Mapping between the sinh-Gordon and Ising models

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The S-matrix of the Ising Model can be obtained as particular limit of the roaming trajectories associated to of the S-matrix of the sinh-Gordon model. Using the form factors of the sinh-Gordon, we analyse the correspondence between the operators of the two theories.

1. Introduction

Given an elastic factorized S-matrix of a 2D system with a mass scale M, we can calculate its ground state energy $E_0(R) \equiv -\pi \widetilde{c}(MR)/6R$ on an infinite strip of width R, by means of the Thermodynamical Bethe Ansatz (TBA) [1,2]. For those models where the S-matrix has a well-defined field theory correspondence [3–5], the scaling function $\widetilde{c}(MR)$ has a smooth behaviour, monotonically decreasing from the limit value $\widetilde{c}(0)$ (where it coincides with the effective central charge of the CFT of the ultraviolet limit) to $\widetilde{c}(\infty) = 0$ (which corresponds to massive regime). However, since the TBA only employs an S-matrix without questioning its field theory interpretation, it can be also used to investigate the finite-size behaviour of any quantum theory axiomatically defined in terms of a scattering amplitude, provided it satisfies the usual constraints of unitarity and crossing symmetry. From this point of view, Zamolodchikov proposed in ref. [6] a simple purely elastic scattering theory which under the TBA analysis reveals a remarkable finite-size behavior. Such theory contains a single particle bosonic state with mass M and two-particle scattering amplitude given by

$$S(\beta) = \frac{\sinh \beta - i \cosh \beta_0}{\sinh \beta + i \cosh \beta_0}, \tag{1.1}$$

where β_0 is a real parameter. $S(\beta)$ has two simple poles in the unphysical sheet at positions $\beta = -\frac{1}{2}i\pi \pm \beta_0$ which correspond to a resonance particle. The presence of a scale β_0 for real values of the rapidities drastically influences the finite-size behaviour of the model. In fact, solving numerically the TBA equations associated to the S-matrix (1.1), for sufficient large values of β_0, $\widetilde{c}(r)$ develops a "staircase" pattern with a series of plateaux at the discrete values $c = 1 - 6/p(p + 1)$ $(p = 3, 4, \ldots)$ which coincide with the central charges of unitary minimal models $\mathcal{M}_p$ [7,8]. Hence the Roaming Trajectory Model (RTM) is a one-parameter family of Renormalization Group flows interpolating between all the fixed points $\mathcal{M}_p$: each trajectory starts from the limiting fixed point $\mathcal{M}_\infty$ and then proceeds on the critical surface through the hopping $\mathcal{M}_p \to \mathcal{M}_{p-1}$ until it arrives in the neighborhood of the fixed point $\mathcal{M}_3$. After this last step, it develops a finite correlation length and gives rise to the usual massive infrared behaviour. From the TBA analysis it also follows that the roaming trajectories spend approximately the same fraction β_0 of the Renormalization Group "time" $x = \log MR/2$ near each fixed point, therefore making

more pronounced the multiple crossover phenomena for large values of β_0. Although a local interpretation of the RTM has been pursued in terms of conformal perturbation of the models $\mathcal{M}_p$ visiting along the trajectories [10], it is worth to consider the RTM as special analytic continuation of the sinh-Gordon model in such a way to take advantage of the recent exact solution of this model [11,12]. Purpose of this letter is to show, as simplest application of this idea, how to relate the operator content of the sinh-Gordon model to that of the Ising model which is the first jump in the staircase series.

2. The sinh-Gordon model

2.1. Main features

The sinh-Gordon Model (SGM) is a 2D Affine Toda Field Theory [13] with one bosonic field $\phi(x)$ and bare action given by

$$A = \int \mathrm{d}^2 x \left[\tfrac{1}{2} (\partial_\mu \phi)^2 - \frac{M_0^2}{g^2} \cosh g\phi(x) \right] . \tag{2.1}$$

The integrability of the model permits the determination of the factorizable elastic S-matrix which is given by [14]

$$S(\beta, B) = \frac{\sinh \beta - \mathrm{i} \sin \tfrac{1}{2} \pi B}{\sinh \beta + \mathrm{i} \sin \tfrac{1}{2} \pi B} , \tag{2.2}$$

where $B(g) = 2g^2/(8\pi + g^2)$. For real values of the coupling constant g, the S-matrix has no poles in the physical sheet and consequently no bound states, but on the contrary it presents two zeroes at the crossing symmetric positions $\mathrm{i}\pi B/2$ and $\mathrm{i}\pi(2-B)/2$. It is easy to see that in the analytical continuation $B \to 1 \pm (2\mathrm{i}/\pi)\beta_0$ the zeros move along a direction parallel to the real β-axis and the S-matrix (2.2) exactly coincides with the scattering amplitude of the RTM [6]. This observation becomes useful in the light of the fact that the SGM has been recently solved by computing the matrix elements of local operators.

2.2. Form factors

A complete knowledge of a QFT is encoded into the matrix elements of local operators $\mathcal{O}_k$ on the asymptotic states, the so-called Form Factors (FF) [15]

$$F_n^k(\beta_1, \ldots, \beta_n) = \langle 0 | \mathcal{O}_k(0) | \beta_1, \ldots, \beta_n \rangle . \tag{2.3}$$

In the case of the SGM at real coupling constant, the FF of local scalar operators have been determined in [11,12]. We briefly recall their main properties, referring the reader to the original references for their detailed discussion. They can be parameterized as

$$F_n^k(\beta_1, \ldots, \beta_n) = H_n^k Q_n^k(x_1, \ldots, x_n) \prod_{i<j} \frac{F_{\min}(\beta_{ij})}{(x_i + x_j)} , \tag{2.4}$$

where $x_i \equiv \mathrm{e}^{\beta_i}$ and $\beta_{ij} = \beta_i - \beta_j$. $F_{\min}(\beta)$ is an analytic function given by

$$F_{\min}(\beta, B) = \mathcal{N}(B) \, \Xi(\beta, B) ,$$

$$\Xi(\beta, B) = \exp \left[8 \int\limits_0^\infty \frac{\mathrm{d}x}{x} \frac{\sinh\left(\tfrac{1}{4}xB\right) \sinh\left(\tfrac{1}{2}x(1 - \tfrac{1}{2}B)\right) \sinh \tfrac{1}{2}x}{\sinh^2 x} \sin^2\left(\frac{x\hat{\beta}}{2\pi}\right) \right] , \tag{2.5}$$

$$\mathcal{N}(B) = \exp\left[-4\int\limits_0^\infty \frac{dx}{x} \frac{\sinh\left(\frac{1}{4}xB\right)\sinh\left(\frac{1}{2}x(1-\frac{1}{2}B)\right)\sinh\frac{1}{2}x}{\sinh^2 x}\right]$$

$(\widetilde{\beta} = i\pi - \beta)$. $F_{\min}(\beta, B)$ has a simple zero at the threshold $\beta = 0$ and no poles in the physical strip $0 \leqslant \operatorname{Im}\beta \leqslant \pi$, with an asymptotic behaviour $\lim_{\beta\to\infty} F_{\min}(\beta, B) = 1$. In eq. (2.4) H_n^k are normalization constants which depend on the operator one is considering. The functions $Q_n^k(x_1, \ldots, x_n)$ are symmetric polynomials in the variables x_i, solutions of the recursion equations which link the n-particle and the $(n+2)$-particle form factors

$$-i\lim_{\widetilde{\beta}\to\beta}(\widetilde{\beta} - \beta)F_{n+2}^k(\widetilde{\beta} + i\pi, \beta, \beta_1, \beta_2 \ldots, \beta_n) = \left(1 - \prod_{i=1}^n S(\beta - \beta_i, B)\right)F_n^k(\beta_1 \ldots, \beta_n). \tag{2.6}$$

For FF of spinless operators, the total degree of Q_n^k is equal to $n(n-1)/2$ whereas their partial degree in each variable x_i depends on the operator $\mathcal{O}_k$ which is considered. It was shown in ref. [12] that a general solution for the Q_n^k can be written in terms of the so-called *elementary solutions* $Q_n(p)$ given by [*1]

$$Q_n(p) = \det M_{ij}(p), \tag{2.7}$$

where $M_{ij}(p)$ is an $(n-1) \times (n-1)$ matrix with entries $M_{ij}(p) = \sigma_{2i-j}[i-j+p]$ (σ_l are the elementary symmetric polynomials [16] and p an arbitrary integer).

2.2.1. Form factors of $\phi(x)$ and $\Theta(x)$

Important operators of the SGM are the elementary field $\phi(x)$ and the trace of the stress-energy tensor $\Theta(x)$. They are odd and even operators respectively under the Z_2 symmetry of the model with normalizations given by $\langle 0 \mid \phi(0) \mid \beta\rangle = 1$ and $\langle\beta \mid \Theta(0) \mid \beta\rangle = 2\pi M^2$, where M is the physical mass. The whole set of FF of the elementary field $\phi(x)$ is given by

$$F_n^\phi(\beta_1, \ldots, \beta_n) = \left(\frac{4\sin(\pi B/2)}{\mathcal{N}(B)}\right)^{(n-1)/2} Q_n(0) \prod_{i<j} \frac{F_{\min}(\beta_{ij})}{x_i + x_j}. \tag{2.8}$$

They are automatically zero for even n (in agreement with the Z_2 parity of the model) whereas for odd n they vanish asymptotically when $\beta_i \to \infty$, as follows from the LSZ reduction formula. Concerning the FF of $\Theta(x)$, $F_{2n+1}^\Theta = 0$ whereas F_{2n}^Θ are given by

$$F_{2n}^\Theta(\beta_1, \ldots, \beta_{2n}) = \frac{2\pi M^2}{\mathcal{N}(B)}\left(\frac{4\sin(\pi B/2)}{\mathcal{N}(B)}\right)^{n-1} Q_{2n}(1) \prod_{i<j} \frac{F_{\min}(\beta_{ij})}{x_i + x_j}, \tag{2.9}$$

and they go to a constant when $\beta_i \to \infty$

2.2.2. Kernel solutions

The general structure of the FF of the SGM is that of a sequence of finite linear spaces whose dimensions grow linearly as n increasing the number $2n - 1$ or $2n$ of external particles. In fact, at each level of the recursive process the space of the FF is enlarged by including the kernel solutions of the recursive equation (2.6), i.e. $Q_n(-x, x, x_1, \ldots, x_{n-2}) = 0$. With the constraint that the total order of the polynomials is $\frac{1}{2}n(n-1)$, the kernel is unique and given by $\Sigma_n(x_1, \ldots, x_n) = \det \sigma_{2i-j}$. This solution gives rise to the half-infinite chain under the recursive pinching $x_1 = -x_2 = x$

$$\cdots \to Q_{n+4}^{(n)} \to Q_{n+2}^{(n)} \to Q_n^{(n)} = \Sigma_n \to 0 \tag{2.10}$$

[*1] We have suppressed the dependence of $Q(p)$ from the variables x_i.

and therefore the whole space of FF presents the foliation structure[#2]

$$
\begin{aligned}
\ldots \to \quad & Q^{(1)}_{n+4} \to Q^{(1)}_{n+2} \to Q^{(1)}_{n} \to Q^{(1)}_{n-2} \to \ldots \to Q^{(1)}_{3} \to 1 \\
\ldots \to \quad & Q^{(3)}_{n+4} \to Q^{(3)}_{n+2} \to Q^{(3)}_{n} \to Q^{(3)}_{n-2} \to \ldots \to \Sigma_{3} \\
& \quad\vdots \qquad\qquad \vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\quad \vdots \\
\ldots \to \quad & Q^{(n-2)}_{n+4} \to Q^{(n-2)}_{n+2} \to Q^{(n-2)}_{n} \to \Sigma_{n-2} \\
\ldots \to \quad & Q^{(n)}_{n+4} \to Q^{(n)}_{n+2} \to \Sigma_{n} \\
\ldots \to \quad & Q^{(n+2)}_{n+4} \to \Sigma_{n+2}
\end{aligned}
\tag{2.11}
$$

The explicit expressions of such solutions can be found by determining the linear combination of $\mathcal{Q}_n(k)$ which reduces to Σ_n at the level n.

3. Violation of the c-theorem sum rule in the RTM

Since the RTM may be seen as the SGM at $B = 1 \pm (2i/\pi)\beta_0$, it is natural to study the behaviour of the FF of the latter model under this analytic continuation. As we show, the presence of a scale β_0 in the rapidity axes may induce a non-uniform convergence in series expansions obtained in the original sinh-Gordon model. Consider for instance the total variation of the central charge $\Delta c = c_{\mathrm{uv}} - c_{\mathrm{ir}}$ going from the short to the large distances. For both the SGM and the RTM, $\Delta c = 1$. Let us try to express it as a sum-rule fulfilled by the two-point function of the trace $\Theta(x)$ [9,17]

$$
\Delta c = \frac{3}{4\pi} \int r^2 \langle \Theta(r)\Theta(0) \rangle \, \mathrm{d}^2 r = \sum_{n=1}^{\infty} \Delta c^{(2n)},
\tag{3.1}
$$

where $\Delta c^{(2n)}$ is the contribution to the variation of the central charge coming from the $2n$-intermediate states. In the original SGM with real coupling constant, the convergence of the series to the value $\Delta c = 1$ is extremely fast and almost saturated by the two-particle contribution $\Delta c^{(2)}$ [11]. This has to be expected, given the massive behaviour of the model and the threshold suppression phenomena analyzed in [18]. Similar behaviour has been also observed in supersymmetric models [19]. However, in the RTM the situation is drastically different. Consider initially the two-particle contribution to the c-theorem sum rule

$$
\Delta c^{(2)}(\beta_0) = \frac{3}{2} \int_0^\infty \mathrm{d}\beta \, \frac{|\Xi(2\beta,\beta_0)|^2}{\cosh^4 \beta}.
\tag{3.2}
$$

The plot of such a quantity (fig. 1) shows that $\Delta c^{(2)}(\beta_0)$ monotonically decreases from the value very close to 1 at $\beta_0 = 0$ (corresponding to the sinh-Gordon self-dual point) to $1/2$ for $\beta_0 \to \infty$. The asymptotic value $1/2$ can be easily obtained analytically by noticing that

$$
\Xi(\beta,\beta_0) = \sinh \frac{\beta}{2} h(\beta,\beta_0), \quad h(\beta,\beta_0) \simeq
\begin{cases}
\exp\left(-\dfrac{\beta-\beta_0}{2}\right), & \beta > \beta_0 \\
-i, & \beta < \beta_0
\end{cases},
$$

and therefore for $\beta_0 \to \infty$ the integral (3.2) simply reduces to

$$
\Delta c^{(2)}(|\beta_0| \to \infty) = \frac{3}{2} \int_0^\infty \mathrm{d}\beta \, \frac{\sinh^2 \beta}{\cosh^4 \beta} = \tfrac{1}{2}.
\tag{3.3}
$$

[#2] This is the structure for FF of odd operators. Analogous structure arises for the FF of even operators.

Fig. 1. Plot of $\Delta c^{(2)}$ as function of β_0.

Concerning the higher particles contributions $\Delta c^{(2n)}$, all of them vanish in the limit $\beta_0 \to \infty$. In fact, the $2n$-particle FF entering the formula (3.1) for $\Delta c^{(2n)}$ is given by eq. (2.9) and after the analytic continuation they may be written as

$$F_{2n}(\beta_1, \ldots, \beta_n) = 2\pi m^2 g_{2n}(\beta_0) \, \mathcal{Q}_{2n}(1) \prod_{i<j} \frac{\sinh \frac{1}{2}\beta_{ij} \, h(\beta_{ij}, \beta_0)}{x_i + x_j}, \qquad (3.4)$$

where $g_{2n}(\beta_0) = (4\cosh\beta_0)^{n-1} \mathcal{N}^{2n(n-1)}(\beta_0)$. Analogously to the two-particle case, the β_0-dependence coming from $h(\beta_{ij})$ is strongly suppressed in the integration over rapidities and the asymptotic behaviour in β_0 of $\Delta c^{(2n)}$ is only determined by the exponential factors contained in g_{2n} and $\mathcal{Q}_{2n}(1)$. In the large β_0 limit, $\mathcal{N}(\beta_0) \sim \exp\left(-\frac{1}{2}|\beta_0|\right)$ and then $g_{2n}(\beta_0) \sim \exp\left\{-(n-1)^2|\beta_0|\right\}$. On the other hand, for $\beta_0 \to \infty$ $\mathcal{Q}_{2n}(1) \sim \exp\{(n-1)(n-2)|\beta_0|\} \mathcal{P}(x_i)$ where $\mathcal{P}(x_i)$ is a symmetric polynomial. So, for $n > 1$ $\Delta c^{(2n)}(|\beta_0| \to \infty) \to 0$ as $\exp\{-(n-1)\beta_0\}$. Therefore the result of the series (3.1) is $\Delta c = 1/2$ instead of $\Delta c = 1$, i.e. a violation of the c-theorem sum rule.

Although striking, the non-uniform convergence of the series has a natural interpretation once the nontrivial interplay between the two scales β and β_0 of the problem is correctly taken into account. In fact, since the n-particle contribution in (3.1) behaves as $e^{-n(Mr)}$, given any length scale r there is always an integer N_r such that the states with a number of particles $n \geqslant N_r$ give a negligible contribution to the series (3.1). This means that any partial sum $\Delta c_N \equiv \sum_{m=1}^{N} \Delta c^{(2m)}$ only reproduces the variation of the c-function from the infrared limit $r = \infty$ up to a certain scale $r^{(N)}$. In usual situations, when $c(r)$ is a smooth function in the deep ultraviolet region, the first few $\Delta c^{(2n)}$ are sufficient to give the correct value of Δc, with high level of precision. But for the RTM this is not the case. Consider a scale r_1 such that $c(r_1, \beta_0 = 0) > 1/2$ (fig. 2). According to the results of the TBA analysis, after the first jump from 0 to 1/2, the function $c(r, \beta_0)$ stays constant at 1/2 until a value r_2 proportional to $\exp\{-|\beta_0|/2\}$ is reached and, only at this point the second jump takes place. The other jumps occur at $r_n \sim \exp\{-|\beta_0|(n-1)/2\}$ and for $\beta_0 \to \infty$, they accumulate to the origin. Truncating the series (3.1)
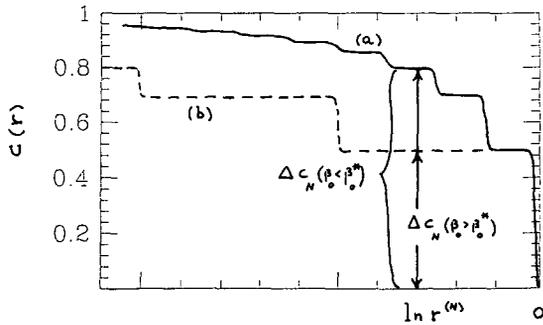
Fig. 2. Roaming trajectories for $\beta_0 < \beta_0^*$ (a) and $\beta_0 > \beta_0^*$ (b). For $\beta_0 > \beta_0^*$, Δc_N measured at the scale $r^{(N)}$ is always $1/2$.

to any N, there is *always* a value β_0^* such that $c(r_1^{(N)}, |\beta_0| > |\beta_0^*|) = 1/2$, i.e. the point of the second jump is always ahead of the corresponding length scale $r_1^{(N)}$, however small $r_1^{(N)}$ may be, and therefore

$$\lim_{N \to \infty} \lim_{|\beta_0| \to \infty} \Delta c_N(\beta_0) = \tfrac{1}{2}. \tag{3.5}$$

4. Collapse of the sinh-Gordon model to the Ising model

Taking the limit $\beta_0 \to \infty$ (keeping β fixed), the S-matrix of the SGM goes to $S = -1$, i.e. to the S-matrix of the thermal perturbed Ising model. Together with (3.6), these results naturally suggest that for $\beta_0 \to \infty$ the Hilbert space of the original SGM collapses to that of the Ising model, spanned in the local sector only by three independent families of fields, those of identity $\{1\}$, magnetization $\{\sigma\}$ and energy $\{\epsilon\}$ operators. It is therefore interesting to find the mapping between the operator content of the two models.

It is easy to see that the elementary field $\phi(x)$ of the SGM is mapped onto the magnetization operator $\sigma(x)$ of the Ising model. In fact, analytically continuing the FF (2.8) and taking the limit $\beta_0 \to \infty$, the β_0 dependences coming from different terms of the original expression compensate each other and we obtain the following finite result

$$F_{2n+1}^\phi(\beta_1, \dots, \beta_{2n+1}) \to i^n \prod_{i<j}^{2n+1} \tanh\left(\frac{\beta_{ij}}{2}\right). \tag{4.1}$$

These are precisely the FF of the magnetization operator $\sigma(x)$ of the thermal perturbed Ising model [20,21]. This field belongs to the interacting sector of the theory and its correlation functions satisfy non-trivial differential equations [22,23]. Notice that in this limit the boundary conditions of the field ϕ have been modified: in the original SGM its FF vanish for large values of β_i whereas in the resulting expression (4.1) they go to a constant.

On the other hand, taking the limit $\beta_0 \to \infty$ for the analytic continuation of the FF of Θ (2.9), all of them vanish but $F_2 = 2\pi m^2 \sinh \beta/2$. Hence the operator $\Theta(x)$ of the original SGM is mapped onto the energy operator $\epsilon(x)$ of the Ising model. This is a free field (a result which is manifest by the absence of higher FF) and its correlators can be easily expressed in terms of Bessel functions. Also in this case the boundary condition of the field Θ has been changed, since originally F_2^Θ goes to a constant for large values of β_i whereas after taking the limit $\beta_0 \to \infty$ it diverges at infinity.

It is also interesting to analyze the behaviour for $\beta_0 \to \infty$ of the kernel solutions. In this limit the recursive equations (2.6) become

$$\begin{aligned} Q_{n+2}(-x, x, x_1, \dots, x_n) &= -x^{n+1} \sigma_n Q_n(x_1, \dots, x_n), \quad n = \text{odd}, \\ Q_{n+2}(-x, x, x_1, \dots, x_n), &= 0 \qquad\qquad\qquad n = \text{even}. \end{aligned} \tag{4.2}$$

The kernel solutions of the Z_2 even operators of the original SGM are therefore mapped onto the free sectors of the Ising model, i.e. those given by the identity and energy operators. Indeed, their FF are different from zero only at a given level n in the number of external particles (where they coincide with Σ_n defined in section 2.2.2) and, due to the second equation in (4.2), they decouple from the rest of the recursive chain. Correlators of the operators defined by such FF can be also expressed in terms of Bessel functions.

Such a decoupling in the recursive chain does not occur, on the contrary, for the kernel solutions of the odd operators of the original SGM. Their explicit expressions may be written as determinants of minors of the matrix Σ_n. In fact, consider the half-infinite chain of FF $Q^{(n)}_{n+2m}$ (n odd and $m = 1, 2, \dots$) satisfying the first equation in (4.2), with the initial condition

$$Q^{(n)}_{n+2} = -x^{n+1}\sigma_n \Sigma_n. \tag{4.3}$$

It is easy to see that[#3] $Q^{(n)}_{n+2} = [\Sigma_{n+2}]_{((n+1)/2, n-1)}$ and in general

$$Q^{(n)}_{n+2m} = \left[\left[\dots [\Sigma_{n+2m}]_{((n+2m-1)/2, n+2m-1)} \dots \right]_{((n+3)/2, n+3)} \right]_{((n+1)/2, n+1)}. \tag{4.4}$$

Such FF define matrix elements of operators belonging to the magnetization sector. For instance $Q^{(1)}_n$ defines the FF of the magnetization operator itself whereas $Q^{(3)}_n$ those of the operator $\mathcal{O}^{(3)} = (\sigma(x) + 1/M^2\partial^2\sigma(x))$ etc. In general such operators have the distinguishing property that their two-point correlation function $\langle \mathcal{O}^{(n)}(r)\mathcal{O}^{(n)}(0)\rangle$ decreases at infinity as $\exp[-nMr]$.

5. Conclusions

The thermal perturbed Ising model is the first model in the staircase series defined by the RTM. Using the analytic continuation which links the sinh-Gordon Model to the RTM, we have seen that in the limit $\beta_0 \to \infty$ the elementary field of the SGM becomes the magnetization operator σ of the Ising model. This is still an interacting field with non-trivial form factors. On the other hand, the field Θ and other even operators of the SGM are mapped into the free sectors of the Ising model.

It would be interesting to extend the analysis of this paper to the higher models of the RTM and to find the correlation functions of the QFT associated to the corresponding massless Renormalization Group flows.

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