Gamma\left(\frac{\Delta_d+1}{2}\right)}\chi_p^{j+1/2}\times \tag{3.24}$$
$$\times\left[{}_2F_1\left(\frac{1}{2};-\frac{1}{2}-j;1;1-\epsilon\right)-\frac{(1-\kappa^2)}{(1-u^2)\chi_p}\,{}_2F_1\left(\frac{1}{2};\frac{1}{2}-j;1;1-\epsilon\right)\right],$$

$$\mathcal{C}_{24}^{RP^3,d}(j) = \frac{c_\Delta^d \pi^{3/2}\sqrt{1-u^2}}{2^{j-1}\kappa}\frac{\Gamma\left(\frac{\Delta_d}{2}\right)}{\Gamma\left(\frac{\Delta_d+1}{2}\right)}\chi_p^{1/2}(1-\chi_p)^j\times \tag{3.25}$$
$$\times\left[F_1\left(\frac{1}{2};-\frac{1}{2},-j;1;1-\epsilon,\frac{\chi_p(\epsilon-1)}{1-\chi_p}\right)-\frac{(1-\kappa^2)}{(1-u^2)\chi_p}F_1\left(\frac{1}{2};\frac{1}{2},-j;1;1-\epsilon,\frac{\chi_p(\epsilon-1)}{1-\chi_p}\right)\right],$$

where χ_p is given by (2.22).

The parameters κ, u, v in (3.24), (3.25) are related to the conserved angular momenta J_1, J_2 and the worldsheet momentum p along with (2.22) by

$$J_1 = \frac{T\sqrt{\chi_p}}{\sqrt{1-u^2}}\left[\frac{1-v^2\kappa^2}{\chi_p}\mathbf{K}(1-\epsilon)-\mathbf{E}(1-\epsilon)\right],$$
$$J_2 = \frac{Tu\sqrt{\chi_p}}{\sqrt{1-u^2}}\mathbf{E}(1-\epsilon),$$
$$p = \frac{2v}{\sqrt{(1-u^2)\chi_p}}\left[\frac{\kappa^2}{1-\chi_p}\mathbf{\Pi}\left(-\frac{\chi_p(1-\epsilon)}{1-\chi_p}|1-\epsilon\right)-\mathbf{K}(1-\epsilon)\right].$$

For given J_1, J_2, p, one can find corresponding κ, u, v, with which the structure constants can be evaluated.

A few comments are in order. The structure constants (3.24) and (3.25) correspond to finite-size dyonic giant magnons living in the RP^3 subspace of CP^3. The case of finite-size giant magnons living in the RP^2 subspace can be obtained by setting $u=0$. For the infinite size case, one can take a limit of $\kappa = 1$ and $\epsilon = 0$. The above results reduce to the zero momentum dilaton with $j = 0$. Small ϵ-expansions for RP^3 are straightforward for these cases since they are either in terms of the hypergeometric ${}_2F_1$ functions or the Appell F_1 functions with special arguments which can be expressed in terms of the elliptic integrals. We will not present detailed expressions here.

4 HHL of primary scalar operators

The vertex for primary scalar operators is given by [12]

$$V_{ab}^{pr}(j) = \left(\frac{x^m x_m + z^2}{z}\right)^{-\Delta_{pr}} (z_a z_b)^j \left[z^{-2}\left(\partial_+ x_m \partial_- x^m - \partial_+ z \partial_- z\right) - \partial_+ X_k \partial_- X^k\right]. \quad (4.1)$$

The scaling dimension is $\Delta_{pr} = j$. This reduces for giant magnons (4.1) to

$$\left[\cosh\left(\kappa\tau_e\right)\right]^{-\Delta_{pr}} (z_a z_b)^j \left[\kappa^2\left(\frac{2}{\cosh^2 \kappa\tau_e} - 1\right) - \partial_+ X_k \partial_- X^k\right]. \quad (4.2)$$

In the case of the CP^1 giant magnons, we consider V_{13}^{pr} with

$$\begin{aligned}(z_1 z_3)^j &= \frac{1}{2^j}(1-\chi)^{j/2}, \\ \kappa^2 + \partial_+ X_k \partial_- X^k &= \frac{1}{2(1-v^2)}\left(\kappa^2 + \chi - 1\right).\end{aligned} \quad (4.3)$$

By using (4.2) and (4.3), we can integrate the vertex over the string worldsheet to obtain the corresponding structure constants as follows:

$$\begin{aligned}\mathcal{C}_{13}^{CP^1,pr}(j) = {}& c_\Delta^{pr} \frac{\pi^{3/2}}{2^j\kappa} \frac{\Gamma\left(\frac{\Delta_{pr}}{2}\right)}{\Gamma\left(\frac{\Delta_{pr}+2}{4}\right)} \chi_p^{-1/2}(1-\chi_p)^{j/2} \times \\ & \times \left\{\left[\frac{\kappa^2\Delta_{pr}}{\Delta_{pr}+1}(1-v^2) + (1-\kappa^2)\right] F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{j}{2}; 1; 1-\epsilon, \frac{\chi_p(\epsilon-1)}{1-\chi_p}\right)\right. \\ & \left. - \chi_p F_1\left(\frac{1}{2}; -\frac{1}{2}, -\frac{j}{2}; 1; 1-\epsilon, \frac{\chi_p(\epsilon-1)}{1-\chi_p}\right)\right\}.\end{aligned} \quad (4.4)$$

The Appell F_1 functions in $\mathcal{C}_{13}^{CP^1,pr}(j)$ have the same arguments as in $\mathcal{C}_{13}^{CP^1,d}(j)$ in (3.7). Therefore, a similar analysis can be done for the leading finite-size corrections for $\mathcal{C}_{13}^{CP^1,pr}(j)$. We present here $j = 2$ which is the simplest case of even integer ($j = 0$ is trivial) after converting F_1 functions into the elliptic functions:

$$\begin{aligned}&\mathcal{C}_{13}^{CP^1,pr}(2) \approx \frac{c_\Delta^{pr}}{6} \csc\frac{p}{2}\left\{2\frac{J_1}{T}\sin\frac{p}{2} - (1-\cos p)\right. \\ &\left. + \left[\cos 2p + 4\left(1+\frac{J_1^2}{T^2}\right)\cos p + 4\frac{J_1^2}{T^2} - 5 + 4\frac{J_1}{T}\left(\sin\frac{p}{2} + \sin\frac{3p}{2}\right)\right] \exp\left(-\frac{J_1}{T\sin\frac{p}{2}} - 2\right)\right\}.\end{aligned} \quad (4.5)$$

For the HHLs with two RP^3 giant magnons, we consider again two primary scalar operators V_{13}^{pr} and V_{24}^{pr} and the results are given by

$$\mathcal{C}_{13}^{RP^3,pr}(j) = c_\Delta^{pr}\frac{\pi^{3/2}\sqrt{1-u^2}}{2^{j-1}\kappa}\frac{\Gamma\left(\frac{\Delta_{pr}}{2}\right)}{\Gamma\left(\frac{\Delta_{pr}+2}{4}\right)}\chi_p^{j-1/2}\times \tag{4.6}$$
$$\times\left\{\left[\frac{\kappa^2\Delta_{pr}}{\Delta_{pr}+1}\frac{1-v^2}{1-u^2}+\frac{1-\kappa^2}{1-u^2}\right]{}_2F_1\left(\frac{1}{2};\frac{1}{2}-j;1;1-\epsilon\right)-\chi_p\ {}_2F_1\left(\frac{1}{2};-\frac{1}{2}-j;1;1-\epsilon\right)\right\},$$

$$\mathcal{C}_{24}^{RP^3,pr}(j) = c_\Delta^{pr}\frac{\pi^{3/2}\sqrt{1-u^2}}{2^{j-1}\kappa}\frac{\Gamma\left(\frac{\Delta_{pr}}{2}\right)}{\Gamma\left(\frac{\Delta_{pr}+2}{4}\right)}\chi_p^{-1/2}(1-\chi_p)^j\times \tag{4.7}$$
$$\times\left\{\left[\frac{\kappa^2\Delta_{pr}}{\Delta_{pr}+1}\frac{1-v^2}{1-u^2}+\frac{1-\kappa^2}{1-u^2}\right]F_1\left(\frac{1}{2};\frac{1}{2},-j;1;1-\epsilon,\frac{\chi_p(\epsilon-1)}{1-\chi_p}\right)\right.$$
$$\left.-\chi_p\ F_1\left(\frac{1}{2};-\frac{1}{2},-j;1;1-\epsilon,\frac{\chi_p(\epsilon-1)}{1-\chi_p}\right)\right\}.$$

These results are quite similar to those of the dilaton vertex in (3.24) and (3.25). As before, the case of RP^2 giant magnons can be obtained by taking $u=0$ limit. One can make small ϵ-expansion by either using series expansions of ${}_2F_1$ for V_{13}^{pr} or the Appell functions which can be expressed by elliptic integrals for V_{24}^{pr}.

5 HHL of singlet scalar operators with string levels

The vertex for singlet scalar operators is given by [12]

$$V_q = [\cosh(\kappa\tau_e)]^{-\Delta_q}\left(\partial_+X_k\partial_-X^k\right)^q, \tag{5.1}$$

where q is related to the string level n by $q=n+1$ and

$$\Delta_q = 2\left(\sqrt{(q-1)\sqrt{\lambda}+1-\frac{1}{2}q(q-1)}+1\right), \tag{5.2}$$

with the 't Hooft coupling λ.

Since the evaluation of $\partial_+X_k\partial_-X^k$ for the giant magnons have been given in previous sections, we just present the results here.

For the CP^1 case,

$$\mathcal{C}^{CP^1}(q) = \frac{c_\Delta^q\pi^{3/2}}{2^{q-1}(1-v^2)^{q-1}\kappa}\frac{\Gamma\left(\frac{\Delta_q}{2}\right)}{\Gamma\left(\frac{\Delta_q+1}{2}\right)}\chi_p^{-1/2}\times \tag{5.3}$$
$$\times\sum_{k=0}^{q}\frac{q!}{k!(q-k)!}\left[\frac{1}{2}(1+v^2)\kappa^2-1\right]^{q-k}\chi_p^k\ {}_2F_1\left(\frac{1}{2};\frac{1}{2}-k;1;1-\epsilon\right),$$

For the RP^3 case,

$$\mathcal{C}^{RP^3}(q) = \frac{c_\Delta^q \pi^{3/2}(1-v^2)}{\sqrt{(1-u^2)\kappa^2}} \frac{\Gamma\left(\frac{\Delta_q}{2}\right)}{\Gamma\left(\frac{\Delta_q+1}{2}\right)} \left[\frac{2(1-u^2)}{1-v^2}\right]^q \chi_p^{-1/2} \times \\ \times \sum_{k=0}^{q} \frac{q!}{k!(q-k)!} \left[\frac{\frac{1}{2}(1+v^2)\kappa^2 - 1}{1-u^2}\right]^{q-k} \chi_p^k \, {}_2F_1\left(\frac{1}{2}; \frac{1}{2} - k; 1; 1-\epsilon\right). \tag{5.4}$$

Again, the RP^2 giant magnons correspond to $u = 0$.

For concrete results, we present small ϵ-expansions for the CP^1 giant magnon with a few lower levels as follows:

$q = 1$ (level $n = 0$)

$$\mathcal{C}^{CP^1}(1) = \frac{c_\Delta^q \sqrt{\pi}\, \Gamma\left(\frac{\Delta_q}{2}\right)}{\Gamma\left(\frac{\Delta_q+1}{2}\right)} \left\{\sin\frac{p}{2} - \frac{J_1}{2T} - \left[\frac{J_1}{T}\left(5 - \cos p + 2\frac{J_1}{T}\csc\frac{p}{2}\right) \right.\right. \\ \left.\left. -2\sin\frac{p}{2}\left(1 + \cos p + \frac{J_1^2}{T^2}\right)\right] \exp\left(-\frac{J_1}{T\sin\frac{p}{2}} - 2\right)\right\}, \tag{5.5}$$

$q = 2$ (level $n = 1$)

$$\mathcal{C}^{CP^1}(2) = \frac{c_\Delta^q \sqrt{\pi}\, \Gamma\left(\frac{\Delta_q}{2}\right)}{24\, \Gamma\left(\frac{\Delta_q+1}{2}\right)} \left\{3\frac{J_1}{T} - 2\sin\frac{p}{2} + 2\left[\frac{J_1}{2T}\left(31 + 13\cos p + 6\frac{J_1}{2T}\csc\frac{p}{2}\right) \right.\right. \\ \left.\left. -2\left(33 + 5\cos p + 3\frac{J_1^2}{T^2}\right)\sin\frac{p}{2}\right] \exp\left(-\frac{J_1}{T\sin\frac{p}{2}} - 2\right)\right\}, \tag{5.6}$$

$q = 3$ (level $n = 2$)

$$\mathcal{C}^{CP^1}(3) = \frac{c_\Delta^q \sqrt{\pi}\, \Gamma\left(\frac{\Delta_q}{2}\right)}{480\, \Gamma\left(\frac{\Delta_q+1}{2}\right)} \left\{38\sin\frac{p}{2} - 15\frac{J_1}{T}\left[12\left(49 + 57\cos p - 5\frac{J_1^2}{T^2}\cot^2\frac{p}{2}\right) \right.\right. \\ \left.\left. -2\frac{J_1}{T}(187 + 97\cos p)\csc\frac{p}{2}\right] \exp\left(-\frac{J_1}{T\sin\frac{p}{2}} - 2\right)\right\}. \tag{5.7}$$

6 Concluding remarks

We obtained here some semiclassical normalized structure constants for strings in $AdS_4 \times CP^3$ dual to $\mathcal{N} = 6$ super Chern-Simons-matter theory in three space-time dimensions (ABJM theory). We considered the cases where the two "heavy" string states are finite-size giant magnons living in the $R_t \times CP^1$, $R_t \times RP^2$ and $R_t \times RP^3$ subspaces and the "light" states are the dilaton operators with non-zero momentum, primary scalar operators, and singlet scalar operators on higher string levels.

Our results can be compared with other computations based on integrability. One of them is to formulate the HHL structure constants as diagonal form factors of the light operators with respect to the on-shell particle states corresponding to the heavy operators [13].

This approach is particularly useful to understand the *finite-size* corrections in the HHL in terms of the underlying integrability structure like the world-sheet S-matrix. In this sense, various HHL functions and their finite-size corrections which we have analysed in this paper can be used to understand new aspects of integrability in the planar limit of AdS_4/CFT_3.

Acknowledgments

This work is supported in part by the Brain Pool program 171S-1-3-1765 from KOFST and by National Research Foundation of Korea (NRF) grant (NRF-2016R1D1A1B02007258) (CA) and the NSF grant DFNI T02/6 (PB). CA also thanks ICTP for a support through Senior Associates program.

A Appell F_1 functions with special arguments

We need to evaluate Appell F_1 functions $F_1(1/2;\pm 1/2,-j;1;x,y)$. Using Mathematica, we have found their relations to the elliptic functions $\mathbf{E},\mathbf{K}$:

$$F_1\left(\frac{1}{2};\pm\frac{1}{2},-j;1;x,y\right)=e_j^{\pm}(x,y)\mathbf{E}(x)+k_j^{\pm}(x,y)\mathbf{K}(x),$$

where the coefficients $e_j^{\pm}$ and $k_j^{\pm}$ $(j=1,2)$ are given by

$$\begin{aligned}
e_1^+(x,y)&=\frac{2y}{\pi x},\quad e_1^-(x,y)=\frac{2(y+3x-6xy)}{3\pi x},\quad k_1^+(x,y)=\frac{2(x-y)}{\pi x},\quad k_1^-(x,y)=\frac{2y(x-1)}{3\pi x},\\
e_2^+(x,y)&=\frac{4y(3x-y-xy)}{3\pi x^2},\quad e_2^-(x,y)=\frac{2(10xy-3xy^2-2y^2+15x^2-20yx^2+8x^2y^2)}{15\pi x^2},\\
k_2^+(x,y)&=\frac{2(3x^2-6x+2y^2+xy^2)}{3\pi x^2},\qquad k_2^-(x,y)=\frac{2y(x-1)(y-5x+2xy)}{15\pi x^2}.
\end{aligned}$$

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