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Cosmological constant behavior in DBI theory

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ABSTRACT

Cosmological constant behavior can be realized as solutions of the Dirac–Born–Infeld (DBI) action within Type IIB string theory and the AdS/CFT correspondence. We derive a family of attractor solutions to the cosmological constant that arise purely from the "relativistic" nature of the DBI action without an explicit false vacuum energy. We also find attractor solutions with values of the equation of state near but with $w \neq -1$; the forms for the potential arising from flux interactions are renormalizable and natural, and the D3-brane tension can be given by the standard throat form. We discuss present and future observational constraints on the theory.

1. Introduction

The cosmological constant is a fundamental puzzle for high energy physics field theory. With the observational discovery of the accelerated expansion of the universe [1,2], this puzzle has become a premier challenge for both theoretical and observational physics. The cosmological constant, or something with similarly negative effective pressure, dominates the energy density of the universe. However, it is not at all clear how it arises naturally within a fundamental theory of physics. In particular, the needed energy density lies 10^{121} times below the Planck energy density, requiring severe fine tuning of high energy physics.

Giving dynamics to the field allows the possibility of ameliorating the fine tuning issue, through scalar fields that exhibit attractor behavior. However, it has been quite difficult to devise fields that both have attractors and can achieve sufficiently negative equations of state (pressure to energy density ratios) by the present epoch.

String theory can impose a non-trivial kinetic behavior through the Dirac–Born–Infeld (DBI) action that arises naturally in consideration of D3-brane motion within a warped compactification. Several articles have considered DBI as a source for inflation [3–5] or dark energy [6], fixing one or another function within the DBI action. We focus on the unusual dynamics (following the pioneering work of [3] for inflation) and find this can have several important consequences with advantages for bringing the theory naturally into close accord with observations. In particular, a near cosmological constant state can be achieved in several ways uniquely distinct from quintessence.

In Section 2 we lay out the foundations of DBI theory and the important parameters in the equations of motion. We find solutions insensitive to initial conditions, i.e. having a large basin of attraction, in Section 3, including for cosmological constant behavior. The most interesting cases arise uniquely from the "relativistic" nature of the DBI action and have quite simple potentials.

2. DBI methodology

We consider the low-energy dynamics of a probe D3-brane in a warped geometry coupled to gravity. It is governed by the DBI action [3],

$$S = -\int d^4x\sqrt{-g}\left[T(\phi)\sqrt{1-\dot{\phi}^2/T(\phi)} + V(\phi) - T(\phi)\right], \qquad (1)$$

where we ignored the spatial derivatives of ϕ. T is the warped brane tension and V is the potential arising from interactions with Ramond–Ramond fluxes or other sectors. The energy–momentum tensor takes a perfect fluid form with energy density ρ_ϕ and pressure p_ϕ given by

$$\rho_\phi = (\gamma-1)T + V; \qquad p_\phi = \left(1-\gamma^{-1}\right)T - V. \qquad (2)$$

The Lorentz factor γ measures the "relativistic" motion of the field,

$$\gamma = \left(1-\dot{\phi}^2/T\right)^{-1/2}. \qquad (3)$$

The equation of state for the DBI field is

$$w \equiv \frac{p_\phi}{\rho_\phi} = -\frac{\gamma^{-1}-1+v}{\gamma-1+v}, \qquad (4)$$

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where $v(\phi) = V(\phi)/T(\phi)$. In the "nonrelativistic" limit, $\gamma \to 1 + K/T$, where $K \equiv \dot\phi^2/2$ is the canonical kinetic energy, and $w \to (K - V)/(K + V)$ as for a quintessence field. However, the noncanonical behavior due to the relativistic corrections will be crucial.

The equation of motion for the field follows from either functional variation of the action or directly from the continuity equation for the energy density,

$$\rho_\phi' = -3(\rho_\phi + p_\phi) = -3\big(\gamma - \gamma^{-1}\big)T, \tag{5}$$

where a prime denotes a derivative with respect to the e-folding parameter, $d/d\ln a$. The necessary ingredients are the tension $T(\phi)$ and potential $V(\phi)$, and initial conditions on the field. Solving for the evolution then delivers the equation of state parameters $w(a)$ and $w'(a)$ for a phase space portrait of the dynamics, and $\Omega_\phi(a) = (8\pi G\rho_\phi)/(3H^2)$ for the dark energy density fraction and computation of the Hubble parameter $H(a) = \dot a/a$ and cosmological distances $d(a) = \int da/(a^2H)$.

For a pure AdS_5 geometry with radius R, the warped tension is given by

$$T(\phi) = \tau\phi^4, \tag{6}$$

with $\tau = 1/(g_s\tilde\lambda)$ where g_s is the string coupling, α' is the inverse string tension, and $\tilde\lambda = R^4/\alpha'^2$ which is identified as the 't Hooft coupling in AdS/CFT correspondence. In general we do not need to take advantage of further degrees of freedom by altering the tension function, although other forms for it lead to similar conclusions as well. In the next section we find that very simple, standard forms of the potential, such as $V(\phi) = m^2\phi^2$ or $V \sim \phi$, have quite interesting behavior. Thus, there is little arbitrariness or unnaturalness needed to find results approaching the cosmological constant behavior.

3. Cosmological constant and other attractors

Solutions to the equations of motion where no special time is picked out in the history of the universe have long been of interest as means to ameliorate the coincidence or fine tuning problems [7–10]. Attractor solutions avoid fine tuning in that the dynamical trajectory of the field lies along a common track despite starting from different initial conditions. In general, only highly specific forms of the potential possess this characteristic in the quintessence case; we find that this is vastly enlarged in the DBI case and in fact many standard potentials such as quadratic and quadratic plus quartic forms exhibit attractor behavior. We identify the origin of this as the relativistic limit where the Lorentz boost factor γ grows large; hence it is an innate characteristic of DBI string theory.

To begin, we define the contributions of the tension and potential to the vacuum energy density relative to the critical density,

$$x^2 = \frac{\kappa^2}{3H^2}(\gamma - 1)T; \qquad y^2 = \frac{\kappa^2}{3H^2}V, \tag{7}$$

where $\kappa^2 = 8\pi G$. The equations of motion are given by

$$x' = -\frac{3}{2\gamma}x\big(1 - x^2\big) - \frac{3}{2}xy^2 + \frac{\sqrt3}{2}\lambda\frac{\sqrt{\gamma+1}}{\gamma}y^2, \tag{8}$$

$$y' = \frac{3}{2\gamma}x^2y + \frac{3}{2}y\big(1 - y^2\big) - \frac{\sqrt3}{2}\lambda\frac{\sqrt{\gamma+1}}{\gamma}xy, \tag{9}$$

where $\kappa\phi' = x\sqrt{3(\gamma+1)}/\gamma$, $\lambda = -(1/\kappa V)\,dV/d\phi$ and

$$\gamma = 1 + \frac{V}{T}\frac{x^2}{y^2}. \tag{10}$$

We are interested in the DBI field as late time accelerating dark energy, not for inflation, so we take the initial conditions in the matter dominated universe and define the present by $\Omega_\phi = 0.72$. The attractor solutions to the equation of motion have the critical values

$$x_{c1}^2 = \frac{\lambda^2}{3(\gamma+1)}; \qquad x_{c2}^2 = \frac{3\gamma^2}{\gamma+1}\frac{1}{\lambda^2}, \tag{11}$$

$$y_{c1}^2 = 1 - \frac{\lambda^2}{3(\gamma+1)}; \qquad y_{c2}^2 = \frac{3\gamma}{\gamma+1}\frac{1}{\lambda^2}, \tag{12}$$

$$\Omega_{\phi,c1} = 1; \qquad \Omega_{\phi,c2} = \frac{3\gamma}{\lambda^2}, \tag{13}$$

$$w_{\phi,c1} = -1 + \frac{\lambda^2}{3\gamma}; \qquad w_{\phi,c2} = 0. \tag{14}$$

A key criterion is whether λ^2/γ is zero, finite, or diverges. For non-negative potentials, the first set of critical values only exists for $\lambda^2 < 3\gamma$. The second set does not lead to acceleration so we do not consider it further.

Note that we have made no assumptions as to whether γ or λ are constant or not in the overall evolution. To obtain the de Sitter behavior of a cosmological constant, we need $\lambda^2/\gamma \to 0$. This requires either $\lambda \to 0$ or $\gamma \to \infty$.

On the attractor x, y will be constant so we can write $\gamma = 1 + kv$, with k a constant, and a key parameter is $v = V/T$. One also has that $\gamma' \to (\gamma - 1)v'/v$ so γ is driven to either 1 or ∞ (unless $v =$ constant). Suppose $\gamma \to 1$. Then we need $\lambda \to 0$. This can be achieved for runaway potentials (where $\phi \to \infty$) of the inverse power law form, $V \sim \phi^{-c}$, similar to the quintessence case [11]. For finite values of ϕ, though, $\lambda \to 0$ can only be realized for $\phi \to 0$ (i.e. potentials without poles) by including nonzero minimum vacuum energy, i.e. an explicit cosmological constant. Therefore we turn to the $\gamma \to \infty$ case.

In the fully relativistic, $\gamma \to \infty$, limit we can obtain the cosmological constant behavior. By Eq. (10) this requires $v \to \infty$. Suppose we take $T \sim \phi^n$, with $n = 4$ giving the quartic brane tension in AdS space. Then following Eq. (14) a simple realization of the cosmological constant attractor is $V \sim \phi^c$ where $0 < c < n - 2$. (Note that the equations of motion guarantee that the field stops at $\phi = 0$ before rolling to negative values of ϕ, so c is not restricted to even integers.)

We illustrate the example of the linear potential, $V \sim \phi$, in Fig. 1. The field indeed goes to the attractor behavior independent of the initial conditions of the field value, ϕ_i, and field velocity, i.e. γ_i (we discuss the evolution further in a later section). At late times the behavior is just that of a cosmological constant, $w = -1$.

An interesting further point is that we can consider the relativistic limit but where $\lambda^2 \to \infty$ also, in such a way that the key ratio λ^2/γ stays finite. In this case, w approaches an asymptotic value with $w \neq -1$, but it can lie close to -1 and certainly in the accelerating regime. This can be realized for $V \sim \phi^c$ with $c = n - 2$. In particular, a quadratic potential with quartic tension leads to such a solution. This is quite interesting as this is naturally predicted by DBI theory in pure AdS geometry.

The potential may arise from the couplings of the D3-brane to fluxes and other sectors involved in a compactification. In the case of pure $AdS_5 \times S^5$ geometry, the potential is quartic. Corrections to the conformal invariance, however, generically create a mass term giving a quadratic contribution [3,4]. In fact, all we require is that the potential looks quadratic near its minimum – a highly generic state. In the $c = n - 2$ case, the equation of state has a negative value

$$w = -1 + \frac{c^2}{6\mu^2}\big[-1 + \sqrt{1 + 12\mu^2/c^2}\big], \tag{15}$$

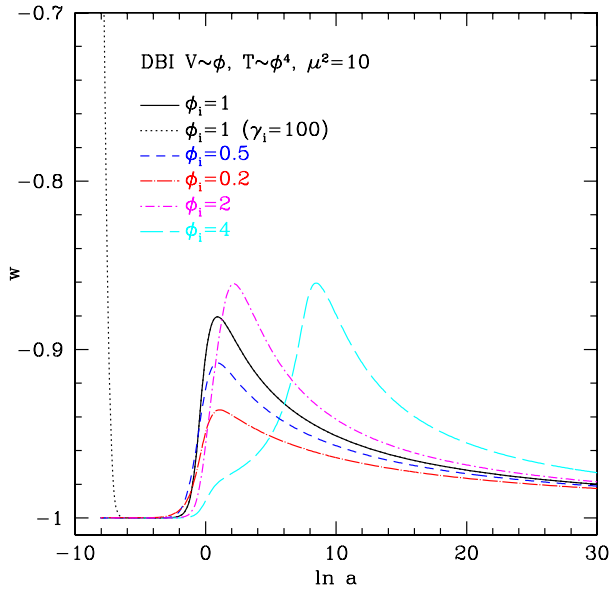

Fig. 1. The DBI dynamics can have an attractor to the cosmological constant state, insensitive to the initial conditions of the field value ϕ_i or boost factor γ_i. During the matter dominated era the field quickly approaches a frozen state with $w=-1$ and $\gamma=1$, and then thaws as the dark energy density starts to become appreciable. In the future, the field joins the attractor solution with $\gamma\to\infty$, and $w\to -1$ as $1/\ln a$.

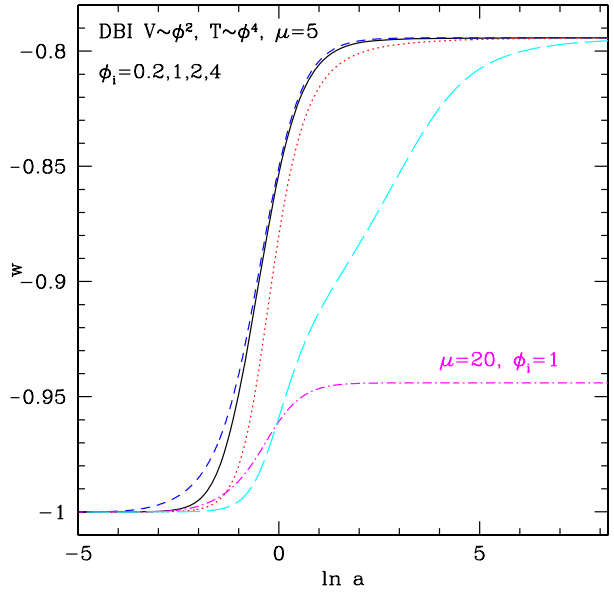

Fig. 2. A natural quadratic potential exhibits attractor behavior in DBI theory due to the relativistic boost factor. The field evolves from a frozen $w=-1$ state in the matter dominated era and goes to a constant equation of state in the future, independent of initial conditions. The larger the effective field mass μ, the closer the behavior approaches a cosmological constant.

where $\mu^2=m^2\kappa^{n-c}/\tau$, with $V=m^2\phi^c$, $T=\tau\phi^n$ ($c=2$, $n=4$ being of special interest). As μ gets large, the behavior looks more and more like a cosmological constant. Note that in the light of Eq. (6) large μ corresponds to the strong coupling regime where DBI analysis can be trusted. The evolution is illustrated in Fig. 2.

Other attractors giving $w\neq -1$ appear in the nonrelativistic limit, $\gamma=1$. Here we want $v\to 0$, and realizations include the exponential potential $V\sim e^{-\lambda\kappa\phi}$, but with any power law or less rapid exponential form for T. In particular, we can keep $T\sim\phi^4$. In this regime,

Table 1
Summary of accelerating attractor properties. The columns give the values of the quantities for the attractor solution, and the stability criteria. For $\lambda^2/(3\gamma)\geqslant 1$, either the field switches to the solution with $w=0$ or no attractor exists. Quintessence attractors can only access the class represented by the last row.

V/T	γ	λ^2/γ	w	Ω_ϕ	Stability
∞	∞	0	-1	1	yes
∞	∞	const	$-1+\lambda^2/(3\gamma)$	1	yes
const	const	const	Eq. (17)	1	$\lambda^2\leqslant 3\gamma$
0	1	const	$-1+\lambda^2/3$	1	$\lambda^2\leqslant 3$

$$w=-1+\frac{\lambda^2}{3}, \tag{16}$$

as for quintessence. However, if we also take $T\sim e^{-\lambda\kappa\phi}$, then $v=$ constant and we can get a finite value of γ different from 1. The equation of state is

$$w=-1+\frac{\lambda^2}{6}\frac{\sqrt{\lambda^4+12(v-1)\lambda^2+36}-\lambda^2}{3+(v-1)\lambda^2}. \tag{17}$$

We approach the cosmological constant value for $\lambda^2\ll 1$ or $v\gg\lambda^2$.

We summarize the accelerating attractor solutions in Table 1.

4. Comparison with observations

While the attractor solutions bring the field to a cosmological constant behavior or near to it, this could be in the future. We need to consider whether DBI theory is consistent with the current observations. Without going into great detail, our conclusion is that generally it is. The boost factor during the matter dominated era is driven toward unity, so from Eq. (4) we have $w\to -1$. Thus we reach an early time "frozen" state, looking like the cosmological constant, for a wide range of initial conditions including relativistic γ (cf. Fig. 1).

The field then evolves away from the frozen state along the same generic thawing trajectory as quintessence, $w'=3(1+w)$ [12,13] – recall that when $\gamma\approx 1$, DBI becomes quintessence-like. Since the attractor solution will pull the field back toward $w\approx -1$, the trajectory often does not deviate far from $w=-1$ for a range of potential parameters.

Taking as a concrete example the potential as in Fig. 1, with $\phi_i=1$, the distance to the cosmic microwave background last scattering surface, $d_{\rm lss}$, agrees with the standard cosmological constant cosmology ΛCDM to 0.67%, for the same present matter density. Other values of ϕ_i give even smaller deviations, and the agreement improves as μ increases. For the potential as in Fig. 2, with $\phi_i=0.2$, the agreement is 1.2%, and again improves as ϕ_i or μ increases. Considering distances to redshifts $z\leqslant 2$, e.g. as measured by Type Ia supernovae (see, for example, [14]), the deviation from ΛCDM is at worst 1.7% and 2.9% respectively. (Note the $\mu=20$ case of Fig. 2 gives 0.28% agreement on $d_{\rm lss}$ and 0.71% on $z\leqslant 2$ distances.) All these deviations are within current observational constraints, although future data will be able to place increasingly tight lower bounds on the effective mass μ.

5. Conclusions

We have shown that DBI string theory can achieve dynamics approaching the cosmological constant and obtaining agreement with cosmological observations. These are attractor solutions that substantially ameliorate the fine tuning of initial conditions. Several of the accelerating classes cannot be realized within quintessence, but instead arise from the relativistic nature of the DBI action with its Lorentz factor γ.

Unlike quintessence, standard renormalizable potentials like those with an $m^2\phi^2$ term exhibit attractor behavior. The linear potential is one example that has an attractor to a future de Sitter state. For a range of reasonable masses and coupling values such models are viable under the current cosmological observations such as distance–redshift data.

Also unlike quintessence this approach starts from a fundamental basis in string theory. The DBI action arises as the low energy effective theory describing the dynamics of a probe D3-brane. The results, including correspondence to the cosmological constant, hold with the natural form of the brane tension $T \sim \phi^4$, but also if it is distorted; they also hold taking into account the breaking of conformal invariance and the generation of a mass term in the potential. The important property is the ratio V/T. Given this, the relativistic kinetic properties of the DBI action allow cosmological constant or $w \approx -1$ states to be realized with some degree of naturalness.

Increasingly accurate cosmological data will be able to test directly aspects of fundamental string theory within the DBI framework. Such connections between string theory and astrophysical data offer exciting prospects for revealing the nature of the cosmological constant and the accelerating universe.

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