Finite-size effects of membranes on $AdS_4 \times S_7$

Changrim Ahn and P. Bozhilov[*]

Department of Physics, Ewha Womans University,
DaeHyun 11-1, Seoul 120-750, South Korea
E-mail: `ahn@ewha.ac.kr`, `bozhilov@inrne.bas.bg`

ABSTRACT: We consider semi-classical solutions of membranes on the $AdS_4 \times S^7$ background. This is supposed to be dual to the $\mathcal{N} = 6$ super Chern-Simons theory with $k = 1$ in a planar limit recently proposed by Aharony, Bergmann, Jafferis, and Maldacena (ABJM). We have identified giant magnon and single spike states on the membrane by reducing them to the Neumamm - Rosochatius integrable system. We also connect these to the complex sine-Gordon integrable model. Based on this approach, we find finite-size membrane solutions and obtain their images in the complex sine-Gordon system along with the leading finite-size corrections to the energy-charge relations.

KEYWORDS: AdS-CFT Correspondence, M-Theory.

[*]On leave from Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Bulgaria.

Contents

1. Introduction

After many interesting developments in the duality between type IIB string theory on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ super Yang-Mills theory, the AdS/CFT correspondence [1–3] is now being extended into the AdS_4/CFT_3. The first three-dimensional conformal field theory, consistent with all known symmetries of M2-branes was found in [4]. It is invariant under 16 supersymmetries, possesses SO(8) R-symmetry and is conformally invariant at the classical level. In this model, the gauge field is nonpropagating. More recent proposal for the worldvolume theory of multiple M2-branes uses a Lorentzian three algebra (constructed from ordinary Lie algebra) [5–7]. There are more related developments recently [8–11]. An alternative proposal for the theory of multiple M2-branes was made by ABJM which is $\mathcal{N} = 6$ super Chern-Simons theory with SU(N) × SU(N) gauge symmetry and level k [12]. In the limit $N, k \to \infty$ with a fixed value of 't Hooft coupling $\lambda = N/k$, this theory is claimed to be dual to the type IIA superstring theory on $AdS_4 \times CP^3$. For a small λ, a leading two-loop perturbation calculation has been studied and a new integrable structure has been discovered [13, 14]. This motivates efforts to discover classical integrability in string theory side. Indeed, BMN-like states [15], integrability [16, 17], and giant magnon state and its finite-size effect [18, 19] in the $k \to \infty$ limit where the target space becomes $AdS_4 \times CP_3$ have been reported.

    With these developments, it is interesting to consider dual to M-theory on $AdS_4 \times S^7$ in $\lambda \gg 1$ limit at $k = 1$ by considering membranes on $AdS_4 \times S^7$. It is already known

that there exist M2-brane configurations on $AdS_4 \times S^7$, which have properties, similar to some string solutions on $AdS_5 \times S^5$. In particular, some of them have description in terms of the Neumamm - Rosochatius (NR) integrable system [20]. The NR system has been proposed for the string theory on $AdS_5 \times S^5$ in [21−23]. It was also established that they can reproduce the continuous limit of the integrable SU(2) spin chain [24]. (See also [25].) Besides, giant magnon (GM) and single spike (SS) like energy-charge relations have been found [26−28]. It is interesting if the above achievements can be extended to include other string analogies. One possible task is to discover membrane configurations which can be related to the complex sine-Gordon (CSG) integrable system. Recently, the finite-size string corrections are actively investigated as a new window for the AdS/CFT correspondence [29−39]. That is why, another direction of research can be to try to find analogous finite-size corrections from M2-branes on $AdS_4 \times S^7$. In this paper, we extend our string results obtained in [39] to the M2-brane case. Namely, we use the reduction of the M2-brane dynamics to the one of the NR integrable model, to map *all* membrane solutions described by this dynamical system onto solutions of the CSG integrable model. In the framework of this NR approach, we find finite size corrections to the membrane energy-charge relations.

The article is organized as follows. In section 2 we introduce the partially gauge fixed M2-brane action and constraints. After reducing to the $R_t \times S^7$ subspace, we propose membrane embedding coordinates appropriate for our purposes. In section 3 we describe how the NR integrable system arises from the M2-brane. In section 4 we find relations between the parameters of the membrane solutions described by this dynamical system and the parameters in the corresponding solutions of the CSG integrable model. GM and SS like solutions are considered as examples. In section 5 we describe finite size membrane solution, its image in the complex sine-Gordon system, and the leading corrections to the energy-charge relations analogous to the GM and SS strings on $R_t \times S^3$. We conclude the paper with some comments in section 6.

2. Membranes on $AdS_4 \times S^7$

We begin with the following membrane action

$$S = \int d^3\xi \left\{ \frac{1}{4\lambda^0} \Big[G_{00} - 2\lambda^j G_{0j} + \lambda^i \lambda^j G_{ij} - \left(2\lambda^0 T_2 \right)^2 \det G_{ij} \Big] + T_2 C_{012} \right\}, \quad (2.1)$$

where

$$G_{mn} = g_{MN}(X)\partial_m X^M \partial_n X^N, \qquad C_{012} = c_{MNP}(X)\partial_0 X^M \partial_1 X^N \partial_2 X^P,$$
$$\partial_m = \partial/\partial\xi^m, \qquad m = (0,i) = (0,1,2),$$
$$(\xi^0, \xi^1, \xi^2) = (\tau, \sigma_1, \sigma_2), \qquad M = (0,1,\ldots,10),$$

are the fields induced on the membrane worldvolume from the background metric g_{MN} and the background 3-form gauge field c_{MNP}, λ^m are Lagrange multipliers, $x^M = X^M(\xi)$ are the membrane embedding coordinates, and T_2 is its tension. As shown in [40], the above

action is classically equivalent to the Nambu-Goto type action

$$S^{\text{NG}} = -T_2 \int d^3\xi \left(\sqrt{-\det G_{mn}} - \frac{1}{6}\varepsilon^{mnp}\partial_m X^M \partial_n X^N \partial_p X^P c_{MNP} \right)$$

and to the Polyakov type action

$$S^P = -\frac{T_2}{2} \int d^3\xi \left[\sqrt{-\gamma}\left(\gamma^{mn} G_{mn} - 1\right) - \frac{1}{3}\varepsilon^{mnp}\partial_m X^M \partial_n X^N \partial_p X^P c_{MNP} \right],$$

where γ^{mn} is the auxiliary worldvolume metric and $\gamma = \det \gamma_{mn}$. In addition, the action (2.1) gives a *unified* description for the tensile and tensionless membranes.

The equations of motion for the Lagrange multipliers λ^m generate the constraints

$$G_{00} - 2\lambda^j G_{0j} + \lambda^i \lambda^j G_{ij} + \left(2\lambda^0 T_2\right)^2 \det G_{ij} = 0, \tag{2.2}$$

$$G_{0j} - \lambda^i G_{ij} = 0. \tag{2.3}$$

Further on, we will work in diagonal worldvolume gauge $\lambda^i = 0$, in which the action (2.1) and the constraints (2.2), (2.3) simplify to

$$S_M = \int d^3\xi \mathcal{L}_M = \int d^3\xi \left\{ \frac{1}{4\lambda^0}\Big[G_{00} - \left(2\lambda^0 T_2\right)^2 \det G_{ij} \Big] + T_2 C_{012} \right\}, \tag{2.4}$$

$$G_{00} + \left(2\lambda^0 T_2\right)^2 \det G_{ij} = 0, \tag{2.5}$$

$$G_{0i} = 0. \tag{2.6}$$

Let us note that the action (2.4) and the constraints (2.5), (2.6) *coincide* with the usually used gauge fixed Polyakov type action and constraints after the following identification of the parameters $2\lambda^0 T_2 = L = const$ (see for instance [41]).

Searching for membrane configurations in $AdS_4 \times S^7$, which correspond to the Neumann or Neumann-Rosochatius integrable systems, we should first eliminate the membrane interaction with the background 3-form field on AdS_4, to ensure more close analogy with the strings on $AdS_5 \times S^5$. To make our choice, let us write down the background. It can be parameterized as follows

$$ds^2 = \mathcal{R}^2 \left[-\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho \left(d\alpha^2 + \sin^2 \alpha d\beta^2 \right) + 4d\Omega_7^2 \right],$$
$$c_{(3)} = \mathcal{R}^3 \sinh^3 \rho \sin \alpha dt \wedge d\alpha \wedge d\beta.$$

Since we want the membrane to have nonzero conserved energy on AdS, the simplest choice for which the interaction with the $c_{(3)}$ field disappears, is to fix the coordinates ρ, α and β: $\rho = 0$, $\alpha, \beta = constants$. Thus, we restrict our considerations to membranes moving on the $R_t \times S^7$ subspace of $AdS_4 \times S^7$ with metric

$$ds^2_{\text{sub}} = \mathcal{R}^2 \left\{ -dt^2 + 4 \left[d\psi_1^2 + \cos^2 \psi_1 d\varphi_1^2 \right. \right.$$
$$\left. \left. + \sin^2 \psi_1 \left(d\psi_2^2 + \cos^2 \psi_2 d\varphi_2^2 + \sin^2 \psi_2 \left(d\psi_3^2 + \cos^2 \psi_3 d\varphi_3^2 + \sin^2 \psi_3 d\varphi_4^2 \right) \right) \right] \right\}.$$

The membrane embedding into $R_t \times S^7$, appropriate for our purposes, is

$$Z_0 = \mathcal{R}e^{it(\xi^m)}, \qquad W_a = 2\mathcal{R}r_a(\xi^m)e^{i\varphi_a(\xi^m)}, \qquad a = (1, 2, 3, 4), \tag{2.7}$$

where r_a are real functions of ξ^m, while φ_a are the isometric coordinates on which the background metric does not depend. The four complex coordinates W_a are restricted by the real embedding condition

$$\delta_{ab}W_a\bar{W}_b - (2\mathcal{R})^2 = 0, \quad \text{or} \quad \delta_{ab}r_ar_b - 1 = 0. \tag{2.8}$$

The coordinates r_a are connected to the initial coordinates, on which the background depends, through the equalities

$$r_1 = \cos\psi_1, \qquad\qquad\qquad r_2 = \sin\psi_1\cos\psi_2,$$
$$r_3 = \sin\psi_1\sin\psi_2\cos\psi_3, \qquad\qquad r_4 = \sin\psi_1\sin\psi_2\sin\psi_3.$$

For the embedding described above, the induced metric is given by

$$\begin{aligned}
G_{mn} &= -\partial_{(m}Z_0\partial_{n)}\bar{Z}_0 + \delta_{ab}\partial_{(m}W_a\partial_{n)}\bar{W}_b \\
&= \mathcal{R}^2\left[-\partial_m t\partial_n t + 4\sum_{a=1}^4\left(\partial_m r_a\partial_n r_a + r_a^2\partial_m\varphi_a\partial_n\varphi_a\right)\right].
\end{aligned} \tag{2.9}$$

Correspondingly, the membrane Lagrangian becomes

$$\mathcal{L} = \mathcal{L}_M + \Lambda_M(\delta_{ab}r_ar_b - 1),$$

where Λ_M is a Lagrange multiplier.

3. NR integrable system from M2-brane

Let us consider the following particular case of the above membrane embedding [20]

$$\begin{aligned}
Z_0 &= \mathcal{R}e^{i\kappa\tau}, & W_a &= 2\mathcal{R}r_a(\xi,\eta)e^{i[\omega_a\tau + \mu_a(\xi,\eta)]}, \\
\xi &= \alpha\sigma_1 + \beta\tau, & \eta &= \gamma\sigma_2 + \delta\tau,
\end{aligned} \tag{3.1}$$

which implies

$$t = \kappa\tau, \quad \varphi_a(\xi^m) = \varphi_a(\tau,\sigma_1,\sigma_2) = \omega_a\tau + \mu_a(\xi,\eta). \tag{3.2}$$

Here κ, ω_a, α, β, γ, δ are parameters. For this ansatz, the membrane Lagrangian takes the form ($\partial_\xi = \partial/\partial\xi$, $\partial_\eta = \partial/\partial\eta$)

$$\begin{aligned}
\mathcal{L} = -\frac{\mathcal{R}^2}{\lambda^0}\Bigg\{ &\left(4\lambda^0 T_2\mathcal{R}\alpha\gamma\right)^2\sum_{a<b=1}^4\Big[(\partial_\xi r_a\partial_\eta r_b - \partial_\eta r_a\partial_\xi r_b)^2 \\
&\quad + (\partial_\xi r_a\partial_\eta\mu_b - \partial_\eta r_a\partial_\xi\mu_b)^2 r_b^2 + (\partial_\xi\mu_a\partial_\eta r_b - \partial_\eta\mu_a\partial_\xi r_b)^2 r_a^2 \\
&\quad + (\partial_\xi\mu_a\partial_\eta\mu_b - \partial_\eta\mu_a\partial_\xi\mu_b)^2 r_a^2 r_b^2\Big] \\
&+ \sum_{a=1}^4\left[\left(4\lambda^0 T_2\mathcal{R}\alpha\gamma\right)^2(\partial_\xi r_a\partial_\eta\mu_a - \partial_\eta r_a\partial_\xi\mu_a)^2 - (\beta\partial_\xi\mu_a + \delta\partial_\eta\mu_a + \omega_a)^2\right] r_a^2 \\
&- \sum_{a=1}^4(\beta\partial_\xi r_a + \delta\partial_\eta r_a)^2 + (\kappa/2)^2\Bigg\} + \Lambda_M\left(\sum_{a=1}^4 r_a^2 - 1\right).
\end{aligned}$$

In order to reduce the above Lagrangian to the NR one, we make the following choice

$$r_1 = r_1(\xi), \qquad r_2 = r_2(\xi), \qquad \omega_3 = \pm\omega_4 = \omega,$$
$$r_3 = r_3(\eta) = r_0 \sin\eta, \qquad r_4 = r_4(\eta) = r_0 \cos\eta, \qquad r_0 < 1, \qquad (3.3)$$
$$\mu_1 = \mu_1(\xi), \qquad \mu_2 = \mu_2(\xi), \qquad \mu_3, \mu_4 = 0,$$

and receive (prime is used for $d/d\xi$)

$$\mathcal{L} = -\frac{\mathcal{R}^2}{\lambda^0} \left\{ \sum_{a=1}^{2} \left[(\tilde{A}^2 - \beta^2) r_a'^2 + (\tilde{A}^2 - \beta^2) r_a^2 \left(\mu_a' - \frac{\beta\omega_a}{\tilde{A}^2 - \beta^2} \right)^2 - \frac{\tilde{A}^2}{\tilde{A}^2 - \beta^2} \omega_a^2 r_a^2 \right] \right.$$
$$\left. + (\kappa/2)^2 - r_0^2(\omega^2 + \delta^2) \right\} + \Lambda_M \left[\sum_{a=1}^{2} r_a^2 - (1 - r_0^2) \right],$$

where $\tilde{A}^2 \equiv \left(4\lambda^0 T_2 \mathcal{R}\alpha\gamma r_0 \right)^2$. A single time integration of the equations of motion for μ_a following from the above Lagrangian gives

$$\mu_a' = \frac{1}{\tilde{A}^2 - \beta^2} \left(\frac{C_a}{r_a^2} + \beta\omega_a \right), \qquad (3.4)$$

where C_a are arbitrary constants. Taking this into account, one obtains the following effective Lagrangian for the coordinates $r_a(\xi)$

$$L_{\mathrm{NR}} = \sum_{a=1}^{2} \left[(\tilde{A}^2 - \beta^2) r_a'^2 - \frac{1}{\tilde{A}^2 - \beta^2} \left(\frac{C_a^2}{r_a^2} + \tilde{A}^2 \omega_a^2 r_a^2 \right) \right] + \Lambda_M \left[\sum_{a=1}^{2} r_a^2 - (1 - r_0^2) \right].$$

This Lagrangian, in full analogy with the string considerations [23], corresponds to particular case of the n-dimensional NR integrable system. For $C_a = 0$ one obtains the Neumann integrable system, which in the case at hand describes two-dimensional harmonic oscillator, constrained to remain on a circle of radius $\sqrt{1 - r_0^2}$.

Let us write down the three constraints (2.5), (2.6) for the present case. To achieve more close correspondence with the string on $AdS_5 \times S^5$, we want the third one, $G_{02} = 0$, to be identically satisfied. To this end, since $G_{02} \sim r_0^2 \gamma\delta$, we set $\delta = 0$, i.e. $\eta = \gamma\sigma_2$. Then, the first two constraints give the conserved Hamiltonian H_{NR} and a relation between the parameters:

$$H_{\mathrm{NR}} = \sum_{a=1}^{2} \left[(\tilde{A}^2 - \beta^2) r_a'^2 + \frac{1}{\tilde{A}^2 - \beta^2} \left(\frac{C_a^2}{r_a^2} + \tilde{A}^2 \omega_a^2 r_a^2 \right) \right] = \frac{\tilde{A}^2 + \beta^2}{\tilde{A}^2 - \beta^2} \left[(\kappa/2)^2 - (r_0\omega)^2 \right],$$

$$\sum_{a=1}^{2} \omega_a C_a + \beta \left[(\kappa/2)^2 - (r_0\omega)^2 \right] = 0. \qquad (3.5)$$

For closed membranes, r_a and μ_a satisfy the following periodicity conditions

$$r_a(\xi + 2\pi\alpha) = r_a(\xi), \quad \mu_a(\xi + 2\pi\alpha) = \mu_a(\xi) + 2\pi n_a, \qquad (3.6)$$

where n_a are integer winding numbers.

In the case at hand, the background metric does not depend on t and φ_a. Therefore, the corresponding conserved quantities are the membrane energy E and four angular momenta J_a, given as spatial integrals of the conjugated to these coordinates momentum densities

$$E = -\int d^2\sigma \frac{\partial \mathcal{L}}{\partial(\partial_0 t)}, \quad J_a = \int d^2\sigma \frac{\partial \mathcal{L}}{\partial(\partial_0 \varphi_a)}, \quad a = 1, 2, 3, 4.$$

E and J_a can be computed by using the expression (2.9) for the induced metric and the ansats (3.1), (3.2). In order to reproduce the string case, we set $\omega = 0$, and thus $J_3 = J_4 = 0$. The energy and the other two angular momenta are given by

$$E = \frac{\pi \mathcal{R}^2 \kappa}{\lambda^0 \alpha} \int d\xi, \quad J_a = \frac{\pi (2\mathcal{R})^2}{\lambda^0 \alpha (\tilde{A}^2 - \beta^2)} \int d\xi \left(\beta C_a + \tilde{A}^2 \omega_a r_a^2 \right), \quad a = 1, 2. \quad (3.7)$$

From here, by using the constraints (3.5), one obtains the energy-charge relation

$$\frac{4}{\tilde{A}^2 - \beta^2} \left[\tilde{A}^2 (1 - r_0^2) + \beta \sum_{a=1}^{2} \frac{C_a}{\omega_a} \right] \frac{E}{\kappa} = \sum_{a=1}^{2} \frac{J_a}{\omega_a}.$$

The corresponding result for strings on $AdS_5 \times S^5$ in conformal gauge is [23]

$$\frac{1}{\alpha^2 - \beta^2} \left(\alpha^2 + \beta \sum_a \frac{C_a}{\omega_a} \right) \frac{E}{\kappa} = \sum_a \frac{J_a}{\omega_a}.$$

Obviously, the above membrane and string energy-charge relations are very similar.

We would like to identically satisfy the embedding condition

$$\sum_{a=1}^{2} r_a^2 - (1 - r_0^2) = 0.$$

To this end we set

$$r_1(\xi) = \sqrt{1 - r_0^2} \sin \theta(\xi), \quad r_2(\xi) = \sqrt{1 - r_0^2} \cos \theta(\xi).$$

Then from (3.5) one finds

$$\theta' = \frac{\pm 1}{\tilde{A}^2 - \beta^2} \left[(\tilde{A}^2 + \beta^2) \tilde{\kappa}^2 - \frac{\tilde{C}_1^2}{\sin^2 \theta} - \frac{\tilde{C}_2^2}{\cos^2 \theta} - \tilde{A}^2 \left(\omega_1^2 \sin^2 \theta + \omega_2^2 \cos^2 \theta \right) \right]^{1/2}, \quad (3.8)$$

$$\sum_{a=1}^{2} \omega_a \tilde{C}_a + \beta \tilde{\kappa}^2 = 0, \quad \tilde{\kappa}^2 = \frac{(\kappa/2)^2}{1 - r_0^2}, \quad \tilde{C}_a^2 = \frac{C_a^2}{(1 - r_0^2)^2}.$$

By replacing the solution for $\theta(\xi)$ received from (3.8) into (3.4), one obtains the solutions for μ_a:

$$\mu_1 = \frac{1}{\tilde{A}^2 - \beta^2} \left(\tilde{C}_1 \int \frac{d\xi}{\sin^2 \theta} + \beta \omega_1 \xi \right), \quad \mu_2 = \frac{1}{\tilde{A}^2 - \beta^2} \left(\tilde{C}_2 \int \frac{d\xi}{\cos^2 \theta} + \beta \omega_2 \xi \right). \quad (3.9)$$

4. Relationship between NR and CSG systems

The CSG system is defined by the Lagrangian

$$\mathcal{L}(\psi) = \frac{\eta^{ab}\partial_a\bar{\psi}\partial_b\psi}{1 - \bar{\psi}\psi} + M^2\bar{\psi}\psi, \quad \eta^{ab} = diag(-1, 1),$$

which give the equation of motion

$$\partial_a\partial^a\psi + \bar{\psi}\frac{\partial_a\psi\partial^a\psi}{1 - \bar{\psi}\psi} - M^2(1 - \bar{\psi}\psi)\psi = 0.$$

If we represent ψ in the form

$$\psi = \sin(\phi/2)\exp(i\chi/2),$$

the Lagrangian can be expressed as

$$\mathcal{L}(\phi, \chi) = \frac{1}{4}\left[\partial_a\phi\partial^a\phi + \tan^2(\phi/2)\partial_a\chi\partial^a\chi + (2M)^2\sin^2(\phi/2)\right],$$

while the equations of motion take the form

$$\partial_a\partial^a\phi - \frac{1}{2}\frac{\sin(\phi/2)}{\cos^3(\phi/2)}\partial_a\chi\partial^a\chi - M^2\sin\phi = 0, \tag{4.1}$$

$$\partial_a\partial^a\chi + \frac{2}{\sin\phi}\partial_a\phi\partial^a\chi = 0. \tag{4.2}$$

The SG system corresponds to a particular case of $\chi = 0$.

To relate the NR with CSG integrable system, we consider the case

$$\phi = \phi(\xi), \quad \chi = A\sigma_1 + B\tau + \tilde{\chi}(\xi),$$

where ϕ and $\tilde{\chi}$ depend on only one variable $\xi = \alpha\sigma_1 + \beta\tau$ in the same way as in our NR ansatz (3.3). Then the equations of motion (4.1), (4.2) reduce to

$$\phi'' - \frac{1}{2}\frac{\sin(\phi/2)}{\cos^3(\phi/2)}\left[\tilde{\chi}'^2 + 2\frac{A\alpha - B\beta}{\alpha^2 - \beta^2}\tilde{\chi}' + \frac{A^2 - B^2}{\alpha^2 - \beta^2}\right] - \frac{M^2\sin\phi}{\alpha^2 - \beta^2} = 0, \tag{4.3}$$

$$\tilde{\chi}'' + \frac{2\phi'}{\sin\phi}\left(\tilde{\chi}' + \frac{A\alpha - B\beta}{\alpha^2 - \beta^2}\right) = 0. \tag{4.4}$$

We further restrict ourselves to the case of $A\alpha = B\beta$. A trivial solution of eq. (4.4) is $\tilde{\chi} = constant$, which corresponds to the solutions of the CSG equations considered in [42, 43] for a GM string on $R_t \times S^3$. More nontrivial solution of (4.4) is

$$\tilde{\chi} = C_\chi \int \frac{d\xi}{\tan^2(\phi/2)}. \tag{4.5}$$

The replacement of the above into (4.3) gives

$$\phi'' = \frac{M^2\sin\phi}{\alpha^2 - \beta^2} + \frac{1}{2}\left[C_\chi^2\frac{\cos(\phi/2)}{\sin^3(\phi/2)} - \frac{A^2}{\beta^2}\frac{\sin(\phi/2)}{\cos^3(\phi/2)}\right]. \tag{4.6}$$

Integrating once, we obtain

$$\phi' = \pm \left[\left(C_\phi - \frac{2M^2}{\alpha^2 - \beta^2} \right) + \frac{4M^2}{\alpha^2 - \beta^2} \sin^2(\phi/2) - \frac{A^2/\beta^2}{1 - \sin^2(\phi/2)} - \frac{C_\chi^2}{\sin^2(\phi/2)} \right]^{1/2} \quad (4.7)$$
$$\equiv \pm\Phi(\phi),$$

from which we get

$$\xi(\phi) = \pm \int \frac{d\phi}{\Phi(\phi)}, \qquad \chi(\phi) = \frac{A}{\beta}\left(\beta\sigma + \alpha\tau\right) \pm C_\chi \int \frac{d\phi}{\tan^2(\phi/2)\Phi(\phi)}.$$

All these solve the CSG system for the considered particular case. It is clear from (4.7) that the expression inside the square root must be positive.

After the reduction of our membrane configuration to a NR-type integrable system, we can establish relationship between its solutions and the solutions of the reduced CSG system, as described above. With this aim, we make the following identification

$$\sin^2(\phi/2) \equiv \frac{\sqrt{-G}}{\tilde{K}^2}, \tag{4.8}$$

where G is the determinant of the metric G_{mn} induced on the membrane worldvolume, computed on the constraints (2.5), (2.6), and $\tilde{K}^2$ is a parameter. For the NR system obtained from the M2-brane, $\sqrt{-G}$ is given by

$$\sqrt{-G} = Q^2 \frac{\mathcal{R}^2 \tilde{A}^2}{\tilde{A}^2 - \beta^2} \left[(\tilde{\kappa}^2 - \omega_1^2) + (\omega_1^2 - \omega_2^2)\cos^2\theta \right], \quad Q^2 = \frac{1 - r_0^2}{(\lambda^0 T_2)^2}. \tag{4.9}$$

To relate the parameters in the solutions of the NR and CSG integrable systems, we put (4.8), (4.9) into (4.7) and receive

$$\tilde{K}^2 = (Q\mathcal{R}M)^2 \left(\frac{1 - \tilde{A}^2/\beta^2}{1 - \alpha^2/\beta^2} \right) \equiv (Q\mathcal{R})^2 \tilde{M}^2,$$

$$C_\phi = \frac{2}{\tilde{A}^2 - \beta^2} \left[3\tilde{M}^2 - 2\left(\tilde{\kappa}^2 + Y - \Omega\right) \right],$$

$$\frac{1}{4}\tilde{M}^4(\tilde{A}^2 - \beta^2)\frac{A^2}{\beta^2} = \tilde{M}^4\left(\tilde{M}^2 - \tilde{\kappa}^2 + \Omega\right) \tag{4.10}$$

$$- Y\left[\tilde{M}^4 + \left(\tilde{M}^2 - Y\right)Y - \left(2\tilde{M}^2 - Y\right)\left(\tilde{\kappa}^2 - \Omega\right) \right]$$

$$- \frac{(\omega_1^2 - \omega_2^2)}{\omega_1^2\left(1 - \frac{\beta^2}{\tilde{A}^2}\right)^3} \left\{ \left[\tilde{M}^2\left(1 - \frac{\beta^2}{\tilde{A}^2}\right) - \tilde{\kappa}^2 \right](\omega_1^2 - \omega_2^2)\hat{C}_2^2 \right.$$

$$\left. - \left[\tilde{M}^2\left(1 - \frac{\beta^2}{\tilde{A}^2}\right) - (\tilde{\kappa}^2 - \omega_1^2) \right]\left[2\frac{\beta}{\tilde{A}}\omega_2\tilde{\kappa}^2\hat{C}_2 + (\tilde{\kappa}^2 - \omega_1^2)\left(\frac{\beta^2}{\tilde{A}^2}\tilde{\kappa}^2 - \omega_1^2\right) \right] \right\},$$

$$\frac{1}{4}\tilde{M}^4(\tilde{A}^2 - \beta^2)C_\chi^2 = Y^2\left(Y + \Omega - \tilde{\kappa}^2\right) + \frac{(\omega_1^2 - \omega_2^2)}{\omega_1^2\left(1 - \frac{\beta^2}{\tilde{A}^2}\right)^3} \left\{ \tilde{\kappa}^2(\omega_1^2 - \omega_2^2)\hat{C}_2^2 - (\tilde{\kappa}^2 - \omega_1^2)\left[2\frac{\beta}{\tilde{A}}\omega_2\tilde{\kappa}^2\hat{C}_2 \right. \right.$$

$$\left. \left. + (\tilde{\kappa}^2 - \omega_1^2)\left(\frac{\beta^2}{\tilde{A}^2}\tilde{\kappa}^2 - \omega_1^2\right) \right] \right\},$$

where

$$Y = \frac{\tilde{\kappa}^2 - \omega_1^2}{1 - \frac{\beta^2}{\tilde{A}^2}}, \quad \Omega = \frac{\omega_2^2}{1 - \frac{\beta^2}{\tilde{A}^2}}, \quad \hat{C}_2 = \tilde{C}_2/\tilde{A}.$$

In this way, we expressed the CSG parameters C_ϕ, A and C_χ through the NR parameters $\tilde{A}$, β, $\tilde{\kappa}$, ω_1, ω_2, $\tilde{C}_2$. The mass parameter $\tilde{M}^2$ remains free. The above equalities give the mapping, which relates all membrane solutions derivable from (3.8), (3.9) with the corresponding solutions of the CSG system.

4.1 Examples: GM and SS analogues

In particular, for the GM-type membrane solutions, for which $\tilde{C}_2 = 0$, $\tilde{\kappa}^2 = \omega_1^2$, the following equalities between the parameters hold

$$C_\phi = \frac{2}{\tilde{A}^2 - \beta^2} \left[3\tilde{M}^2 - 2\left(\omega_1^2 - \frac{\omega_2^2}{1 - \beta^2/\tilde{A}^2} \right) \right], \qquad \tilde{K}^2 = (Q\mathcal{R})^2 \tilde{M}^2,$$

$$A^2 = \frac{4}{\tilde{A}^2/\beta^2 - 1} \left(\tilde{M}^2 - \omega_1^2 + \frac{\omega_2^2}{1 - \beta^2/\tilde{A}^2} \right), \qquad C_\chi = 0. \qquad (4.11)$$

For the case of SS-type membrane solutions, when $\tilde{C}_2 = 0$, $\tilde{\kappa}^2 = \omega_1^2 \tilde{A}^2/\beta^2$, one has

$$C_\phi = \frac{2}{\beta^2 - \tilde{A}^2} \left[2\left(2\omega_1^2 \tilde{A}^2/\beta^2 + \frac{\omega_2^2}{\beta^2/\tilde{A}^2 - 1} \right) - 3\tilde{M}^2 \right],$$

$$A^2 = \frac{4}{\tilde{M}^4(1 - \tilde{A}^2/\beta^2)} \left(\omega_1^2 \tilde{A}^2/\beta^2 - \tilde{M}^2 \right)^2 \left(\frac{\omega_2^2}{\beta^2/\tilde{A}^2 - 1} - \tilde{M}^2 \right), \qquad (4.12)$$

$$C_\chi = \frac{2\omega_1^2 \omega_2 \tilde{A}^3}{\tilde{M}^2(\beta^2 - \tilde{A}^2)\beta^2}, \quad \tilde{K}^2 = (Q\mathcal{R})^2 \tilde{M}^2.$$

Let us give the membrane configurations which are analogous to the GM and SS string solutions on $R_t \times S^3$.

For the GM-like case by using that $\tilde{C}_2 = 0$, $\tilde{\kappa}^2 = \omega_1^2$ in (3.8), (3.9), one finds

$$\cos\theta = \frac{\cos\tilde{\theta}_0}{\cosh(D_0\xi)}, \qquad \sin^2\tilde{\theta}_0 = \frac{\beta^2 \omega_1^2}{\tilde{A}^2(\omega_1^2 - \omega_2^2)}, \qquad D_0 = \frac{\tilde{A}\sqrt{\omega_1^2 - \omega_2^2}}{\tilde{A}^2 - \beta^2} \cos\tilde{\theta}_0,$$

$$\mu_1 = \arctan\left[\cot\tilde{\theta}_0 \tanh(D_0\xi) \right], \qquad \mu_2 = \frac{\beta\omega_2}{\tilde{A}^2 - \beta^2}\xi.$$

Then, the corresponding membrane configuration is given by

$$Z_0 = \mathcal{R}\exp\left(2i\sqrt{1 - r_0^2}\,\omega_1\tau \right),$$

$$W_1 = 2\mathcal{R}\sqrt{1 - r_0^2}\sqrt{1 - \frac{\cos^2\tilde{\theta}_0}{\cosh^2(D_0\xi)}}\exp\left\{ i\omega_1\tau + i\arctan\left[\cot\tilde{\theta}_0 \tanh(D_0\xi) \right] \right\},$$

$$W_2 = 2\mathcal{R}\sqrt{1 - r_0^2}\frac{\cos\tilde{\theta}_0}{\cosh(D_0\xi)}\exp\left[i\omega_2\left(\tau + \frac{\beta}{\tilde{A}^2 - \beta^2}\xi \right) \right], \qquad (4.13)$$

$$W_3 = 2\mathcal{R}r_0 \sin(\gamma\sigma_2),$$
$$W_4 = 2\mathcal{R}r_0 \cos(\gamma\sigma_2).$$

For the SS-like solutions when $\tilde{C}_2 = 0$, $\tilde{\kappa}^2 = \omega_1^2 \tilde{A}^2 / \beta^2$, by solving the equations (3.8), (3.9), one arrives at

$$\cos\theta = \frac{\cos\tilde{\theta}_1}{\cosh(D_1\xi)}, \qquad \sin^2\tilde{\theta}_1 = \frac{\tilde{A}^2\omega_1^2}{\beta^2(\omega_1^2 - \omega_2^2)}, \quad D_1 = \frac{\tilde{A}\sqrt{\omega_1^2 - \omega_2^2}}{\tilde{A}^2 - \beta^2}\cos\tilde{\theta}_1,$$
$$\mu_1 = -\frac{\omega_1}{\beta}\xi - \arctan\left[\cot\tilde{\theta}_1 \tanh(D_1\xi)\right], \qquad \mu_2 = \frac{\beta\omega_2}{\tilde{A}^2 - \beta^2}\xi.$$

Now, the shape of the membrane is described by

$$Z_0 = \mathcal{R}\exp\left(2i\sqrt{1 - r_0^2}\frac{\tilde{A}}{\beta}\omega_1\tau\right),$$
$$W_1 = 2\mathcal{R}\sqrt{1 - r_0^2}\sqrt{1 - \frac{\cos^2\tilde{\theta}_1}{\cosh^2(D_1\xi)}}\exp\left\{-i\omega_1\frac{\alpha}{\beta}\sigma_1 - i\arctan\left[\cot\tilde{\theta}_1 \tanh(D_1\xi)\right]\right\},$$
$$W_2 = 2\mathcal{R}\sqrt{1 - r_0^2}\frac{\cos\tilde{\theta}_1}{\cosh(D_1\xi)}\exp\left[i\omega_2\left(\tau + \frac{\beta}{\tilde{A}^2 - \beta^2}\xi\right)\right], \tag{4.14}$$
$$W_3 = 2\mathcal{R}r_0 \sin(\gamma\sigma_2),$$
$$W_4 = 2\mathcal{R}r_0 \cos(\gamma\sigma_2).$$

The energy-charge relations computed on the above membrane solutions were found in [28], and in our notations read

$$\sqrt{1 - r_0^2}E - \frac{J_1}{2} = \sqrt{\left(\frac{J_2}{2}\right)^2 + \frac{\tilde{\lambda}}{\pi^2}\sin^2\frac{p}{2}}, \quad \frac{p}{2} = \frac{\pi}{2} - \tilde{\theta}_0, \tag{4.15}$$

for the GM-like case, and

$$\sqrt{1 - r_0^2}E - \frac{\sqrt{\tilde{\lambda}}}{2\pi}\Delta\varphi_1 = \frac{\sqrt{\tilde{\lambda}}}{\pi}\frac{p}{2}, \quad \frac{J_1}{2} = \sqrt{\left(\frac{J_2}{2}\right)^2 + \frac{\tilde{\lambda}}{\pi^2}\sin^2\frac{p}{2}}, \quad \frac{p}{2} = \frac{\pi}{2} - \tilde{\theta}_1, \tag{4.16}$$

for the SS-like solution, where

$$\tilde{\lambda} = \left[(4\pi)^2 T_2 \mathcal{R}^3 r_0(1 - r_0^2)\gamma\right]^2. \tag{4.17}$$

For the obtained membrane solutions (4.13), (4.14), one has

$$|W_1|^2 + |W_2|^2 = (2\mathcal{R})^2(1 - r_0^2), \quad W_3^2 + W_4^2 = (2\mathcal{R}r_0)^2.$$

That is why, these membrane configurations live in the $R_t \times S^3 \times S^1$ subspace of $AdS_4 \times S^7$. Besides, the radii of the three-sphere S^3 and the circle S^1 are functions of the parameter r_0. When r_0 approaches 1 from below, the radius of S^3 decreases, while the radius of S^1 increases. For $r_0 \to 0$, we observe the opposite behavior.

One may ask what happens when $r_0 = 0$. As is seen from (4.13) and (4.14), the membrane shrinks to a string in this case, because the dependence on σ_2 disappears. However, this string has completely different properties. Indeed, by solving the eqs. (3.8), (3.9), one finds the following string solution (now $\tilde{A} = 0$)

$$Z_0 = \mathcal{R} \exp\left(2i\tilde{\kappa}\tau\right),$$

$$W_1 = 2\mathcal{R}\sqrt{1 - \left(1 - \frac{\tilde{\kappa}^2}{\omega_1^2}\right)\sin^2\left(\frac{\tilde{\kappa}}{\beta}\xi\right)} \exp\left[i\omega_1\tau + i\frac{\tilde{\kappa}}{\omega_1}F\left(\frac{\tilde{\kappa}}{\beta}\xi\middle|1 - \frac{\tilde{\kappa}^2}{\omega_1^2}\right)\right],$$

$$W_2 = 2\mathcal{R}\sqrt{1 - \frac{\tilde{\kappa}^2}{\omega_1^2}}\sin\left(\frac{\tilde{\kappa}}{\beta}\xi\right)\exp\left(-i\omega_2\frac{\alpha}{\beta}\sigma_1\right),$$

where $F(n|m)$ is the elliptic integral of first kind. The computation of the corresponding conserved quantities gives

$$E - \frac{J_1}{2} = \frac{(2\pi\mathcal{R})^2}{\lambda^0}\tilde{\kappa}\left(1 - \frac{\tilde{\kappa}}{\omega_1}\right), \quad J_2 = 0.$$

If we set here $\tilde{\kappa} = \omega_1$ as for the GM case, we obtain $E - J_1/2 = 0$, i.e. the vacuum state. The same result can be obtained directly from (4.15), taking into account (3.7), since for $r_0 = 0$ we have $\tilde{\lambda} = 0$ too. The SS case corresponds to $\tilde{\kappa} = 0$, which leads to trivial solution with $E = J_1 = J_2 = 0$.

Let us explain the obvious differences between the M2-brane energy-charge relation (4.15) and the one for dyonic GM strings on $R_t \times S^3$, which as is well known is given by

$$E - J_1 = \sqrt{J_2^2 + \frac{\lambda}{\pi^2}\sin^2\frac{p}{2}}.$$

The factor $\sqrt{1 - r_0^2}$ comes from the fact that the NR system for membranes is defined to live on a circle with radius $\sqrt{1 - r_0^2}$, while for the strings this radius is one. The factor $1/2$ appears as a consequence of the background geometry. While the radii of AdS_5 and S^5 in the type IIB background $AdS_5 \times S^5$ are equal, the radius of AdS_4 is half the S^7 radius in the $AdS_4 \times S^7$ target space. The same applies to the SS case. Note however that such coefficients in the dispersion relation can also appear for strings on $AdS_5 \times S^5$ as shown in [26, 27].

Let us also write down the images of these M2-brane solutions in the CSG system. In order to derive them, we replace (4.11) for the GM-like case and correspondingly (4.12) for the SS-like case into (4.7), and then use the obtained field ϕ in (4.5) in order to find χ. The results of the integrations are as follows:

$$\Psi_{GM-like} = \frac{\sqrt{\omega_1^2 - \omega_2^2/(1 - \beta^2/\tilde{A}^2)}}{\tilde{M}\cosh\left[\sqrt{\frac{\omega_1^2 - \omega_2^2/(1 - \beta^2/\tilde{A}^2)}{1 - \beta^2/\tilde{A}^2}}(\xi/\tilde{A})\right]}$$

$$\times \exp\left[i\sqrt{\frac{\tilde{M}^2 - \omega_1^2 + \omega_2^2/(1 - \beta^2/\tilde{A}^2)}{\tilde{A}^2/\beta^2 - 1}}\left(\sigma_1 + \frac{\alpha}{\beta}\tau\right)\right],$$

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$$\Psi_{SS-like} = \sqrt{\tanh^2{(D\xi)} + \frac{\omega_2^2}{\omega_1^2\left(1 - \tilde{A}^2/\beta^2\right)\cosh^2{(D\xi)}}}$$

$$\times \exp\left\{i\arctan\left[\frac{\omega_1}{\omega_2}\sqrt{1 - \tilde{A}^2/\beta^2 - \omega_2^2/\omega_1^2}\tanh{(D\xi)}\right]\right\},$$

where

$$D = \frac{\tilde{A}\omega_1\sqrt{1 - \tilde{A}^2/\beta^2 - \omega_2^2/\omega_1^2}}{\beta^2\left(1 - \tilde{A}^2/\beta^2\right)}.$$

We point out that the obtained $\Psi_{SS-like}$ corresponds to $\tilde{M}^2 = \tilde{\kappa}^2 = \omega_1^2\tilde{A}^2/\beta^2$, when the parameter A in (4.12) becomes zero.

5. Finite-size effects

In this section we will find finite-size membrane solution, its image in the CSG system, and the leading corrections to the energy-charge relations analogous to the ones for the GM and SS strings on $R_t \times S^3$.

For $C_2 = 0$, eq. (3.8) can be written as

$$(\cos\theta)' = \mp\frac{\tilde{A}\sqrt{\omega_1^2 - \omega_2^2}}{\tilde{A}^2 - \beta^2}\sqrt{(z_+^2 - \cos^2\theta)(\cos^2\theta - z_-^2)}, \tag{5.1}$$

where

$$z_\pm^2 = \frac{1}{2(1 - \frac{\omega_2^2}{\omega_1^2})}\left\{q_1 + q_2 - \frac{\omega_2^2}{\omega_1^2} \pm \sqrt{(q_1 - q_2)^2 - \left[2(q_1 + q_2 - 2q_1q_2) - \frac{\omega_2^2}{\omega_1^2}\right]\frac{\omega_2^2}{\omega_1^2}}\right\},$$

$$q_1 = 1 - \tilde{\kappa}^2/\omega_1^2, \quad q_2 = 1 - \beta^2\tilde{\kappa}^2/\tilde{A}^2\omega_1^2.$$

The solution of (5.1) is

$$\cos\theta = z_+ dn\left(C\xi|m\right), \quad C = \mp\frac{\tilde{A}\sqrt{\omega_1^2 - \omega_2^2}}{\tilde{A}^2 - \beta^2}z_+, \quad m \equiv 1 - z_-^2/z_+^2. \tag{5.2}$$

The solutions of eqs. (3.9) now read

$$\mu_1 = \frac{2\beta/\tilde{A}}{z_+\sqrt{1 - \omega_2^2/\omega_1^2}}\left[F\left(am(C\xi)|m\right) - \frac{\tilde{\kappa}^2/\omega_1^2}{1 - z_+^2}\Pi\left(am(C\xi), -\frac{z_+^2 - z_-^2}{1 - z_+^2}\Big|m\right)\right],$$

$$\mu_2 = \frac{2\beta\omega_2/\tilde{A}\omega_1}{z_+\sqrt{1 - \omega_2^2/\omega_1^2}}F\left(am(C\xi)|m\right),$$

where $\Pi(k, n|m)$ is the elliptic integral of third kind. Therefore, the full membrane solution is given by

$$Z_0 = \mathcal{R} \exp\left(2i\sqrt{1 - r_0^2}\tilde{\kappa}\tau\right),$$

$$W_1 = 2\mathcal{R}\sqrt{1 - r_0^2}\sqrt{1 - z_+^2 dn^2\left(C\xi|m\right)} \exp\left\{i\omega_1\tau + \frac{2i\beta/\tilde{A}}{z_+\sqrt{1 - \omega_2^2/\omega_1^2}}\right.$$
$$\left. \times \left[F\left(am(C\xi)|m\right) - \frac{\tilde{\kappa}^2/\omega_1^2}{1 - z_+^2}\Pi\left(am(C\xi), -\frac{z_+^2 - z_-^2}{1 - z_+^2}\Big|m\right)\right]\right\},$$

$$W_2 = 2\mathcal{R}\sqrt{1 - r_0^2}z_+ dn\left(C\xi|m\right) \exp\left[i\omega_2\tau + \frac{2i\beta\omega_2/\tilde{A}\omega_1}{z_+\sqrt{1 - \omega_2^2/\omega_1^2}}F\left(am(C\xi)|m\right)\right], \qquad (5.3)$$

$$W_3 = 2\mathcal{R}r_0\sin(\gamma\sigma_2),$$

$$W_4 = 2\mathcal{R}r_0\cos(\gamma\sigma_2).$$

We note that (5.3) contains both cases: $\tilde{A}^2 > \beta^2$ and $\tilde{A}^2 < \beta^2$ corresponding to GM and SS respectively.

To find the CSG solution related to (5.3), we insert (5.2) into (4.8) and (4.9) to get

$$\sin^2(\phi/2) = \frac{\omega_1^2/\tilde{M}^2}{\beta^2/\tilde{A}^2 - 1}\left[\left(1 - \tilde{\kappa}^2/\omega_1^2\right) - \left(1 - \omega_2^2/\omega_1^2\right)\left(z_+^2 cn^2(C\xi|m) + z_-^2 sn^2(C\xi|m)\right)\right]. \quad (5.4)$$

After that, we put (5.4) into (4.5) and integrate. The result is as follows

$$\chi = \frac{A}{\beta}(\beta\sigma_1 + \alpha\tau) - C_\chi(\alpha\sigma_1 + \beta\tau) + \frac{C_\chi}{CD}\Pi\left(am(C\xi), n|m\right), \qquad (5.5)$$

where A/β and C_χ are given in (4.10) ($\hat{C}_2 = 0$), and

$$D = \frac{\omega_1^2/\tilde{M}^2}{\beta^2/\tilde{A}^2 - 1}\left[\left(1 - \tilde{\kappa}^2/\omega_1^2\right) - \left(1 - \omega_2^2/\omega_1^2\right)z_+^2\right], \quad n = \frac{\left(1 - \omega_2^2/\omega_1^2\right)\left(z_+^2 - z_-^2\right)}{\left(1 - \tilde{\kappa}^2/\omega_1^2\right) - \left(1 - \omega_2^2/\omega_1^2\right)z_+^2}.$$

Hence for the present case, the CSG field $\psi = \sin(\phi/2)\exp(i\chi/2)$ is determined by (5.4) and (5.5).

Our next task is to find out what kind of energy-charge relations can appear for the M2-brane solution (5.3) in the limit when the energy $E \to \infty$. It turns out that the semiclassical behavior depends crucially on the sign of the difference $\tilde{A}^2 - \beta^2$.

5.1 The GM analogue

We begin with the GM analogue, i.e. $\tilde{A}^2 > \beta^2$. In this case, one obtains from (3.7) the following expressions for the conserved energy E and the angular momenta J_1, J_2

$$\mathcal{E} = \frac{2\tilde{\kappa}(1 - \beta^2/\tilde{A}^2)}{\omega_1 z_+\sqrt{1 - \omega_2^2/\omega_1^2}}\mathbf{K}\left(1 - z_-^2/z_+^2\right),$$

$$\mathcal{J}_1 = \frac{2z_+}{\sqrt{1 - \omega_2^2/\omega_1^2}}\left[\frac{1 - \beta^2\tilde{\kappa}^2/\tilde{A}^2\omega_1^2}{z_+^2}\mathbf{K}\left(1 - z_-^2/z_+^2\right) - \mathbf{E}\left(1 - z_-^2/z_+^2\right)\right], \qquad (5.6)$$

$$\mathcal{J}_2 = \frac{2z_+\omega_2/\omega_1}{\sqrt{1 - \omega_2^2/\omega_1^2}}\mathbf{E}\left(1 - z_-^2/z_+^2\right).$$

Here, we have used the notations

$$\mathcal{E} = \frac{2\pi}{\sqrt{\tilde{\lambda}}} \sqrt{1 - r_0^2} E, \quad \mathcal{J}_1 = \frac{2\pi}{\sqrt{\tilde{\lambda}}} \frac{J_1}{2}, \quad \mathcal{J}_2 = \frac{2\pi}{\sqrt{\tilde{\lambda}}} \frac{J_2}{2}, \tag{5.7}$$

where $\tilde{\lambda}$ is defined in (4.17). The computation of $\Delta\varphi_1$ gives

$$p \equiv \Delta\varphi_1 = 2 \int_{\theta_{\min}}^{\theta_{\max}} \frac{d\theta}{\theta'} \mu_1' \tag{5.8}$$

$$= -\frac{2\beta/\tilde{A}}{z_+ \sqrt{1 - \omega_2^2/\omega_1^2}} \left[\frac{\tilde{\kappa}^2/\omega_1^2}{1 - z_+^2} \Pi\left(-\frac{z_+^2 - z_-^2}{1 - z_+^2} \Big| 1 - z_-^2/z_+^2 \right) - \mathbf{K}\left(1 - z_-^2/z_+^2 \right) \right].$$

In the above expressions, $\mathbf{K}(m)$, $\mathbf{E}(m)$ and $\Pi(n|m)$ are the complete elliptic integrals.

Let us introduce the new parameters

$$u \equiv \omega_2^2/\omega_1^2, \quad v \equiv -\beta/\tilde{A}, \quad \epsilon \equiv z_-^2/z_+^2.$$

This will allow us to eliminate $\tilde{\kappa}/\omega_1$ and $z_\pm$ from the coefficients in (5.6), (5.8) and rewrite them as functions of u, v and ϵ only:

$$\begin{aligned}
\mathcal{E} &= 2K_e \mathbf{K}\left(1 - \epsilon \right), \\
\mathcal{J}_1 &= 2K_{11} \left[K_{12} \mathbf{K}\left(1 - \epsilon \right) - \mathbf{E}\left(1 - \epsilon \right) \right], \\
\mathcal{J}_2 &= 2K_2 \mathbf{E}\left(1 - \epsilon \right), \\
p &= 2K_{\varphi 1} \left[K_{\varphi 2} \Pi\left(K_{\varphi 3} | 1 - \epsilon \right) - \mathbf{K}\left(1 - \epsilon \right) \right].
\end{aligned} \tag{5.9}$$

We are interested in the behavior of these quantities in the limit $\epsilon \to 0$. To establish it, we will use the expansions for the elliptic integrals and $K_e, \ldots, K_{\varphi 3}$ given in appendix A.

Our approach is as follows. First, we expand $\mathcal{E}$, $\mathcal{J}_1$, $\mathcal{J}_2$ and p about $\epsilon = 0$ keeping u and v independent of ϵ. Second, we introduce $u(\epsilon)$ and $v(\epsilon)$ according to the rule

$$u(\epsilon) = u_0 + u_1\epsilon + u_2\epsilon \log(\epsilon), \quad v(\epsilon) = v_0 + v_1\epsilon + v_2\epsilon \log(\epsilon) \tag{5.10}$$

and expand again. Requiring $\mathcal{J}_2$ and p to be finite, we find

$$u_0 = \frac{\mathcal{J}_2^2}{\mathcal{J}_2^2 + 4\sin^2(p/2)}, \quad v_0 = \frac{\sin(p)}{\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}}$$

$$u_1 = \frac{u_0}{2(1 - v_0^2)} \left\{ 1 - 4v_0^2 + 3v_0^4 - (1 - v_0^2)^2 \log(16) \right.$$
$$\left. - u_0 \left[1 - \log(16) + 3v_0^2 \left(\log(16) - 3 \right) \right] \right\}$$

$$v_1 = \frac{v_0}{4(1 - u_0)(1 - v_0^2)} \left\{ u_0^2 (1 + 3v_0^2) \left(\log(16) - 3 \right) + (1 - v_0^2)^2 \left(\log(16) - 1 \right) \right.$$
$$\left. - u_0(1 - v_0^2) \left[v_0^2 \left(\log(16) - 3 \right) + \log(256) - 4 \right] \right\}$$

$$u_2 = \frac{u_0}{2(1 - v_0^2)} \left[(1 - v_0^2)^2 - u_0(1 - 3v_0^2) \right]$$

$$v_2 = -\frac{1}{4} v_0 \left[u_0 + \frac{(1 - 2u_0 - v_0^2)(1 - u_0 - (1 + u_0)v_0^2)}{(1 - u_0)(1 - v_0^2)} \right].$$

The parameter ϵ can be obtained from the expansion for $\mathcal{J}_1$ to be

$$\epsilon = 16 \exp\left[-\frac{\sqrt{1 - u_0 - v_0^2}\mathcal{J}_1 + 2\left(1 - v_0^2/(1 - u_0)\right)}{1 - v_0^2}\right].$$

Using all of the above in the expansion for $\mathcal{E} - \mathcal{J}_1$, one arrives at

$$\mathcal{E} - \mathcal{J}_1 = \sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)} - \frac{16\sin^4(p/2)}{\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}} \tag{5.11}$$

$$\times \exp\left[-\frac{2\left(\mathcal{J}_1 + \sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}\right)\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}\sin^2(p/2)}{\mathcal{J}_2^2 + 4\sin^4(p/2)}\right].$$

It is easy to check that the energy-charge relation (5.11) coincides with the one found in [44], describing the finite-size effects for dyonic GM. The difference is that in the string case the relations between $\mathcal{E}$, $\mathcal{J}_1$, $\mathcal{J}_2$ and E, J_1, J_2 are given by

$$\mathcal{E} = \frac{2\pi}{\sqrt{\lambda}}E, \quad \mathcal{J}_1 = \frac{2\pi}{\sqrt{\lambda}}J_1, \quad \mathcal{J}_2 = \frac{2\pi}{\sqrt{\lambda}}J_2,$$

while for the M2-brane they are written in (5.7).

5.2 The SS analogue

Let us turn our attention to the SS analogue, when $\tilde{A}^2 < \beta^2$. The computation of the conserved quantities (3.7) and $\Delta\varphi_1$ now gives

$$\mathcal{E} = \frac{2\tilde{\kappa}(\beta^2/\tilde{A}^2 - 1)}{\omega_1\sqrt{1 - \omega_2^2/\omega_1^2}z_+}\mathbf{K}\left(1 - z_-^2/z_+^2\right),$$

$$\mathcal{J}_1 = \frac{2z_+}{\sqrt{1 - \omega_2^2/\omega_1^2}}\left[\mathbf{E}\left(1 - z_-^2/z_+^2\right) - \frac{1 - \beta^2\tilde{\kappa}^2/\tilde{A}^2\omega_1^2}{z_+^2}\mathbf{K}\left(1 - z_-^2/z_+^2\right)\right],$$

$$\mathcal{J}_2 = -\frac{2z_+\omega_2/\omega_1}{\sqrt{1 - \omega_2^2/\omega_1^2}}\mathbf{E}\left(1 - z_-^2/z_+^2\right),$$

$$\Delta\varphi_1 = -\frac{2\beta/\tilde{A}}{\sqrt{1 - \omega_2^2/\omega_1^2}z_+}\left[\frac{\tilde{\kappa}^2/\omega_1^2}{1 - z_+^2}\Pi\left(-\frac{z_+^2 - z_-^2}{1 - z_+^2}\Big|1 - z_-^2/z_+^2\right) - \mathbf{K}\left(1 - z_-^2/z_+^2\right)\right].$$

Our next step is to introduce the new parameters

$$u \equiv \omega_2^2/\omega_1^2, \quad v \equiv \beta/\tilde{A}, \quad \epsilon \equiv z_-^2/z_+^2,$$

and to rewrite $\mathcal{E}$, $\mathcal{J}_1$, $\mathcal{J}_2$, $\Delta\varphi_1$ in the form (5.9). The explicit ϵ-expansions of the coefficients $K_e, \ldots, K_{\varphi3}$ are given as functions of u and v in appendix A. The ϵ-expansions for u and v are the same as before. Now, the coefficients in these expansions can be determined by

the condition that $\mathcal{J}_1$ and $\mathcal{J}_2$ should be finite,

$$v_0 = \frac{2\mathcal{J}_1}{\sqrt{\left(\mathcal{J}_1^2 - \mathcal{J}_2^2\right)\left[4 - \left(\mathcal{J}_1^2 - \mathcal{J}_2^2\right)\right]}}, \quad u_0 = \frac{\mathcal{J}_2^2}{\mathcal{J}_1^2},$$

$$v_1 = \frac{(1-u_0)v_0^2 - 1}{4(u_0-1)(v_0^2-1)v_0}\left\{(u_0-1)v_0^4(1+\log(16)) - 2\right.$$
$$\left. + v_0^2\left[3 + \log(16) + u_0(\log(4096) - 5)\right]\right\},$$

$$v_2 = -\frac{v_0\left[1 - (1-u_0)v_0^2\right]\left[1 + 3u_0 - (1-u_0)v_0^2\right]}{4(1-u_0)(v_0^2-1)},$$

$$u_1 = \frac{u_0\left[1 - (1-u_0)v_0^2\right]\log(16)}{v_0^2-1}, \quad u_2 = -\frac{u_0\left[1 - (1-u_0)v_0^2\right]}{v_0^2-1}.$$

The parameter ϵ can be obtained from $\Delta\varphi_1$ as follows:

$$\epsilon = 16\exp\left(-\frac{\sqrt{(1-u_0)v_0^2 - 1}}{v_0^2-1}\left[\Delta\varphi_1 + \arcsin\left(\frac{2\sqrt{(1-u_0)v_0^2-1}}{(1-u_0)v_0^2}\right)\right]\right).$$

Taking the above results into account, $\mathcal{E} - \Delta\varphi_1$ can be derived as

$$\mathcal{E} - \Delta\varphi_1 = \arcsin N(\mathcal{J}_1, \mathcal{J}_2) + 2\left(\mathcal{J}_1^2 - \mathcal{J}_2^2\right)\sqrt{\frac{4}{\left[4 - \left(\mathcal{J}_1^2 - \mathcal{J}_2^2\right)\right]} - 1} \tag{5.12}$$

$$\times \exp\left[-\frac{2\left(\mathcal{J}_1^2 - \mathcal{J}_2^2\right)N(\mathcal{J}_1, \mathcal{J}_2)}{\left(\mathcal{J}_1^2 - \mathcal{J}_2^2\right)^2 + 4\mathcal{J}_2^2}\left[\Delta\varphi_1 + \arcsin N(\mathcal{J}_1, \mathcal{J}_2)\right]\right],$$

$$N(\mathcal{J}_1, \mathcal{J}_2) \equiv \frac{1}{2}\left[4 - \left(\mathcal{J}_1^2 - \mathcal{J}_2^2\right)\right]\sqrt{\frac{4}{\left[4 - \left(\mathcal{J}_1^2 - \mathcal{J}_2^2\right)\right]} - 1}.$$

Finally, by using the SS relation between the angular momenta

$$\mathcal{J}_1 = \sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)},$$

we obtain

$$\mathcal{E} - \Delta\varphi_1 = p + 8\sin^2\frac{p}{2}\tan\frac{p}{2}\exp\left(-\frac{\tan\frac{p}{2}(\Delta\varphi_1 + p)}{\tan^2\frac{p}{2} + \mathcal{J}_2^2\csc^2 p}\right). \tag{5.13}$$

This is our final expression for the leading finite-size correction to the "$E - \Delta\varphi$" relation for the membrane analogue of the SS string with two angular momenta. It coincides with the string result found in [39]. As in the GM case, the difference is in the identification (5.7).

6. Concluding remarks

In this paper, by using the possibility to reduce the M2-brane dynamics to the one of the NR integrable system, we gave an explicit mapping connecting the parameters of *all* membrane solutions described by this dynamical system and the parameters in the corresponding solutions of the CSG integrable model. Based on this NR approach, we found finite-size

M2-brane solution, its image in the CSG system, and the leading finite-size corrections to the energy-charge relations analogous to the ones for the GM and SS strings on $R_t \times S^3$.

An evident direction for further investigations is to consider more general membrane configurations, which could describe finite-size effects corresponding to GM and SS strings on $R_t \times S^5$. i.e. with three angular momenta. One can also try to include the energy dependence on the spin $\mathcal{S}$, arising from the AdS part of the full $AdS_4 \times S^7$ background. Note however, that such general cases are not considered yet even for strings on $AdS_5 \times S^5$.

Another interesting problem is to find the M2-brane analogues of the semiclassical GM and SS scattering [45–47]. To this end, one can use the established correspondence between the membrane solutions (4.13), (4.14) and the CSG model. Alternatively, one may apply the dressing method as is done in [47–49].

Acknowledgments

A. ϵ-expansions

We use the following expansions for the elliptic functions

$$\mathbf{K}(1-\epsilon) \propto -\frac{1}{2}\log\epsilon\,(1+O(\epsilon)) + \log(4)\,(1+O(\epsilon)),$$

$$\mathbf{E}(1-\epsilon) \propto 1 - \epsilon\left(\frac{1}{4} - \log(2)\right)(1+O(\epsilon)) - \frac{\epsilon}{4}\log\epsilon\,(1+O(\epsilon))$$

$$\Pi(n|1-\epsilon) \propto \frac{\log\epsilon}{2(n-1)}(1+O(\epsilon)) + \frac{\sqrt{n}\log\left(\frac{1+\sqrt{n}}{1-\sqrt{n}}\right) - \log(16)}{2(n-1)}(1+O(\epsilon)).$$

The expansions for the coefficients in (5.9) for $\tilde{A}^2 > \beta^2$ are

$$K_e \propto \frac{1-v^2}{\sqrt{1-u-v^2}} - \frac{(1-u)^2 - v^2}{2(1-u)\sqrt{1-u-v^2}}\,\epsilon,$$

$$K_{11} \propto -\frac{\sqrt{1-u-v^2}}{1-u} - \frac{\sqrt{1-u-v^2}\left(1-2u-v^2\right)v^2}{2(1-u)^2(1-v^2)}\,\epsilon,$$

$$K_{12} \propto \frac{(1-u)(1-v^2)}{1-u-v^2} + \frac{u^2v^2}{(1-u-v^2)(1-v^2)}\,\epsilon,$$

$$K_2 \propto \frac{\sqrt{u(1-u-v^2)}}{1-u} + \frac{\sqrt{u(1-u-v^2)}\left(1-2u-v^2\right)v^2}{2(1-u)^2(1-v^2)}\,\epsilon,$$

$$K_{\varphi 1} \propto \frac{v}{\sqrt{1-u-v^2}} - \frac{\left(1-2u-v^2\right)v^3}{2(1-u)(1-v^2)\sqrt{1-u-v^2}}\,\epsilon,$$

$$K_{\varphi 2} \propto \frac{1-u}{v^2} - \frac{u\left(1-u-v^2\right)}{(1-v^2)v^2}\,\epsilon,$$

$$K_{\varphi 3} \propto 1 - \frac{1-u}{v^2} + \frac{2u\left(1-u-v^2\right)}{(1-v^2)v^2}\,\epsilon.$$

The expansions for the coefficients in (5.9) for $\tilde{A}^2 < \beta^2$ are given by

$$K_e \propto \frac{v^2-1}{\sqrt{v^2(1-u)-1}} - \frac{v^2(1-u)^2-1}{2\sqrt{v^2(1-u)-1}(1-u)}\,\epsilon,$$

$$K_{11} \propto \sqrt{\frac{v^2(1-u)-1}{v^2(1-u)^2}} - \frac{\sqrt{v^2(1-u)-1}\left(1+v^2(2u-1)\right)}{2v^3(v^2-1)(1-u)^2}\,\epsilon,$$

$$K_{12} \propto \left(1 - \frac{v^2 u}{v^2-1}\right)\epsilon,$$

$$K_2 \propto -\sqrt{\frac{\left(v^2(1-u)-1\right)u}{v^2(1-u)^2}} + \frac{\sqrt{\frac{(v^2(1-u)-1)u}{v^2(1-u)^2}}\left(1+v^2(2u-1)\right)}{2v^2(v^2-1)(1-u)}\,\epsilon,$$

$$K_{\varphi 1} \propto -\frac{v}{\sqrt{1-1/v^2-u}} - \frac{1+v^2(2u-1)}{2(v^2-1)\sqrt{v^2(1-u)-1}(1-u)}\,\epsilon,$$

$$K_{\varphi 2} \propto 1-u + \frac{\left(1-v^2(1-u)\right)u}{v^2-1}\,\epsilon,$$

$$K_{\varphi 3} \propto 1 - v^2(1-u) + 2v^2 u\left(1 - \frac{v^2 u}{v^2-1}\right)\epsilon.$$

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