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Neumann-Rosochatius integrable system for strings on $AdS_4 \times \mathbb{CP}^3$

Changrim Ahn and P. Bozhilov*

Department of Physics, Ewha Womans University,
DaeHyun 11-1, Seoul 120-750, South Korea
E-mail: ahn@ewha.ac.kr, bozhilov@inrne.bas.bg

R.C. Rashkov†

Institute for Theoretical Physics, Vienna University of Technology,
Wiedner Haupsstr. 8-10, 1040 Vienna, Austria
E-mail: rash@hep.itp.tuwien.ac.at

Abstract: We use the reduction of the string dynamics on $AdS_4 \times \mathbb{CP}^3$ to the Neumann-Rosochatius integrable system. All constraints can be expressed simply in terms of a few parameters. We analyze the giant magnon and single spike solutions on $R_t \times \mathbb{CP}^3$ with two angular momenta in detail and find the energy-charge relations. The finite-size effects of the giant magnon and single spike solutions are analyzed.

Keywords: AdS-CFT Correspondence, String Duality.

*On leave from Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Bulgaria.

†On leave from Dept. of Physics, Sofia University, Bulgaria.

Contents

1. Introduction

The AdS/CFT correspondence [1–3], which has led to many exciting developments in the duality between type IIB string theory on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ super Yang-Mills theory, is now being extended into AdS_4/CFT_3. A most promising candidate is $\mathcal{N} = 6$ super Chern-Simons theory with $\mathrm{SU}(N) \times \mathrm{SU}(N)$ gauge symmetry and level k. This model, which was first proposed by Aharony, Bergman, Jafferis, and Maldacena [4], is believed to be dual to M-theory on $AdS_4 \times S^7/Z_k$. Furthermore, in the planar limit of $N, k \to \infty$ with a fixed value of 't Hooft coupling $\lambda = N/k$, the $\mathcal{N} = 6$ Chern-Simons is believed to be dual to type IIA superstring theory on $AdS_4 \times \mathbb{CP}^3$.

Quantum integrability of the planar $\mathcal{N} = 6$ Chern-Simons theory was first discovered by Minahan and Zarembo in the leading two-loop-order perturbative computation of the anomalous dimensions of gauge-invariant composite operators [5]. (See also [6].) Its excitation spectrum and symmetry have been studied in [7] and all-loop Bethe ansatz, first conjectured in [8], was confirmed by the exact S-matrix first proposed in [9].

Integrability in the string theory side is also under active investigation. The Penrose limit of the type IIA string and BMN-like spectrum have been studied in [10]. Various aspects of classical integrability in the $\lambda \gg 1$ limit have been found in [11–14]. The giant magnon (GM) solution [7, 15] and its finite-size effect [16, 17] have been computed. The GM and single spike (SS) solutions of membranes on $AdS_4 \times S^7$ background and their finite-size effects have been worked out in [18] and such string solutions as circular and pulsating strings [19] and spiky strings and finite-size effects on $AdS_4 \times \mathbb{CP}^3$ [20] have been found. Also recently, one-loop quantum correction to the GMs has been computed [21].

All these solutions, however, are restricted to the strings moving in $R_t \times S^2 \times S^2$ with one angular momentum. The configuration in the target space is in such a way that the azimuthal angle of the string coordinates in the first S^2 is opposite to that in the second sphere. The purpose of this article is to find classical solutions with two angular momenta in $\mathbb{CP}^3$. Our string solutions develop spikes in the two spheres $S^2 \times S^2 \subset \mathbb{CP}^3$ with a certain dynamics in U(1) fiber. The picture is analogous to the dyonic GM in $AdS_5 \times S^5$ [22]. We want to emphasize that the Neumann-Rosochatius (NR) integrable system is very effective for dealing with the strings on $\mathbb{CP}^3$. This integrable system is obtained by reformulating the problem in a conformal gauge using the Polyakov action and assuming a particular ansatz for string coordinates. This approach has been previously developed and applied to find classical solutions such as the GM [23] and SS [24] of type IIB string theory on $AdS_5 \times S^5$ in [25–27]. The application of the NR system to the SS in S^3 has been worked out in [28], the most general case of GM and SS has been considered in [29] and to the finite-size effects in [30]. The NR integrable system in AdS_4/CFT_3 was used to find the GM solution for the membrane on $AdS_4 \times S^7$ [31] and to compute the finite-size effects in [18]. In this article we apply this system to the strings moving in the $R_t \times \mathbb{CP}^3$ background. The space $\mathbb{CP}^3$ can be thought as a U(1) fibration over $S^2 \times S^2$ (see the appendix for basic facts about $\mathbb{CP}^3$). Our ansatz for string coordinates allows motion in $S^2 \times S^2$ subspace and U(1) fiber as well. The solutions we find contain the GM solutions for motion in $S^2 \times S^2$ found in [15] as a special case. Using this formulation, we compute the finite-size effects of the GM and the SS strings.

The paper is organized as follows. In section 2 we introduce the classical string action on $R_t \times \mathbb{CP}^3$ and the corresponding NR system. We provide explicit GM and SS solutions moving in $R_t \times \mathbb{CP}^3$ and provide an analysis of the finite-size effects in section 3. We conclude in section 4 with a brief discussion of our results.

2. Strings on $R_t \times \mathbb{CP}^3$ and the NR integrable system

Let us start with the Polyakov string action

$$S^P = -\frac{T}{2}\int d^2\xi \sqrt{-\gamma}\gamma^{mn} G_{mn}, \qquad G_{mn} = g_{MN}\partial_m X^M \partial_n X^N, \tag{2.1}$$
$$\partial_m = \partial/\partial\xi^m, \quad m,n = (0,1), \qquad (\xi^0,\xi^1) = (\tau,\sigma), \quad M,N = (0,1,\ldots,9),$$

and choose *conformal gauge* $\gamma^{mn} = \eta^{mn} = diag(-1,1)$, in which the Lagrangian and the Virasoro constraints take the form

$$\mathcal{L}_s = \frac{T}{2}\left(G_{00} - G_{11}\right) \tag{2.2}$$
$$G_{00} + G_{11} = 0, \qquad G_{01} = 0. \tag{2.3}$$

where T is the string tension.

The background metric g_{MN} for $AdS_4 \times \mathbb{CP}^3$ is given by

$$ds^2 = g_{MN}dx^M dx^N = R^2\left(\frac{1}{4}ds^2_{AdS^4} + ds^2_{\mathbb{CP}^3}\right), \quad R^2 = \sqrt{32\pi^2\lambda},$$

where $\lambda \equiv N/k$ is the 't Hooft coupling. With $\alpha' = 1$ convention, this coupling is related to the string tension by

$$\frac{TR^2}{2} = \sqrt{2\lambda},$$

which is different from the case of $AdS_5 \times S^5$.

The coordinates describing the background can be chosen such that

$$\sum_{i,j=0}^{4} \eta_{ij} y^i y^j + \left(\frac{R}{2}\right)^2 = 0, \qquad \eta_{ij} = diag(-1,1,1,1,-1),$$

for the AdS part and

$$\sum_{i=1}^{8} (x^i)^2 - R^2 = 0, \qquad \sum_{i=1,3,5,7} \left(x^i \partial_m x^{i+1} - x^{i+1} \partial_m x^i\right) = 0,$$

for the $\mathbb{CP}^3$ part [15]. Further on, we restrict ourselves to the $R_t \times \mathbb{CP}^3$ subspace for which $y^1 = y^2 = y^3 = 0$, and introduce the complex coordinates

$$z = y^0 + iy^4, \quad w_1 = x^1 + ix^2, \quad w_2 = x^3 + ix^4, \quad w_3 = x^5 + ix^6, \quad w_4 = x^7 + ix^8.$$

Now, we can embed the string as follows

$$z = Z(\tau,\sigma) = \frac{R}{2} e^{it(\tau,\sigma)}, \qquad w_a = W_a(\tau,\sigma) = R r_a(\tau,\sigma) e^{i\varphi_a(\tau,\sigma)}.$$

These complex coordinates should satisfy

$$\sum_{a=1}^{4} W_a \bar{W}_a = R^2,$$

which corresponds to S^7 and further more

$$\sum_{a=1}^{4} \left(W_a \partial_m \bar{W}_a - \bar{W}_a \partial_m W_a\right) = 0, \tag{2.4}$$

which reduces the embedding to $\mathbb{CP}^3$. Here t is the AdS time. In terms of the embedding coordinates, the $\mathbb{CP}^3$ condition (2.4) becomes

$$\sum_{a=1}^{4} r_a^2 \partial_m \varphi_a = 0, \quad m = 0,1. \tag{2.5}$$

For this embedding, the metric induced on the string worldsheet is given by

$$\begin{aligned} G_{mn} &= -\partial_{(m} Z \partial_{n)} \bar{Z} + \sum_{a=1}^{4} \partial_{(m} W_a \partial_{n)} \bar{W}_a \\ &= R^2 \left[-\frac{1}{4} \partial_m t \partial_n t + \sum_{a=1}^{4} \left(\partial_m r_a \partial_n r_a + r_a^2 \partial_m \varphi_a \partial_n \varphi_a\right)\right]. \end{aligned}$$

The corresponding string Lagrangian becomes

$$\mathcal{L} = \mathcal{L}_s + \sqrt{8\lambda}\Lambda\left(\sum_{a=1}^{4} r_a^2 - 1\right) + \sqrt{8\lambda}\Lambda_0 \sum_{a=1}^{4} r_a^2 \partial_0 \varphi_a + \sqrt{8\lambda}\Lambda_1 \sum_{a=1}^{4} r_a^2 \partial_1 \varphi_a,$$

where Λ, Λ_0, Λ_1 are Lagrange multipliers.

In the case at hand, the background metric does not depend on t and φ_a. Therefore, the conserved quantities are the string energy E_s and four angular momenta J_a, given by

$$E_s = -\int d\sigma \frac{\partial \mathcal{L}}{\partial(\partial_0 t)}, \qquad J_a = \int d\sigma \frac{\partial \mathcal{L}}{\partial(\partial_0 \varphi_a)}. \tag{2.6}$$

In order to reduce the string dynamics on $R_t \times \mathbb{CP}^3$ to the NR integrable system, we use the ansatz [25–27]

$$t(\tau,\sigma) = \kappa\tau, \qquad r_a(\tau,\sigma) = r_a(\xi), \qquad \varphi_a(\tau,\sigma) = \omega_a \tau + f_a(\xi), \tag{2.7}$$
$$\xi = \alpha\sigma + \beta\tau, \qquad \kappa, \omega_a, \alpha, \beta = \text{constants}.$$

Then the Lagrangian $\mathcal{L}$ takes the form (prime is used for $\partial/\partial\xi$)

$$\begin{aligned}\mathcal{L} \;=\; & -\sqrt{2\lambda}(\alpha^2-\beta^2)\sum_{a=1}^{4}\left[r_a'^2 + r_a^2\left(f_a' - \frac{\beta\omega_a}{\alpha^2-\beta^2}\right)^2 - \frac{\alpha^2\omega_a^2}{(\alpha^2-\beta^2)^2} r_a^2\right] \\ & + \sqrt{8\lambda}\Lambda\left(\sum_{a=1}^{4} r_a^2 - 1\right) + \sqrt{8\lambda}\Lambda_0 \sum_{a=1}^{4} \omega_a r_a^2 + \sqrt{8\lambda}\Lambda_1 \sum_{a=1}^{4} f_a' r_a^2.\end{aligned}$$

Now we can integrate the equations of motion for f_a to get

$$f_a' = \frac{1}{\alpha^2-\beta^2}\left(\frac{C_a}{r_a^2} + \beta\omega_a + \Lambda_1\right), \tag{2.8}$$

where C_a are integration constants. By using (2.8), the equations of motion for r_a can be written as

$$\left(\alpha^2-\beta^2\right) r_a'' - \frac{1}{\alpha^2-\beta^2}\frac{C_a}{r_a^3} + \left[\omega_a^2 + 2\left(\Lambda + \Lambda_0\omega_a\right) + \frac{\left(\Lambda_1 + \beta\omega_a\right)^2}{\alpha^2-\beta^2}\right] r_a = 0.$$

These can be obtained from the Lagrangian

$$\begin{aligned}L \;=\; & \sum_{a=1}^{4}\left[(\alpha^2-\beta^2) r_a'^2 - \frac{1}{\alpha^2-\beta^2}\frac{C_a}{r_a^2} - \omega_a^2 r_a^2\right] \\ & -2\Lambda\left(\sum_{a=1}^{4} r_a^2 - 1\right) - 2\Lambda_0 \sum_{a=1}^{4}\omega_a r_a^2 - \frac{1}{\alpha^2-\beta^2}\sum_{a=1}^{4}\left(\Lambda_1 + \beta\omega_a\right)^2 r_a^2.\end{aligned}$$

From the equations of motion for the Lagrange multipliers it follows that $\Lambda_1 = 0$. Thus, we end up with the following effective Lagrangian for the coordinates r_a

$$\begin{aligned}L_{\rm NR} \;=\; & (\alpha^2-\beta^2)\sum_{a=1}^{4}\left[r_a'^2 - \frac{1}{(\alpha^2-\beta^2)^2}\left(\frac{C_a^2}{r_a^2} + \alpha^2\omega_a^2 r_a^2\right)\right] \\ & -2\Lambda\left(\sum_{a=1}^{4} r_a^2 - 1\right) - 2\Lambda_0\sum_{a=1}^{4}\omega_a r_a^2.\end{aligned} \tag{2.9}$$

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This is the Lagrangian for the NR integrable system [27] with one more embedding condition which comes from the $\mathbb{CP}^3$ condition (2.5),

$$\sum_{a=1}^{4} \omega_a r_a^2 = 0. \tag{2.10}$$

In addition, from (2.8) and the constraint $\sum_{a=1}^4 f_a' r_a^2 = 0$, one can find $\Lambda_1 = -\sum_{a=1}^4 C_a$. Since Λ_1 should be zero, this leads to

$$\sum_{a=1}^{4} C_a = 0. \tag{2.11}$$

These two extra conditions for the NR system of $\mathbb{CP}^3$ are the main difference from that of the sphere geometry. In other words, strings moving on $R_t \times \mathbb{CP}^3$ should satisfy these two conditions additionally.

The Virasoro constraints (2.3) give the conserved Hamiltonian $H_{\rm NR}$ and a relation between the embedding parameters and the arbitrary constants C_a:

$$H_{\rm NR} = (\alpha^2-\beta^2) \sum_{a=1}^{4} \left[r_a'^2 + \frac{1}{(\alpha^2-\beta^2)^2} \left(\frac{C_a^2}{r_a^2} + \alpha^2 \omega_a^2 r_a^2 \right) \right] = \frac{\alpha^2+\beta^2}{\alpha^2-\beta^2} \frac{\kappa^2}{4}, \tag{2.12}$$

$$\sum_{a=1}^{4} C_a \omega_a + \beta (\kappa/2)^2 = 0. \tag{2.13}$$

For closed strings, r_a and f_a satisfy the following periodicity conditions

$$r_a(\xi + 2\pi\alpha) = r_a(\xi), \qquad f_a(\xi + 2\pi\alpha) = f_a(\xi) + 2\pi n_a, \tag{2.14}$$

where n_a are integer winding numbers.

The conserved charges can be computed from the definition (2.6). Using the ansatz (2.7), one can express the angular momenta as

$$J_a = \frac{2\sqrt{2\lambda}}{\alpha} \int d\sigma \; r_a(\xi)^2 \partial_0 \varphi_a, \quad a = 1, 2, 3, 4. \tag{2.15}$$

Inserting the solutions (2.8) into these, we can find

$$E_s = \frac{\kappa\sqrt{2\lambda}}{2\alpha} \int d\xi, \qquad J_a = \frac{2\sqrt{2\lambda}}{\alpha^2 - \beta^2} \int d\xi \left(\frac{\beta}{\alpha} C_a + \alpha \omega_a r_a^2 \right). \tag{2.16}$$

In view of (2.10) and (2.11), one arrives at

$$\sum_{a=1}^{4} J_a = 0. \tag{2.17}$$

This condition, first noticed in [15], appears naturally in our NR approach.

3. Two angular momenta solutions

In this section we are interested in finding string configurations corresponding to the following particular solution of (2.10) and (2.11)

$$r_1 = r_3, \quad r_2 = r_4, \quad \omega_1 = -\omega_3, \quad \omega_2 = -\omega_4.$$

Two angular velocities ω_1, ω_2 are independent and lead to strings moving in $\mathbb{CP}^3$ with two angular momenta. A special case $\omega_2 = 0$ corresponds to the cases considered in [15, 16].

3.1 Explicit solutions

We will use the parametrization

$$r_1 = r_3 = \frac{1}{\sqrt{2}}\sin\theta, \quad r_2 = r_4 = \frac{1}{\sqrt{2}}\cos\theta.$$

From the NR Hamiltonian (2.12) one finds

$$\theta'^2(\xi) = \frac{1}{(\alpha^2-\beta^2)^2}\left[\frac{\kappa^2}{4}(\alpha^2+\beta^2)-2\left(\frac{C_1^2+C_3^2}{\sin^2\theta}+\frac{C_2^2+C_4^2}{\cos^2\theta}\right)-\alpha^2\left(\omega_1^2\sin^2\theta+\omega_2^2\cos^2\theta\right)\right].$$

We further restrict ourselves to $C_2 = C_4 = 0$ to search for GM and SS solutions. Eqs. (2.11) and (2.13) give

$$C_1 = -C_3 = -\frac{\beta\kappa^2}{8\omega_1}.$$

In this case, the above equation for θ' can be rewritten in the form

$$(\cos\theta)' = \mp\frac{\alpha\sqrt{\omega_1^2-\omega_2^2}}{\alpha^2-\beta^2}\sqrt{(z_+^2-\cos^2\theta)(\cos^2\theta-z_-^2)}, \tag{3.1}$$

where

$$z_\pm^2 = \frac{1}{2(1-\frac{\omega_2^2}{\omega_1^2})}\left\{y_1+y_2-\frac{\omega_2^2}{\omega_1^2}\pm\sqrt{(y_1-y_2)^2-\left[2\left(y_1+y_2-2y_1y_2\right)-\frac{\omega_2^2}{\omega_1^2}\right]\frac{\omega_2^2}{\omega_1^2}}\right\},$$

$$y_1 = 1-\frac{\kappa^2}{4\omega_1^2}, \quad y_2 = 1-\frac{\beta^2}{\alpha^2}\frac{\kappa^2}{4\omega_1^2}.$$

The solution of (3.1) is given by

$$\cos\theta = z_+ dn\left(C\xi|m\right), \quad C = \mp\frac{\alpha\sqrt{\omega_1^2-\omega_2^2}}{\alpha^2-\beta^2}z_+, \quad m \equiv 1 - z_-^2/z_+^2, \tag{3.2}$$

where $dn\left(C\xi|m\right)$ is one of the elliptic functions.

To find the full string solution, we also need to obtain the explicit expressions for the functions f_a from (2.8)

$$f_a = \frac{1}{\alpha^2-\beta^2}\int d\xi\left(\frac{C_a}{r_a^2}+\beta\omega_a\right).$$

Using the solution for $\theta(\xi)$ in (3.2), we can find

$$f_1 = -f_3 = \frac{\beta/\alpha}{z_+\sqrt{1-\omega_2^2/\omega_1^2}}\left[C\xi - \frac{2(\kappa/2)^2/\omega_1^2}{1-z_+^2}\Pi\left(am(C\xi), -\frac{z_+^2-z_-^2}{1-z_+^2}|m\right)\right],$$
$$f_2 = -f_4 = \frac{\beta\omega_2}{\alpha^2-\beta^2}\xi.$$

Here, Π is the elliptic integrals of the third kind. As a consequence, the full string solution is given by

$$\begin{aligned}
W_1 &= \frac{R}{\sqrt{2}}\sqrt{1-z_+^2 dn^2\left(C\xi|m\right)}\; e^{i(\omega_1\tau+f_1)},\\
W_2 &= \frac{R}{\sqrt{2}}z_+ dn\left(C\xi|m\right)\; e^{i(\omega_2\tau+f_2)},\\
W_3 &= \frac{R}{\sqrt{2}}\sqrt{1-z_+^2 dn^2\left(C\xi|m\right)}\; e^{-i(\omega_1\tau+f_1)},\\
W_4 &= \frac{R}{\sqrt{2}}z_+ dn\left(C\xi|m\right)\; e^{-i(\omega_2\tau+f_2)}.
\end{aligned} \tag{3.3}$$

Let us also note that (3.3) contains both cases: $\alpha^2 > \beta^2$ and $\alpha^2 < \beta^2$, which correspond to the GM and SS strings respectively as mentioned in [24].

The geometric meaning of the explicit solutions (3.3) is as follows. Each pairs of complex coordinates, (W_1, W_2) and (W_3, W_4), describe a spiky solutions in S^2 sphere geometry but with dynamics at opposite points in the U(1) fiber. The two phases in (W_1, W_2) are exactly opposite to those of (W_3, W_4) which, together with the dynamics in U(1), ensures vanishing of the total momentum. This behavior has been also noticed for the string in $R_t \times S^2 \times S^2$ in [15].

3.2 Infinite volume limit

The GM and SS in the infinite volume can be obtained by taking $z_- \to 0$. In this limit, the solution reduces to

$$\cos\theta = \frac{\sin\frac{p}{2}}{\cosh(C\xi)},$$

where the constant $z_+ \equiv \sin p/2$ is given by

$$z_+^2 = \frac{y_2-\omega_2^2/\omega_1^2}{1-\omega_2^2/\omega_1^2} \quad (\text{GM}), \qquad \text{and} \qquad z_+^2 = \frac{y_1-\omega_2^2/\omega_1^2}{1-\omega_2^2/\omega_1^2} \quad (\text{SS}).$$

One angular momentum solutions are given by $\omega_2 = 0$. Inserting these into (2.16), one can find energy-charge dispersion relation. For the GM, the energy and angular momentum J_1 become infinite but their difference remain finite:

$$E_s - J_1 = \sqrt{J_2^2 + 8\lambda\sin^2\frac{p}{2}}. \tag{3.4}$$

While this is exactly same as that of the dyonic GM in the $AdS_5 \times S^5$, the result arises from quite different string dynamics in the $\mathbb{CP}^3$. Similarly, the dispersion relation for the SS becomes

$$E_s - \sqrt{2\lambda}\Delta\varphi = \sqrt{2\lambda}p.$$

3.3 Finite-size effects

Using the most general solutions (3.3), we can calculate the finite-size corrections to the energy-charge relation (3.4) in the limit when the string energy $E_s \to \infty$. This analysis depends crucially on the sign of the difference $\alpha^2 - \beta^2$. The GM solution corresponds to $\alpha^2 > \beta^2$ while the SS to $\alpha^2 < \beta^2$. While the string dynamics are quite different, computations are identical to the cases in the sphere geometries. Therefore, we will provide only the results here, referring technical details to [30, 18].

3.3.1 Giant magnon

We begin with the GM case, i.e. $\alpha^2 > \beta^2$. Then, one obtains from (2.16) the following expressions for the conserved string energy E_s and the angular momenta J_a

$$\begin{aligned} \mathcal{E} &= \frac{\kappa(1-\beta^2/\alpha^2)}{\omega_1 z_+ \sqrt{1-\omega_2^2/\omega_1^2}} \mathbf{K}\left(1 - z_-^2/z_+^2\right), \\ \mathcal{J}_1 &= \frac{2z_+}{\sqrt{1-\omega_2^2/\omega_1^2}} \left[\frac{1-\beta^2(\kappa/2)^2/\alpha^2\omega_1^2}{z_+^2} \mathbf{K}\left(1 - z_-^2/z_+^2\right) - \mathbf{E}\left(1 - z_-^2/z_+^2\right)\right], \qquad (3.5) \\ \mathcal{J}_2 &= \frac{2z_+\omega_2/\omega_1}{\sqrt{1-\omega_2^2/\omega_1^2}} \mathbf{E}\left(1 - z_-^2/z_+^2\right), \quad \mathcal{J}_3 = -\mathcal{J}_1, \quad \mathcal{J}_4 = -\mathcal{J}_2. \end{aligned}$$

As a result, the condition (2.17) is identically satisfied. Here, we introduced the notations

$$\mathcal{E} = \frac{E_s}{\sqrt{2\lambda}}, \qquad \mathcal{J}_a = \frac{J_a}{\sqrt{2\lambda}}. \qquad (3.6)$$

The computation of $\Delta\varphi_1$ gives

$$\begin{aligned} p \equiv \Delta\varphi_1 &= 2\int_{\theta_{\min}}^{\theta_{\max}} \frac{d\theta}{\theta'} f_1' \qquad (3.7) \\ &= -\frac{2\beta/\alpha}{z_+\sqrt{1-\omega_2^2/\omega_1^2}} \left[\frac{(\kappa/2)^2/\omega_1^2}{1-z_+^2} \Pi\left(-\frac{z_+^2-z_-^2}{1-z_+^2}\middle|1-z_-^2/z_+^2\right) - \mathbf{K}\left(1-z_-^2/z_+^2\right)\right]. \end{aligned}$$

In the above expressions, $\mathbf{K}(m)$, $\mathbf{E}(m)$ and $\Pi(n|m)$ are the complete elliptic integrals.

Expanding the elliptic integrals, we obtain

$$\begin{aligned} \mathcal{E} - \mathcal{J}_1 &= \sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)} - \frac{16\sin^4(p/2)}{\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}} \qquad (3.8) \\ &\times \exp\left[-\frac{2\left(\mathcal{J}_1 + \sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}\right)\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}\sin^2(p/2)}{\mathcal{J}_2^2 + 4\sin^4(p/2)}\right]. \end{aligned}$$

It is easy to check that the energy-charge relation (3.8) coincides with the results in [32] (and in [33] for $J_2 = 0$), which describes the finite-size effects for dyonic GM on $R_t \times S^3$ subspace of $AdS_5 \times S^5$. The difference is that in the last case the relations between $\mathcal{E}$, $\mathcal{J}_1$, $\mathcal{J}_2$ and E, J_1, J_2 are given by

$$\mathcal{E} = \frac{2\pi}{\sqrt{\lambda}} E, \qquad \mathcal{J}_1 = \frac{2\pi}{\sqrt{\lambda}} J_1, \qquad \mathcal{J}_2 = \frac{2\pi}{\sqrt{\lambda}} J_2,$$

while for strings on $R_t \times \mathbb{CP}^3$ they are written in (3.6).

3.3.2 Single spike

Now, we turn our attention to the SS case, when $\alpha^2 < \beta^2$. The computation of the conserved quantities (2.16) and $\Delta\varphi_1$ gives

$$\begin{aligned}
\mathcal{E} &= \frac{\kappa(\beta^2/\alpha^2-1)}{\omega_1\sqrt{1-\omega_2^2/\omega_1^2}z_+}\mathbf{K}\left(1-z_-^2/z_+^2\right), \\
\mathcal{J}_1 &= \frac{2z_+}{\sqrt{1-\omega_2^2/\omega_1^2}}\left[\mathbf{E}\left(1-z_-^2/z_+^2\right)-\frac{1-\beta^2(\kappa/2)^2/\alpha^2\omega_1^2}{z_+^2}\mathbf{K}\left(1-z_-^2/z_+^2\right)\right], \\
\mathcal{J}_2 &= -\frac{2z_+\omega_2/\omega_1}{\sqrt{1-\omega_2^2/\omega_1^2}}\mathbf{E}\left(1-z_-^2/z_+^2\right), \\
\Delta\varphi_1 &= -\frac{2\beta/\alpha}{\sqrt{1-\omega_2^2/\omega_1^2}z_+}\left[\frac{(\kappa/2)^2/\omega_1^2}{1-z_+^2}\Pi\left(-\frac{z_+^2-z_-^2}{1-z_+^2}|1-z_-^2/z_+^2\right)-\mathbf{K}\left(1-z_-^2/z_+^2\right)\right].
\end{aligned}$$

From these, we obtain

$$\mathcal{J}_1 = \sqrt{\mathcal{J}_2^2+4\sin^2(p/2)},$$

and

$$\mathcal{E}-\Delta\varphi_1 = p+8\sin^2\frac{p}{2}\tan\frac{p}{2}\exp\left(-\frac{\tan\frac{p}{2}(\Delta\varphi_1+p)}{\tan^2\frac{p}{2}+\mathcal{J}_2^2\csc^2 p}\right). \tag{3.9}$$

This is the leading finite-size correction to the "$E-\Delta\varphi$" relation for the SS string with two angular momenta on $R_t\times\mathbb{CP}^3$. It coincides with the string result for $R_t\times S^3$ found in [30]. As in the GM case, the difference is in the identification (3.6).

4. Concluding remarks

We have shown that the NR integrable system is particularly effective to find classical string solutions for $AdS_4\times\mathbb{CP}^3$. The extra constraints arising from $\mathbb{CP}^3$ geometry can be naturally reformulated into simple conditions under the NR ansatz. In addition to the GM and SS solutions moving in $R_t\times S^2\times S^2$ with a single angular momentum, the NR system can be used to study more complicated string dynamics. As shown in this paper, the GM and SS solutions in $\mathbb{CP}^3$ with two angular momenta solutions can be described in the same way as the cases in the sphere geometries. These solutions describe the strings moving in $R_t\times\mathbb{CP}^3$ where the two angular momenta in one S^2 are opposite to those in the other S^2 executing motion on S^1 in the U(1) Hopf fibration over $S^2\times S^2$. Of course, the extra constraints are limiting the possible string configurations. It would be interesting to find other configurations which could be found within the context of the NR system (some solutions are given in the appendix). Another interesting feature would be the relation of the NR integrable system to other classical integrable systems such as complex sine-Gordon model as has been shown for the type IIB string theory on $AdS_5\times S^5$ in [30].

The two angular momenta string states are related to the composite operators in the gauge theory side. The BPS state corresponding to the string vacuum is $\mathrm{tr}\left[(A_1B_1)^L\right]$ where A_1 and B_1 are the scalar fields of $\mathcal{N}=6$ Chern-Simons theory in the bifundamental representation $(\mathbf{N}, \bar{\mathbf{N}})$ and $(\bar{\mathbf{N}}, \mathbf{N})$, respectively. The excited states are obtained by replacing these fields with fields in the theory. The composite operators dual to the string with two angular momenta with the dispersion relation (3.4) should be

$$\mathrm{tr}\left[(A_1 B_1)^{J_1}(A_2 B_2)^{J_2}\right] + \cdots ,$$

where the ellipsis represents the permutations of the fields while maintaining the alternating spin chain structure and A_2 and B_2 are another scalar fields.

It would be interesting to compare the energy-charge dispersion relation we have obtained here with the solutions of all-loop Bethe ansatz equations recently proposed in [8]. The finite-size corrections can not be derived solely from the Bethe ansatz equations. Instead, one can use the Lüscher correction formulation based on exact S-matrix which has been particularly powerful in the AdS/CFT correspondence [34]. It would be interesting to compute the corrections based on a recently proposed S-matrix [9] and compare with our results in the large 't Hooft coupling limit.

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A. More string solutions on $R_t \times \mathbb{CP}^3$

Basic facts about $\mathbb{CP}^3$. Let us explain briefly the basic properties of the $\mathbb{CP}^3$ spaces. It is most convenient to define an n-dimensional complex projective space $\mathbb{CP}^n$ as the family of one-dimensional subspaces in $\mathbb{C}^{n+1}$, i.e. this is the quotient $\mathbb{C}^{n+1}/(\mathbb{C}\setminus\{0\})$. The equivalence relation is defined as

$$\alpha Z_1 : \cdots : \alpha Z_{n+1} = Z_1 : \cdots : Z_{n+1}.$$

The space $\mathbb{CP}^n$ itself is covered by patches $U_i \, : \, \{Z_1 : \cdots : Z_{n+1} \in \mathbb{CP}^n \,|\, Z_i \neq 0\}$, $i = 1, \ldots, n+1$. One can see that each patch U_i is isomorphic to $\mathbb{CP}^n$, where the isomorphism is defined by $W_j^{(i)} = Z_j/Z_i$, $j \neq i$. One can choose local coordnates $W = (W_1, W_2, \ldots, W_n)^t \in \mathbb{C}^{n+1}$ with $W_j \equiv W_j^{(n+1)}$. The Fubini-Study metric then is given by the line element

$$ds^2 = \frac{(1+|W|^2)|dW|^2 - |W^\dagger dW|^2}{(1+|W|^2)^2}.$$

One can think of $\mathbb{CP}^n$ as the homogeneous space $\mathbb{CP}^n = \mathrm{U}(n+1)/(\mathrm{U}(n)\times\mathrm{U}(1))$. The $u(n+1)$ Lie algebra $\mathfrak{f}$ can be realized as anti-hermitian matrices and splits into two parts: $\mathfrak{p} = u(n) \oplus u(1)$ and its orthogonal completion $\mathfrak{cp}(n)$ with respect to the $\mathrm{U}(n+1)$ Killing for

$$\begin{aligned} \mathfrak{p} &= u(n) \oplus u(1) = \{iM \in u(n+1) \,|\, [\Gamma, M] = 0\} \\ \mathfrak{cp}(n) &= \{iM \in u(n+1) \,|\, \{\Gamma, M\} = 0\}, \end{aligned}$$

where M is traceless and hermitian and

$$\Gamma = \begin{pmatrix} \mathbf{1}_n & \\ & -1 \end{pmatrix}.$$

A generator of $\mathfrak{cp}(n)$ part, $\mathbf{B}$ then is given by

$$\mathbf{B} = \begin{pmatrix} & W^\dagger \\ -W & \end{pmatrix}.$$

Then one can write schematically

$$\mathfrak{f} = \mathfrak{p} \oplus \mathfrak{cp}, \quad [\mathfrak{p}, \mathfrak{p}] \subset \mathfrak{p}, \quad [\mathfrak{p}, \mathfrak{cp}] \subset \mathfrak{cp}, \quad [\mathfrak{cp}, \mathfrak{cp}] \subset \mathfrak{cp}.$$

More strings on $R_t \times \mathbb{CP}^3$ and NR system. The next step is to consider concrete solutions, taking into account all of the existing constraints, which can be summarized as follows

$$\sum_{a=1}^{4} r_a^2 = 1, \quad \sum_{a=1}^{4} \omega_a r_a^2 = 0, \quad \sum_{a=1}^{4} C_a = 0, \quad \sum_{a=1}^{4} C_a \omega_a + \beta(\kappa/2)^2 = 0.$$

From the first two equalities we can express two of the r_a coordinates through the remaining ones. Then, we are left only with relations between the parameters. In order to be able to compare with the known particular solutions, we choose to express $r_{1,3}^2$ as functions of $r_{2,4}^2$. The general solution is

$$r_1^2 = \frac{\omega_3(1 - r_2^2 - r_4^2) + \omega_2 r_2^2 + \omega_4 r_4^2}{\omega_3 - \omega_1}, \qquad r_3^2 = \frac{\omega_1(1 - r_2^2 - r_4^2) + \omega_2 r_2^2 + \omega_4 r_4^2}{\omega_1 - \omega_3}. \tag{A.1}$$

In particular, for $\omega_1 = -\omega_3$, $\omega_2 = \omega_4 = 0$, we have $r_1^2 = r_3^2$. The case considered in [15, 16] is reached after fixing $r_2^2 = r_4^2$, $r_1^2 + r_2^2 = 1/2$.

Denoting dynamical variables as

$$r_1 = r_3 = \frac{r}{\sqrt{2}}, \quad r_2 = r_4 = \frac{\sqrt{1 - r^2}}{\sqrt{2}},$$

one can describe the system by only one independent variable r. The first order differential equation for r can be obtained either from the equations of motion (integrating them once) or from the Virasoro constraints. It goes as follows

$$\sum_a \left[(\alpha^2 - \beta^2) {r'_a}^2 + \frac{C_a^2}{\alpha^2 - \beta^2} \frac{1}{r_a^2} + \frac{\alpha^2}{\alpha^2 - \beta^2} \omega_a^2 r_a^2 + \frac{2\beta C_a \omega_a}{\alpha^2 - \beta^2} \right] = (\kappa/2)^2$$

$$\Rightarrow \quad (1 - \beta^2) \frac{r'^2}{1 - r^2} + \frac{4}{1 - \beta^2} \frac{C^2}{r^2} + \frac{1}{1 - \beta^2} \omega^2 r^2 + \frac{4\beta C\omega}{1 - \beta^2} = (\kappa/2)^2 \tag{A.2}$$

Here, without loss of generality, we set $\alpha = 1$. Using the constraint

$$2C\omega + \beta(\kappa/2)^2 = 0$$

we find

$$
\begin{aligned}
(1-\beta^2)^2 r'^2 &= (1-r^2)\left\{(1+\beta^2)(\kappa/2)^2 - \tfrac{4C^2}{r^2} - \omega^2 r^2\right\} \\
&= -\omega^2 \tfrac{(1-r^2)}{r^2}\left\{\tfrac{4C^2}{\omega^2} - (1+\beta^2)\tfrac{(\kappa/2)^2}{\omega^2} r^2 + r^4\right\}
\end{aligned} \tag{A.3}
$$

The right hand side determines the turning points $r'^2 = 0$ and they are three. In order the string to extends to the equator of the sphere, one must choose $r = 1$. To find a solution of the type we are looking for, $r^2 = 1$ has to be double zero of the right hand side of (A.3). The latter conditions leads to the following constraints

$$
(1+\beta^2)(\kappa/2)^2 = \omega^2 + 4C^2, \quad 2C\omega + \beta(\kappa/2)^2 = 0
$$

which can be obtained either by substituting $r = 1$ in the right hand side of (A.3) or from the Virasoro constraints. The correct choice for the parameters solving the above equation and giving GM type string solutions is

$$
\kappa/2 = \omega, \quad \alpha = 1, \quad \beta = -\frac{2C}{\omega} \tag{A.4}
$$

Let us turn to the solutions developing a SS in the $R_t \times S^2 \times S^2$ subspace. This configuration can be realized in terms of the NR integrable system with specific choice of the parameters. The solutions we are looking for are characterized by large quantum numbers, especially large energy. The careful analysis shows that in order to have such solutions one has to choose the parameters in a specific way. The "spiky" choice for the parameters, namely the choice giving solutions with a SS but infinitely wound around the equator, is slightly different from the case of the GM. In fact the constraints on the parameters are the same, but instead of the choice (A.4), now we choose the other solution to the constraint

$$
\kappa = 2C, \quad \beta = -\frac{2\omega C}{\kappa^2} = -\frac{\omega}{2C}
$$

The equation for the variable r is the same, but the parameters are fixed differently

$$
\frac{du}{d\xi} = u' = \frac{2\omega}{1-\beta^2}(1-u)\sqrt{u-\bar{u}}.
$$

Above we use the following notations

$$
\begin{aligned}
u &= r^2 = \sin^2\theta, \quad \bar{u} = \frac{4C^2}{\omega^2}, \\
d\xi &= \frac{du}{u'} = \frac{(1-\beta^2)\,du}{2\omega(1-u)\sqrt{u-\bar{u}}} = \frac{(4C^2-\omega^2)\,du}{8C^2\omega(1-u)\sqrt{u-\bar{u}}}
\end{aligned} \tag{A.5}
$$

The conserved quantities are

$$
\begin{aligned}
E &= \kappa T \int d\xi \\
J &= \frac{C\beta}{(1-\beta^2)} \int d\xi + \frac{\omega}{(1-\beta^2)} T \int u\, d\xi \\
\Delta\phi &= \frac{C}{(1-\beta^2)} \int \frac{d\xi}{u} + \frac{\beta\omega}{(1-\beta^2)} \int d\xi.
\end{aligned}
$$

To find finite results (which is so for $E - J$ in the GM case) we consider

$$E - T\Delta\phi = \frac{2CT}{\omega} \arccos \sqrt{\bar{u}} = \frac{\sqrt{\lambda}}{\pi} \bar{\theta}$$

where

$$\bar{\theta} = \frac{\pi}{2} - \theta_0$$

For the total spin J we get

$$J = \frac{2T\omega}{\omega} \cos\theta_0 = 2T \sin\bar{\theta}$$

All this implies finally

$$\Delta = (E - T\Delta\phi) - J = \frac{\sqrt{\lambda}}{\pi}(\bar{\theta} - \sin\bar{\theta})$$

which completes our result on SS case with this ansatz. We note that the solution in this case is

$$\sin\theta = \tanh\left(\frac{\omega\bar{z}}{1-\beta^2}(\xi - \xi_0)\right). \tag{A.6}$$

Now we consider more general solutions. It is reasonable to ask for symmetric motion on the subspace $S^2 \times S^2$ and therefore to set

$$r_1^2 + r_2^2 = \frac{1}{2}, \quad r_3^2 + r_4^2 = \frac{1}{2}.$$

Then one can use the parametrization

$$r_1^2 + r_3^2 = r^2, \quad r_2^2 + r_4^2 = 1 - r^2.$$

Having in mind that the total worldshhet momentum has to be zero, one can set

$$C_1 = -C_3, \quad \omega_1 = -\omega_3 = \omega.$$

It follows then that

$$C_2 = -C_4.$$

Using the above constraints one can find

$$\left(1 - \frac{\omega_2}{\omega}\right) r_2^2 - \left(\frac{\omega_4}{\omega} + 1\right) r_4^2 = 0.$$

There are two cases:

a) $\omega_2 = \omega$ which entails $\omega_4 = -\omega$ — this choice slightly generalizes the case considered above;

b) r_2 and r_4 are proportional

$$r_2^2 = \frac{\omega_4 + \omega}{\omega - \omega_2} r_4^2 := \Gamma^2 r_4^2.$$

Let us consider the last possibility in more details. The constraints tells us that

$$\begin{aligned} r_2^2 &= \frac{\Gamma^2}{1+\Gamma^2}(1-r^2), \quad r_4^2 = \frac{1}{1+\Gamma^2}(1-r^2) \\ r_1^2 &= \frac{\omega(1+\Gamma^2) + \omega_2\Gamma^2 + \omega_4}{\omega(1+\Gamma^2)} r^2 - \frac{\omega_2\Gamma^2 + \omega_4}{\omega(1+\Gamma^2)}. \end{aligned} \tag{A.7}$$

Rewriting the last equality as

$$r_1^2 = (1+b^2)r^2 - b^2, \quad b^2 = \frac{\omega_2\Gamma^2 + \omega_4}{\omega(1+\Gamma^2)},$$

one can find the lower bound

$$r_{\min} \le r, \quad r_{\min} = \frac{b}{\sqrt{1+b^2}}.$$

It is better to use $y^2 = (1+b^2)x^2 = (1+b^2)(1-r^2)$ and then the radial dynamical variables become

$$r_1^2 = 1 - y^2, \quad r_2^2 = c^2 y^2, \quad r_3^2 = d^2 y^2, \quad r_4^2 = (1-c^2)y^2, \tag{A.8}$$

where

$$c^2 = \frac{\Gamma^2}{(1+b^2)(1+\Gamma^2)}, \quad d^2 = \frac{b^2}{(1+b^2)}.$$

The Virasoro constraints give

$$(\alpha^2-\beta^2)\frac{y'^2}{1-y^2} + \frac{1}{\alpha^2-\beta^2}\left[\frac{C_1^2}{1-y^2} + \frac{\tilde{C}^2}{y^2}\right] + \frac{\alpha^2}{\alpha^2-\beta^2}\left[\omega_1^2 + \tilde{\omega}^2 y^2\right] = \frac{\alpha^2+\beta^2}{\alpha^2-\beta^2}(\kappa/2)^2, \tag{A.9}$$

where

$$\begin{aligned} \tilde{\omega}^2 &= \frac{(\omega_2^2-\omega_1^2)\Gamma^2 + (\omega_4^2-\omega_1^2)}{(1+b^2)(1+\Gamma^2)}, \qquad \tilde{C}^2 = \tilde{C}_2^2 + \tilde{C}_3^2 + \tilde{C}_4^2, \qquad \tilde{C}_2^2 = \frac{C_2^2(1+b^2)}{c^2}, \\ \tilde{C}_3^2 &= \frac{C_3^2(1+b^2)}{b^2}, \qquad \tilde{C}_4^2 = \frac{C_4^2(1+b^2)}{1-c^2}. \end{aligned}$$

The system we obtained has the same type solutions as in the previous considerations and can be solved in terms of elliptic functions. Note that all the spins are different, so we find multi-spin solutions.

It is a standard procedure to bring the above equation into a Weierstrass form. To do that we define

$$\tilde{\xi} = \frac{\tilde{\omega}}{1-\beta^2}\xi, \quad \zeta = \frac{\tilde{a}}{3} + y^2,$$

and rewrite the equation (A.9) as

$$\left(\frac{d\zeta}{d\tilde{\xi}}\right)^2 = 4\zeta^3 - g_2\zeta - g_3, \tag{A.10}$$

with

$$g_2 = \frac{\tilde{a}^2}{3} - \tilde{b}, \quad g_3 = \tilde{C}^2 = \frac{\tilde{a}\tilde{b}}{3} + \frac{2}{27}\tilde{a}^3,$$

where

$$\tilde{a} = \frac{\omega_1^2}{\tilde{\omega}^2} - 1 - \frac{1+\beta^2}{\tilde{\omega}^2}(\kappa/2)^2, \quad \tilde{b} = \frac{1+\beta^2}{\tilde{\omega}^2}(\kappa/2)^2 + \frac{\tilde{C}^2 - \omega_1^2 - C_1^2}{\tilde{\omega}^2}.$$

The solution is

$$\zeta = e_3 - e_{31}\mathrm{dn}^2\left(\sqrt{e_{31}}\tilde{\xi}, \tilde{\kappa}\right) \tag{A.11}$$

where e_i are the roots of the rhs of (A.10), $e_{mn} = e_m - e_n$ and the modulus is defined by $\tilde{\kappa} = e_{21}/e_{31}$. Going back to the variable r we get

$$r^2 = 1 - \frac{1}{1+\beta^2}\left(e_3 - \frac{a}{3}\right) + \frac{e_{31}}{1+\beta^2}\mathrm{dn}^2\left(\sqrt{e_{31}}\frac{\tilde{\omega}}{1-\beta^2}\xi, \kappa\right).$$

Making specific choice of the parameters as in the above, one can get either GM or SS solutions. The dispersion relations can be obtained using the explicit form of the charges (2.16) and the relevant constraints.

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