

# Finite volume matrix elements in integrable QFT: Some exact results

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- **B. Pozsgay: Form factor approach to diagonal finite volume matrix elements in Integrable QFT**  
arXiv:1305.3373

### Earlier works:

- B. Pozsgay, G. Takacs: Characterization of resonances using finite size effects  
arXiv:hep-th/0604022
- B. Pozsgay, G. Takacs: Form factors in finite volume I: form factor bootstrap and truncated conformal space  
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- B. Pozsgay: Mean values of local operators in highly excited Bethe states  
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Object: Finite volume matrix elements of local operators

- 1+1 D Integrable QFT
- Massive theories, diagonal scattering
- IR approach (using infinite volume quantities)

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- One particle with mass  $m$ .
- Exact scattering phase shift:  $S(\theta) = e^{i\delta(\theta)}$
- Example: sinh-Gordon theory

$$S(\theta) = e^{i\delta(\theta)} = \frac{\sinh \theta - i \sin b\pi}{\sinh \theta + i \sin b\pi} \quad b > 0$$

(no bound states)

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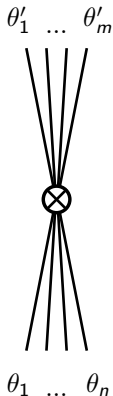
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- Form factors:

$$F_{mn}^{\mathcal{O}}(\theta'_1, \dots, \theta'_m | \theta_1, \dots, \theta_n) = \langle \theta'_1, \dots, \theta'_m | \mathcal{O}(0, 0) | \theta_1, \dots, \theta_n \rangle$$

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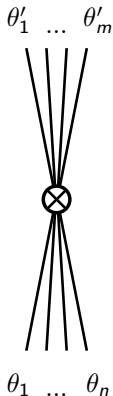


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- Watson's equations: (exchange and periodicity)

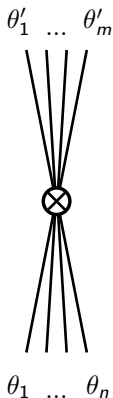
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$$F_n^{\mathcal{O}}(\theta_1 + 2i\pi, \theta_2, \dots, \theta_n) = F_n^{\mathcal{O}}(\theta_2, \dots, \theta_n, \theta_1)$$

- Kinematical pole:

$$\begin{aligned} \text{Res}_{\theta_1 = \theta'_1} F_{n+1}^{\mathcal{O}}(\theta'_1 + i\pi, \theta_1, \theta_2, \dots, \theta_n) \\ = i \left( 1 - \prod_{k=2}^n S(\theta'_1 - \theta_k) \right) F_n^{\mathcal{O}}(\theta_2, \dots, \theta_n). \end{aligned}$$

- Bound state structure...
- Analytic properties...



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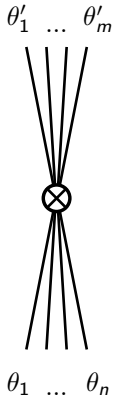
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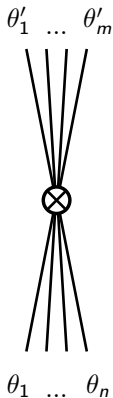
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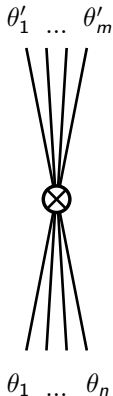
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Question:

$${}_L\langle m|\mathcal{O}|n\rangle_L = ?$$

1. Step: Bethe Ansatz in large volume

All orders in  $1/L$

2. Step: Lüscher's corrections

$$\mathcal{O}(e^{-mL})$$

3. Step: Exact results?

Possible for the diagonal elements  ${}_L\langle n|\mathcal{O}|n\rangle_L$

*with a number of educated guesses...*

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$$Q_j(\theta_1, \dots, \theta_n) = mL \sinh \theta_j + \sum_{k \neq j} \delta(\theta_j - \theta_k) = 2\pi I_j \quad j = 1 \dots n$$

- Off-diagonal matrix elements:

$$\langle \{l'_1, \dots, l'_m\} | \mathcal{O}(0,0) | \{l_1, \dots, l_n\} \rangle_L = \frac{F_{m+n}^{\mathcal{O}}(\tilde{\theta}'_m, \dots, \tilde{\theta}'_1 | \tilde{\theta}_1, \dots, \tilde{\theta}_n)}{\sqrt{\rho_n(\tilde{\theta}_1, \dots, \tilde{\theta}_n) \rho_m(\tilde{\theta}'_1, \dots, \tilde{\theta}'_m)}} + O(e^{-\mu L}),$$

where  $\rho_N$  is the density of states:

$$\rho_n(\theta_1, \dots, \theta_n) = \det \mathcal{J}^{(n)}, \quad \mathcal{J}_{jk}^{(n)} = \frac{\partial Q_j}{\partial \theta_k}, \quad \rho_n \sim L^n$$

B. Pozsgay, G. Takacs: Form factors in finite volume I: form factor bootstrap and truncated conformal space

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- Diagonal limit ill-defined:

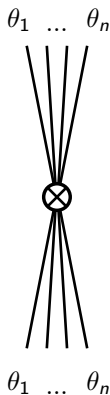
$$F_{2n}^{\mathcal{O}}(\theta_n + \varepsilon_n, \dots, \theta_1 + \varepsilon_1 | \theta_1, \dots, \theta_n) = ?$$

Terms with combinations of  $\varepsilon_j/\varepsilon_k$

- Connected diagonal form factor:

$$F_{2n,c}^{\mathcal{O}}(\tilde{\theta}_1, \dots, \tilde{\theta}_n)$$

is the finite part of the diagonal limit.



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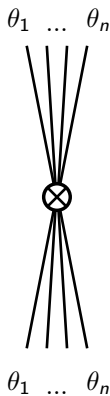
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$$\langle \theta_1, \dots, \theta_K | \mathcal{O} | \theta_1, \dots, \theta_K \rangle_L = \sum_{\{\theta_+\} \cup \{\theta_-\}} F_{2n,c}^{\mathcal{O}}(\{\theta_-\}) \frac{\tilde{\rho}_{K-n}(\{\theta_+\} | \{\theta_-\})}{\rho_K(\theta_1, \dots, \theta_K)} + \mathcal{O}(e^{-\mu L})$$

Sum over partitions:

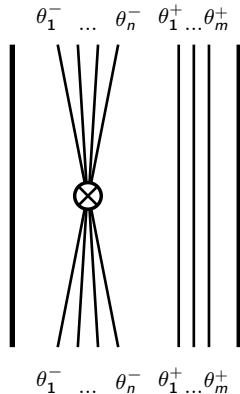
$$\{\theta\}_K = \{\theta_+\}_m \cup \{\theta_-\}_n$$

with the sub-determinants

$$\tilde{\rho}_{K-n}(\{\theta_+\} | \{\theta_-\}) = \det \mathcal{J}_+$$

Important:  $\tilde{\rho}$  depends on both sets!

B. Pozsgay, G. Takacs: Form factors in finite volume II: disconnected terms and finite temperature correlators  
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## 2. Step: Lüscher's corrections

Main idea: Small perturbation towards non-integrability

$$H' = H + \lambda \int dx \mathcal{O}(x) \quad \rightarrow \quad \langle n | \mathcal{O} | n \rangle = \frac{1}{L} \frac{dE_n}{d\lambda}$$

Lüscher's correction for  $E_n$  can be computed for one-particle states.  
Even the  $\mathcal{O}(\lambda)$  perturbation:

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Result for a standing one-particle state:

$$\begin{aligned}\langle \theta = 0 | \mathcal{O} | \theta = 0 \rangle_L &= \frac{F_2^\mathcal{O}}{mL} + \langle \mathcal{O} \rangle_L + \\ &+ \frac{1}{mL} \int \frac{d\theta}{2\pi} F_{4,c}^\mathcal{O}(\theta, -i\pi/2) S(\theta + i\pi/2) e^{-mL \cosh \theta} \\ &- \frac{1}{mL} F_2^\mathcal{O} \int \frac{d\theta}{2\pi} i\varphi(\theta + i\pi/2) S(\theta + i\pi/2) \sinh \theta e^{-mL \cosh \theta} \\ &+ F_2^\mathcal{O} \int \frac{d\theta}{2\pi} (S(\theta + i\pi/2) - 1) e^{-mL \cosh \theta}\end{aligned}$$

### 3. Step: Exact results

- Strategy: VEV's first, excited states afterwards
- Vacuum energy from Thermodynamic Bethe Ansatz (TBA)

$$E_0(L) = -m \int \frac{d\theta}{2\pi} \cosh(\theta) \log(1 + e^{-\varepsilon_0(\theta)})$$

where

$$\varepsilon_0(\theta) = mL \cosh(\theta) - \int_{-\infty}^{\infty} \frac{d\theta'}{2\pi} \varphi(\theta - \theta') \log(1 + e^{-\varepsilon_0(\theta')})$$

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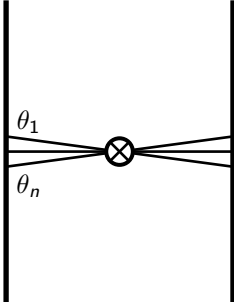
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- LeClair-Mussardo series for the VEVs:

$$\langle \mathcal{O} \rangle_L = \sum_{n=0}^{\infty} \frac{1}{n!} \int \frac{d\theta_1}{2\pi} \cdots \frac{d\theta_n}{2\pi} \left( \prod_j \frac{1}{1 + e^{\varepsilon_0(\theta_j)}} \right) F_{2n,c}^{\mathcal{O}}(\theta_1, \dots, \theta_n)$$

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- No summation!

Exception: Non-relativistic 1D Bose gas, see

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- Proven using the rotated channel (finite temperature  $L = 1/T$ ) in

B. Pozsgay: Mean values of local operators in highly excited Bethe states  
arXiv:1009.4662

- No summation!

Exception: Non-relativistic 1D Bose gas, see

M. Kormos, G. Mussardo, A. Trombettoni: Expectation Values in the Lieb-Liniger Bose Gas  
arXiv:0909.1336



### 3. Step: Exact results

- Excited state TBA in sinh-Gordon for  $|\bar{\theta}_1, \dots, \bar{\theta}_K\rangle_L$ :

$$\varepsilon(\theta) = mL \cosh \theta + \sum_{j=1}^K \log S(\theta - \bar{\theta}_j - i\pi/2) - \int \frac{d\theta'}{2\pi} \varphi(\theta - \theta') \log(1 + e^{-\varepsilon(\theta')})$$

Condition for the Bethe roots:

$$1 + e^{\varepsilon(\bar{\theta}_j + i\pi/2)} = 0, \quad j = 1 \dots K$$

Exact finite size energies:

$$E = \sum_{j=1}^K m \cosh \bar{\theta}_j - \int \frac{d\theta}{2\pi} m \cosh(\theta) \log(1 + e^{-\varepsilon(\theta)})$$

- Derived in:

J. Teschner: On the spectrum of the Sinh-Gordon model in finite volume  
arXiv:hep-th/0702214

### 3. Step: Exact results

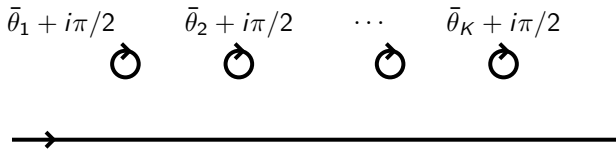
- Excited state TBA in sinh-Gordon:

$$\varepsilon(\theta) = mL \cosh(\theta) - \int_C \frac{d\theta'}{2\pi} \varphi(\theta - \theta') \log(1 + e^{-\varepsilon(\theta')})$$

and

$$E(L) = -m \int_C \frac{d\theta}{2\pi} \cosh(\theta) \log(1 + e^{-\varepsilon(\theta)})$$

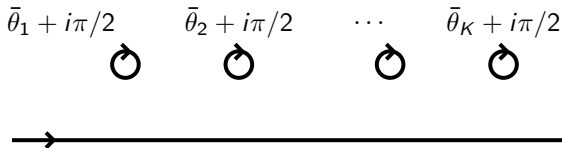
New contour C:



### 3. Step: Exact results

- Big conjecture: LeClair-Mussardo series for excited states

$$\langle \bar{\theta}_1, \bar{\theta}_2, \dots, \bar{\theta}_K | \mathcal{O} | \bar{\theta}_1, \bar{\theta}_2, \dots, \bar{\theta}_K \rangle_L =$$
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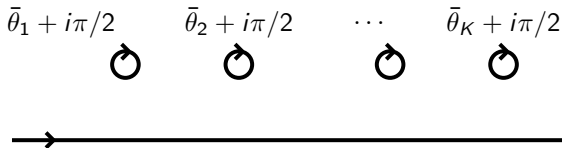


- Strategy: Evaluate the residues and do a partial resummation...

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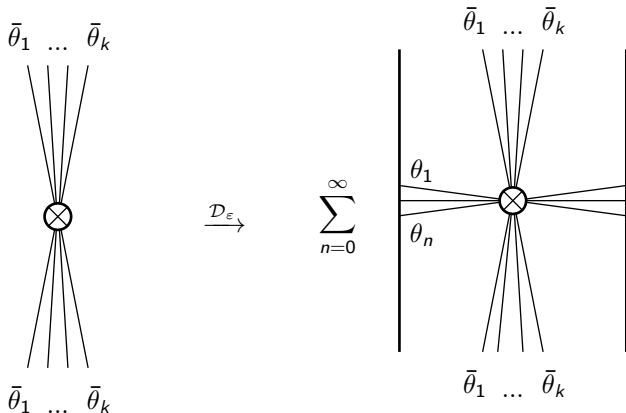


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### 3. Step: Exact results

- For a form factor  $F_{2k}^{\mathcal{O}}(\bar{\theta}_1, \dots, \bar{\theta}_k)$  define the „dressed version” as

$$\mathcal{D}_\varepsilon^{\mathcal{O}}(\bar{\theta}_1, \dots, \bar{\theta}_k) \equiv \sum_{n=0}^{\infty} \frac{1}{n!} \left( \prod_j \int_{-\infty}^{\infty} \frac{d\theta_j}{2\pi(1 + e^{\varepsilon(\theta_j)})} \right) F_{2(n+k),c}^{\mathcal{O}}(\bar{\theta}_1 + i\pi/2, \dots, \bar{\theta}_k + i\pi/2, \theta_1, \dots, \theta_n)$$



### 3. Step: Exact results

- After partial resummation:

$${}_L\langle \bar{\theta}_1, \dots, \bar{\theta}_K | \mathcal{O} | \bar{\theta}_1, \dots, \bar{\theta}_K \rangle_L = \sum_{\{\bar{\theta}_+\} \cup \{\bar{\theta}_-\}} \mathcal{D}_\varepsilon^\mathcal{O}(\{\bar{\theta}_-\}) \frac{\bar{\rho}_{K-n}(\{\bar{\theta}_+\} | \{\bar{\theta}_-\})}{\bar{\rho}_K(\bar{\theta}_1, \dots, \bar{\theta}_K)}$$

Here

$$\bar{\rho}_K(\bar{\theta}_1, \dots, \bar{\theta}_K) = \det \mathcal{K}_{jk} \quad \mathcal{K}_{jk} = \frac{d\varepsilon(\bar{\theta}_j + i\pi/2)}{d\bar{\theta}_k}$$

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$$\bar{\rho}_{K-n}(\{\bar{\theta}_+\} | \{\bar{\theta}_-\}) = \det \mathcal{K}_+,$$

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# Checks and To Do list

## Checks:

- Agrees with the Lüscher's correction for the one-particle states
- Proven for  $\mathcal{O} = \Theta$
- Numerical checks in

B. Pozsgay, I.M. Szecsenyi, G. Takacs: Exact finite volume expectation values of local operators in excited states

arXiv:1412.8436

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- Resummation?
- UV limit
- Other derivation?



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Thank you for your attention!