Giant magnonlike solution in $Sch_5 \times S^5$

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In this paper we derive a giant magnonlike solution for a string theory in a Schrödinger spacetime $Sch_5 \times S^5$ which is holographic dual to a dipole-deformed super-Yang-Mills theory. This classical string state is pointlike in Sch_5 but stringy in a S^2 subspace of S^5. We find the string solution and the energy-charge relation exactly for an infinite angular momentum as well as for a finite one which shows an explicit *finite-size correction*.

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I. INTRODUCTION

Integrability is a key feature in the AdS/CFT duality. It plays an important role in finding exact solutions of both string theories in the anti–de Sitter (AdS) space and gauge theories on their boundaries. It is powerful enough to determine the complete spectrum nonperturbatively for certain well-known cases. Efforts have been made to find new dualities which have more general applicability. Some of these maintain the integrability, and hence, potential to find exact solutions. In these cases, various classical string solutions and their energy spectrum allow for quantitative understanding of the duality in the strong coupling regime.

Dipole deformations of $\mathcal{N} = 4$ super-Yang-Mills theory are interesting developments in this direction. With minimal nonlocality, imposed by a null dipole deformation in the light-cone direction, the resulting theory maintains a nonrelativistic conformal symmetry in three-dimensional perpendicular directions [1–4]. Furthermore, a dual supergravity background has been worked out and identified with the "Schrödinger" spacetime geometry.

An interesting issue is if this duality can be established in the context of the integrability. One can think of the dipole deformations as the integrable Yang-Baxter deformations of $AdS_5 \times S^5$ string [5] as shown in Refs. [6,7]. A unified point of view on these integrability structures has been provided in Ref. [8]. More recently, this issue was addressed again in Ref. [9] from the view point of the Drinfeld twist [10,11] in the same way as other $\star$-product deformations [12]. Considering a specific dipole deformation with the $Sch_5 \times S^5$ target space, the authors have obtained semiclassical solutions such as spinning Berenstein-Maldacena-Nastase (BMN)-like strings [9] (see also Ref. [13]).

In this paper, we focus on a giant magnon in $Sch_5 \times S^5$ space. The original giant magnon is constructed in a $R \times S^2$ subspace of $AdS_5 \times S^5$ and is mapped to a classical sine-Gordon soliton solution [14]. The energy and other conserved charges give strong support for the conjectured all-loop spin chain. This solution also exists in several deformed AdS/CFT dualities and provides quantitative understandings in the strong coupling limit [15]. We look for a giant magnonlike solution in $Sch_5 \times S^5$ along with the energy-charge relation, which reduces to the original one when the deformation is turned off. Also, in the supergravity limit, our solution reproduces the supersymmetric BMN-like solution. Furthermore, we have also computed the *finite-size* correction to the energy spectrum derived from the exact classical string solution with *a finite angular momentum*. We hope these results can be useful to clarify the AdS/CFT duality of the dipole-deformed theory.

This paper is organized as follows. In Sec. II we introduce the metric of the "Schrödinger" spacetime and impose conformal gauge in the Polyakov string action along with Virasoro constraints. In Sec. III we obtain the giant magnonlike solution and corresponding energy-charge relation when the angular momentum in S^2 is infinite. More general results of the solution for the case of a finite angular momentum are derived in Sec. IV where an explicit finite-size correction is presented. We conclude the paper in Sec. V with some comments and future research directions.

II. THE STRING LAGRANGIAN AND VIRASORO CONSTRAINTS

According to Ref. [9], the metric on $Sch_5 \times S^5$ in global coordinates can be written as

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$$\frac{ds^2}{l^2} = -\left(1 + \frac{\mu^2}{Z^4}\right)dt^2 + \frac{2dtdV - \vec{X}^2dt^2 + d\vec{X}^2 + dZ^2}{Z^2} + (d\psi + P)^2 + ds^2_{CP^2}, \tag{2.1}$$

where

$$ds^2_{CP^2} = d\theta_1^2 + \frac{1}{4}\sin^2\theta_1[\cos^2\theta_1(d\phi_1 + \cos\theta_2 d\phi_2)^2 + d\theta_2^2 + \sin^2\theta_2 d\phi_2^2], \tag{2.2}$$

$$P = \frac{1}{2}\sin^2\theta_1(d\phi_1 + \cos\theta_2 d\phi_2), \tag{2.3}$$

and the metric on S^5 is given as a $U(1)$ Hopf fibre over CP^2 [13]. The parameter μ deforms the usual AdS space. The B field is given by

$$\alpha' B = l^2 \frac{\mu}{Z^2} dt \wedge (d\psi + P). \tag{2.4}$$

We set the AdS radius l and the inverse string tension α' to $l = 1, \alpha' = 1$ which fixes the string tension to $T = 1/(2\pi)$ from now on.

We focus on a S^2 subspace of S^5 by choosing

$$\phi_1 = 0, \qquad \theta_2 = \frac{\pi}{2} \tag{2.5}$$

which lead to

$$P = 0; \qquad ds^2_{CP^2} \to ds^2 = d\theta_1^2 + \frac{1}{4}\sin^2\theta_1 d\phi_2^2. \tag{2.6}$$

We further restrict the string solutions in a subspace of Sch_5 by setting $\vec{X} = 0, Z = Z_0$. The background then simplifies to

$$\begin{aligned} ds^2 &= g_{MN}dX^M dX^N \\ &= -\left(1 + \frac{\mu^2}{Z_0^4}\right)dt^2 + \frac{2}{Z_0^2}dtdV + d\psi^2 + d\theta^2 + \sin^2\theta d\phi^2, \\ B &= b_{MN}dX^M dX^N = \frac{\mu}{Z_0^2}dt \wedge d\psi, \end{aligned} \tag{2.7}$$

with the new definition of the angular coordinates $\theta \equiv \theta_1$, $\phi \equiv \phi_2/2$.

In our consideration we will use a conformal gauge in the Polyakov string action, in which the string Lagrangian has the form

$$\mathcal{L}_s = \frac{T}{2}(G_{00} - G_{11} + 2B_{01}) \tag{2.8}$$

along with the Virasoro constraints

$$G_{00} + G_{11} = 0, \tag{2.9}$$

$$G_{01} = 0, \tag{2.10}$$

where the induced metric and the B fields are given by

$$G_{mn} = g_{MN}\partial_m X^M \partial_n X^N, \qquad B_{mn} = b_{MN}\partial_m X^M \partial_n X^N,$$
$$m, n = 0, 1; \qquad M, N = 0, 1, \ldots, 9,$$

where the derivatives are with respect to world-sheet coordinates $\eta^0 = \tau, \eta^1 = \sigma$.

We consider the following string embedding for the coordinates in Eq. (2.7):

$$\begin{aligned} &t = \kappa\tau, \qquad V = \mu^2 m\tau, \qquad \psi = \omega_1\tau, \\ &\phi = \omega\tau + f(\xi), \qquad \theta = \theta(\xi), \qquad \xi = \sigma - v\tau. \end{aligned} \tag{2.11}$$

This choice implies that the string moves as a point particle in the subspace of Sch_5 while a stringlike motion appears only in S^2. We assume that the speed v of the string in S^2 satisfies $v < 1$. In a particular limit where the string collapses to a point even in S^2, the configuration reduces to the spinning BMN-like solution considered in Ref. [9] if the constant coordinate Z_0 is fixed to

$$Z = Z_0 = \sqrt{\frac{\kappa}{m}}. \tag{2.12}$$

Inserting Eq. (2.11) into Eq. (2.8), one can reduce the string Lagrangian to an effective one-dimensional one (a prime is used for $d/d\xi$)

$$L = -\frac{T}{2}((1 - v^2)\theta'^2 + \kappa^2 - \beta^2\omega^2 + [(1 + v)f' - \omega] \times [(1 - v)f' + \omega]\sin^2\theta). \tag{2.13}$$

Here we have introduced a deformation parameter β defined by

$$\beta^2 \equiv \frac{\omega_1^2 + \mu^2 m^2}{\omega^2}, \tag{2.14}$$

which implies a relative speed of the string motion in Sch_5 with respect to that in S^2. The equation of motion from Eq. (2.13) gives a solution for f [1]

$$f'(\xi) = \frac{1}{1 - v^2}\left(\frac{C}{\sin^2\theta} - v\omega\right), \tag{2.15}$$

where the integration constant C is to be fixed shortly.

[1] Instead of the effective Lagrangian, the same result for f can be obtained from the Virasoro constraints (2.9)–(2.10) based on the full string Lagrangian and along with the embedding coordinates (2.7).

From Eqs. (2.11) and (2.15), the Virasoro constraints (2.9)–(2.10) can be written as

$$\theta'^2 = \frac{1}{1+v^2}\left[\kappa^2 - \beta^2\omega^2 + \frac{4Cv\omega}{(1-v^2)^2} - \frac{(1+v^2)(C^2\csc^2\theta + \omega^2\sin^2\theta)}{(1-v^2)^2}\right], \tag{2.16}$$

$$C\omega = v(\kappa^2 - \beta^2\omega^2). \tag{2.17}$$

The first Virasoro constraint (2.16) is equivalent to the first integral of the equation of motion for θ as shown in general case [16]. The second constraint determines the integration constant C. After inserting it into Eq. (2.16) one finds that the nonisometric coordinate θ satisfies a first-order ordinary differential equation:

$$\theta'^2 = \frac{(\kappa^2 - \beta^2\omega^2 - \omega^2\sin^2\theta)(\omega^2\sin^2\theta - v^2(\kappa^2 - \beta^2\omega^2))}{(1-v^2)^2\omega^2\sin^2\theta}. \tag{2.18}$$

We will solve this equation in infinite and finite volumes in the next two sections.

III. THE GIANT MAGNONLIKE SOLUTION IN INFINITE VOLUME

A. Solution

The giant magnon was introduced in Ref. [14] as a string image on S^2, which is dual to the magnon excitation of the spin chains arising from the gauge theory. The geometric meaning of the momentum carried by the magnon is a deficit angle of ϕ for the infinite-size string on the equator $\theta = \pi/2$ in S^2. Therefore, we impose the following condition on a giant magnonlike solution in infinite volume:

$$\theta'^2 = 0 \quad \text{for } \theta = \frac{\pi}{2}. \tag{3.1}$$

For the case of the finite volume, this condition should be relaxed as we will see in the next section.

Along with Eq. (2.18), this condition requires

$$(\kappa^2 - \beta^2\omega^2 - \omega^2)(\omega^2 - v^2(\kappa^2 - \beta^2\omega^2)) = 0. \tag{3.2}$$

Among two solutions of this, we choose

$$\kappa^2 = (1+\beta^2)\omega^2, \tag{3.3}$$

because this choice is consistent with that of the undeformed giant magnon $\kappa = \omega$ upon setting $\beta = 0$ ($\omega_1 = 0, \mu = 0$). The condition (3.3) also reproduces the Virasoro constraint ($\kappa^2 = \omega_1^2 + \mu^2 m^2$) for the BMN-like solution in the $\omega \to 0$ limit [9]. Along with Eq. (2.17), this also fixes $C = v\omega$.

With this choice, Eqs. (2.18) and (2.15) are simplified to

$$\theta'^2 = \frac{\omega^2\cos^2\theta(\sin^2\theta - v^2)}{(1-v^2)^2\sin^2\theta},$$
$$f'(\xi) = \frac{v\omega}{1-v^2}\left(\frac{1}{\sin^2\theta} - 1\right). \tag{3.4}$$

The solution for θ is given by

$$\cos\theta(\xi) = \sqrt{1-v^2}\,\mathrm{sech}\left(\frac{\omega}{\sqrt{1-v^2}}\xi\right). \tag{3.5}$$

Inserting this into Eq. (2.15), one can find the solution for the isometric coordinate ϕ on S^2:

$$\phi = \omega\tau + \mathrm{arccot}\left[\frac{v}{\sqrt{1-v^2}}\coth\left(\frac{\omega}{\sqrt{1-v^2}}\xi\right)\right]. \tag{3.6}$$

These are exactly the Hofman-Maldacena solution for the infinite-size giant magnon [14]. The only difference is the deformed parametric relation (3.3).

B. The energy-charge relation

The conserved charges associated with the isometric coordinates t, V, ψ and ϕ are the string energy E_s, spin M and two angular momenta J_1 and J, respectively. In the infinite-volume limit $L \to \infty$, these conserved charges are given by

$$E_s = \int_{-L/2}^{L/2} d\sigma \frac{\partial\mathcal{L}_s}{\partial(\partial_0 t)} = T\kappa\int_{-L/2}^{L/2} d\sigma = T\kappa L,$$
$$M = \int_{-L/2}^{L/2} d\sigma \frac{\partial\mathcal{L}_s}{\partial(\partial_0 V)} = Tm\int_{-L/2}^{L/2} d\sigma = TmL,$$
$$J_1 = \int_{-L/2}^{L/2} d\sigma \frac{\partial\mathcal{L}_s}{\partial(\partial_0 \psi)} = T\omega_1\int_{-L/2}^{L/2} d\sigma = T\omega_1 L,$$
$$J = \int_{-L/2}^{L/2} d\sigma \frac{\partial\mathcal{L}_s}{\partial(\partial_0 \phi)} = T\omega\left[\int_{-L/2}^{L/2} d\sigma - \frac{1}{(1-v^2)}\int_{-L/2}^{L/2}\cos^2\theta d\sigma\right] = T[\omega L - 2\sqrt{1-v^2}]. \tag{3.7}$$

While each of these charges diverges, we consider a combination which generates a finite quantity as follows:

$$
\begin{aligned}
E_s-\sqrt{\mu^2M^2+J_1^2+J^2}
&=T\kappa L-\sqrt{(\mu TmL)^2+(T\omega_1L)^2+(TL\omega)^2\left[1-\frac{2\sqrt{1-v^2}}{L\omega}\right]^2}\\
&=TL\left\{\kappa-\sqrt{1+\beta^2}\omega+\frac{2}{\sqrt{1+\beta^2}}\frac{\sqrt{1-v^2}}{L}\right\}\\
&=\frac{2T}{\sqrt{1+\beta^2}}\sqrt{1-v^2}, \qquad (3.8)
\end{aligned}
$$

where we used Eq. (3.3) at the last line. This particular combination is chosen in such a way that it reduces to the undeformed energy-charge relation when the deformation is turned off and to the supersymmetric BMN-like solution in the supergravity limit.

The deficit angle in the ϕ variable can be computed as

$$\Delta\phi=\int_{-\infty}^{\infty}f'(\xi)d\xi=\arccos v\to v=\cos\frac{\Delta\phi}{2}. \qquad (3.9)$$

In the undeformed AdS/CFT, the deficit angle $\Delta\phi$ is identified with the momentum p of the excitations on the world sheet or the magnon excitations of the corresponding spin chain [14]. It is not clear whether this identification ($\Delta\phi\equiv p$) is valid in this deformed case since the physical excitations are yet unknown. Even so, we will still refer to the deficit angle as "momentum" p whose real physical meaning may be clarified in future works. Then, the energy-momentum dispersion relation becomes

$$E_s-\sqrt{\mu^2M^2+J_1^2+J^2}=\frac{2T}{\sqrt{1+\beta^2}}\left|\sin\frac{p}{2}\right|. \qquad (3.10)$$

This result shows how the deformation affects the energy-momentum dispersion relation of the giant magnon-like string state in $Sch_5\times S^5$. In the undeformed limit $\beta=0$ ($\omega_1=0,\mu=0$), this reduces to that of the ordinary giant magnon. In the pointlike limit ($p=0$, $J=0$), this relation reduces to that of the spinning BMN-like strings considered in Ref. [9].

IV. THE GIANT MAGNONLIKE SOLUTION IN FINITE VOLUME

A. Solution

One can solve Eq. (2.18) by relaxing the condition (3.1). For this purpose, we introduce the new variables

$$\chi=\cos^2\theta, \qquad W=\frac{\kappa^2}{\omega^2}-\beta^2, \qquad (4.1)$$

with which we can rewrite Eq. (2.18) as

$$\chi'^2=\frac{4\omega^2}{(1-v^2)^2}\chi(\chi_p-\chi)(\chi-\chi_m), \qquad (4.2)$$

where χ_m and χ_p are given by

$$\chi_p=1-v^2W, \qquad \chi_m=1-W. \qquad (4.3)$$

The solution of Eq. (4.2) is then given by

$$
\begin{aligned}
\xi&=\frac{1-v^2}{2\omega}\int\frac{d\chi}{\sqrt{\chi(\chi_p-\chi)(\chi-\chi_m)}}\\
&=\frac{1-v^2}{\omega}\frac{1}{\sqrt{\chi_p}}\mathbf{F}\left(\arcsin\sqrt{\frac{\chi_p-\chi}{\chi_p-\chi_m}},1-\frac{\chi_m}{\chi_p}\right), \qquad (4.4)
\end{aligned}
$$

where $\mathbf{F}$ is the incomplete elliptic integral of the first kind. Inserting Eq. (4.4) into Eq. (2.15), one can find the solution for the isometric coordinate ϕ on S^2:

$$\phi=\omega\tau+\frac{v\omega\xi}{v^2-1}+\frac{vW\Pi\left[\frac{\chi_p-\chi_m}{\chi_p-1},\mathbf{am}\left[\frac{\sqrt{\chi_p}\omega\xi}{-1+v^2},1-\frac{\chi_m}{\chi_p}\right],1-\frac{\chi_m}{\chi_p}\right]\mathbf{dn}\left[\frac{\sqrt{\chi_p}\omega\xi}{-1+v^2},1-\frac{\chi_m}{\chi_p}\right]}{(\chi_p-1)\sqrt{\chi_p+(\chi_m-\chi_p)\mathbf{sn}\left[\frac{\sqrt{\chi_p}\omega\xi}{-1+v^2},1-\frac{\chi_m}{\chi_p}\right]^2}}, \qquad (4.5)$$

where $\mathbf{am},\mathbf{dn},\mathbf{sn}$ are the JacobiAmplitude, JacobiDN, and JacobiSN functions, respectively, and $\mathbf{\Pi}$ is the complete elliptic integral of the third kind.

B. The conserved quantities

Again, the conserved charges associated with the isometric coordinates t, V, ψ and ϕ are the string energy E_s, spin M and two angular momenta J_1 and J as introduced in Eq. (3.7). By changing the integration variable from $d\xi$ to $d\chi/\chi'$ and the integration range from $(-L/2,L/2)$ to (χ_m,χ_p) for the finite-volume L, we can express the charges by

$$E_s=2T\frac{(1-v^2)\kappa}{\omega\sqrt{\chi_p}}\mathbf{K}(1-\epsilon), \qquad (4.6)$$

$$M=2T\frac{(1-v^2)m}{\omega\sqrt{\chi_p}}\mathbf{K}(1-\epsilon), \qquad (4.7)$$

$$J_1 = 2T \frac{(1-v^2)\omega_1}{\omega\sqrt{\chi_p}} \mathbf{K}(1-\epsilon), \tag{4.8}$$

$$J = 2T\sqrt{\chi_p}[\mathbf{K}(1-\epsilon) - \mathbf{E}(1-\epsilon)], \tag{4.9}$$

where we have introduced the useful parameter

$$\epsilon = \frac{\chi_m}{\chi_p}, \tag{4.10}$$

and $\mathbf{K}$, $\mathbf{E}$ are the complete elliptic integrals of the first and the second kinds. These charges diverge in the large-volume limit since $\mathbf{K}(1-\epsilon) \to \infty$ as $\epsilon \to 0$ from Eqs. (4.1) and (4.3). But the combination in Eq. (3.8) should remain finite. The deficit angle $\Delta\phi$ can be similarly obtained as

$$\Delta\phi = \frac{2v}{\sqrt{\chi_p}} \left[\frac{1}{v^2} \Pi\left(-\frac{\chi_p}{1-\chi_p}(1-\epsilon), 1-\epsilon \right) - \mathbf{K}(1-\epsilon) \right]. \tag{4.11}$$

C. The energy-charge relation

From the explicit expressions for the charges, the energy dispersion relation of the giant magnon given in Eq. (3.10) can be expressed by

$$E_s - \sqrt{\mu^2 M^2 + J_1^2 + J^2} = 2T \frac{(1-v^2)}{\sqrt{1-v^2 W}} \mathbf{K}(1-\epsilon) \left[\sqrt{\beta^2 + W} - \sqrt{\beta^2 + \left[\frac{1-v^2 W}{1-v^2} \left(1 - \frac{\mathbf{E}(1-\epsilon)}{\mathbf{K}(1-\epsilon)} \right) \right]^2} \right]. \tag{4.12}$$

So far, all results are exact. For any J and Δp, one can solve Eqs. (4.9) and (4.11) to get χ_p, χ_m (or v, W) which can be inserted into Eq. (4.12) to find the energy-charge relation.

For explicit analytic expressions, we consider a large but finite angular momentum. From Eq. (4.9), $J \gg T$ means $\epsilon \ll 1$. Therefore, Eq. (4.12) can be first expanded in the very small ratio $\mathbf{E}/\mathbf{K}$ as

$$E_s - \sqrt{\mu^2 M^2 + J_1^2 + J^2} \approx 2T \frac{(1-v^2)}{\sqrt{1-v^2 W}} \left\{ \left[\sqrt{\beta^2 + W} - \sqrt{\beta^2 + \left(\frac{1-v^2 W}{1-v^2} \right)^2} \right] \mathbf{K}(1-\epsilon) + \frac{\left(\frac{1-v^2 W}{1-v^2}\right)^2}{\sqrt{\beta^2 + \left(\frac{1-v^2 W}{1-v^2}\right)^2}} \mathbf{E}(1-\epsilon) \right\}. \tag{4.13}$$

Now we assume that the parameters are expanded for a small ϵ as follows:

$$v = v_0 + (v_1 + v_2 \log\epsilon)\epsilon, \tag{4.14}$$

$$W = W_0 + (W_1 + W_2 \log\epsilon)\epsilon. \tag{4.15}$$

By identifying $\Delta\phi = p$ in Eq. (4.11) and using Eq. (4.3) along with Eq. (4.10), one can find the coefficients as

$$v_0 = \cos\frac{p}{2}, \qquad v_1 = \frac{1-\log 16}{4}\cos\frac{p}{2}\sin^2\frac{p}{2},$$
$$v_2 = \frac{1}{4}\cos\frac{p}{2}\sin^2\frac{p}{2}, \tag{4.16}$$

$$W_0 = 1, \qquad W_1 = -\sin^2\frac{p}{2}, \qquad W_2 = 0. \tag{4.17}$$

The coefficient in front of $\mathbf{K}(1-\epsilon)$ in Eq. (4.13) is as small as $\mathcal{O}(\epsilon)$ so that the logarithmic-divergent term disappears.

With these coefficients and ϵ expansions of the elliptic functions, we find

$$E_s - \sqrt{\mu^2 M^2 + J_1^2 + J^2} = \frac{2T\sin\frac{p}{2}}{\sqrt{1+\beta^2}} \left[1 - \frac{\sin^2\frac{p}{2} + \beta^2(1 - 5\cos^2\frac{p}{2})}{4(1+\beta^2)}\epsilon + \mathcal{O}(\epsilon^2) \right]. \tag{4.18}$$

The ϵ expansion of J in Eq. (4.9) is

$$J \approx T\sin\frac{p}{2}\left(-2 - \log\frac{\epsilon}{16} \right), \tag{4.19}$$

from which the parameter ϵ can be expressed for $J \gg T$ as

$$\epsilon = 16\exp\left(-\frac{J}{T\sin\frac{p}{2}} - 2 \right). \tag{4.20}$$

Combining these together, we obtain the leading finite-size correction of the energy-charge relation

$$E_s - \sqrt{\mu^2 M^2 + J_1^2 + J^2} = \frac{2T\sin\frac{p}{2}}{\sqrt{1+\beta^2}}\left[1 - 4\frac{\sin^2\frac{p}{2} + \beta^2(1-5\cos^2\frac{p}{2})}{1+\beta^2}e^{-\frac{J}{T\sin\frac{p}{2}}-2}\right]. \quad (4.21)$$

The first term on the rhs is the energy dispersion relation in infinite volume which we have obtained in Eq. (3.10). The second term is the finite-size correction to the energy and is our main result in this paper. The coefficient in front of the exponential factor and its dependence on the deformation parameter β defined in Eq. (2.14) may contain important information on the interaction between the giant magnon states. The exponent in the finite-size part is independent of β, since it comes from the small-ϵ expansion of the angular momentum J as shown in Eq. (4.20). It originates from our choice of the S^2 subspace in Eq. (2.5) where the isometric angle ϕ couples with the nonisometric θ independently of β.

For the undeformed case of $\beta \to 0$, this result reduces to

$$E_s - J = 2T\sin\frac{p}{2}\left[1 - 4\sin^2\frac{p}{2}e^{-\frac{J}{T\sin\frac{p}{2}}-2}\right], \quad (4.22)$$

which was obtained previously in Refs. [17–19].

V. CONCLUDING REMARKS

In this paper, we have found a classical giant magnonlike solution moving in the $Sch_5 \times S^5$ target space. We have considered the string configuration similar to the ordinary giant magnons, namely, pointlike in the Sch_5 space and stringlike in the S^2 subspace of the S^5. The conserved charges and the corresponding energy-charge relations are expressed in terms of the elliptic integrals in the finite volume. We have confirmed that these results are consistent with previously known results in the pointlike and undeformed limits. In the same way as the giant magnon dual to a magnonic excitation of the $su(2)$ spin-chain operators of the $N = 4$ super-Yang-Mills (SYM) theory, we conjectured that this giant magnonlike string state is holographically dual to a similar magnon state in a composite spin chain operator of the dipole-deformed $N = 4$ SYM theory considered in Ref. [9].

A possible generalization is to consider a dyonic giant magnon solution in $Sch_5 \times S^5$. This solution can live in S^3 where additional angular momentum should be introduced. Another direction is to deform S^5 in addition to the dipole deformation of Sch_5. For this purpose, one needs to find string solutions for the integrable systems with two or more deformations. Our eventual goal is to utilize our results to identify the excitation spectrum on the string world sheet and the corresponding spin chain with the momentum related to the deficit angle. We hope to report on these in the near future.

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Note added.—After the first version of this paper was posted to the arXiv, a new paper appeared where classical string solutions were derived [20]. The main difference from ours is that it considered string-like solutions even in the Sch_5 space.

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