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New Integrable Matrix Field Theories from Strongly Twisted AdS/CFT

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Outline

- AdS/CFT correspondence and integrability of planar N=4 SYM are well tested hypothesis, but they were never proven. Strong coupling regime of N=4 SYM looks almost as complicated as usual QCD and any direct computation is hardly possible
- N=4 SYM and ABJM admit various integrable (β-, γ-, η-…) deformations (twists)
- We propose new chiral field theory (χFT) in specific double scaling (DS) limit of gamma-deformed N=4 SYM: large (imaginary) gamma-deformation parameters at weak coupling. It is integrable in 't Hooft limit in 4D!
- In particular case this χFT reduces to two interacting complex scalars. It has very limited set of Feynman graphs for most interesting quantities (essentially – one graph at a given loop order). Typical graphs are of "fishnet" type (square lattice of massless propagators), shown to be explicitly integrable by A.Zamolodchikov. This reveals the origins of AdS/CFT integrability at any coupling!
- Integrability allows to compute multi-loop Feynman graphs of "wheel" and "multi-spiral" types, related to BMN vacuum 2-point correlator and operators with magnons. I will show how the asymptotic Bethe ansatz (ABA) computes the anomalous dimensions of multi-magnon operators in χFT at first few loops
- The general approach to computation of dimensions in planar twisted N=4 SYM, called Qantum Spectral Curve (QSC), has yet to be adopted to this DS limit. Next lecture will devoted to general properties of twisted QSC

Gamma-twisted N=4 SYM

Leigh, Strassler
Beisert, Roiban

- Gamma-twisted N=4 SYM Lagrangian (integrable in planar llimit!)

$$\mathcal{L} = N_c \mathrm{Tr}[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D^\mu \phi_i^\dagger D_\mu \phi^i + i\bar{\psi}_A^{\dot{\alpha}} D_{\dot{\alpha}}^{\alpha} \psi_\alpha^A] + \mathcal{L}_{\mathrm{int}}$$

$$\mathcal{L}_{\mathrm{int}} = N_c g\ \mathrm{Tr}[\frac{g}{4}\{\phi_i^\dagger, \phi^i\}\{\phi_j^\dagger, \phi^j\} - g\, e^{-i\epsilon^{ijk}\gamma_k}\phi_i^\dagger \phi_j^\dagger \phi^i \phi^j$$

$$-e^{-\frac{i}{2}\gamma_j^-}\bar{\psi}_j \phi^j \bar{\psi}_4 + e^{+\frac{i}{2}\gamma_j^-}\bar{\psi}_4 \phi^j \bar{\psi}_j + i\epsilon_{ijk} e^{\frac{i}{2}\epsilon_{jkm}\gamma_m^+}\psi^k \phi^i \psi^j$$

$$-e^{+\frac{i}{2}\gamma_j^-}\psi_4 \phi_j^\dagger \psi_j + e^{-\frac{i}{2}\gamma_j^-}\psi_j \phi_j^\dagger \psi_4 + i\epsilon^{ijk} e^{\frac{i}{2}\epsilon_{jkm}\gamma_m^+}\bar{\psi}_k \phi_i^\dagger \bar{\psi}_j]\,.$$

$$\gamma_1^\pm = -\frac{\gamma_3 \pm \gamma_2}{2}$$

$$\gamma_2^\pm = -\frac{\gamma_1 \pm \gamma_3}{2}$$

$$\gamma_3^\pm = -\frac{\gamma_2 \pm \gamma_1}{2}$$

- Product of fields replaced by star product:

$$F_a F_b \to F_a \star F_b e^{-\frac{1}{2}Q_a \wedge Q_b}$$

$$Q_a \wedge Q_b = Q_a^T \begin{pmatrix} 0 & -\gamma_3 & \gamma_2 \\ \gamma_3 & 0 & -\gamma_1 \\ -\gamma_2 & \gamma_1 & 0 \end{pmatrix} Q_b$$

R-charge matrix:

	A	ψ^1	ψ^2	ψ^3	ψ^4	ϕ^1	ϕ^2	ϕ^3
Q_a^1	0	$+1/2$	$-1/2$	$-1/2$	$+1/2$	1	0	0
Q_a^2	0	$-1/2$	$+1/2$	$-1/2$	$+1/2$	0	1	0
Q_a^3	0	$-1/2$	$-1/2$	$+1/2$	$+1/2$	0	1	0

- Breaks all supersymmetry and R-symmetry: $PSU(2,2|4) \to SU(2,2)\times U(1)^3$

Strongly twisted N=4 SYM in double scaling limit

- Gamma-twisted N=4 SYM Lagrangian

$$\mathcal{L} = N_c \mathrm{Tr}[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D^\mu\phi_i^\dagger D_\mu\phi^i + i\bar\psi_A^{\dot\alpha}D_{\dot\alpha}^{\alpha}\psi_\alpha^A] + \mathcal{L}_{\mathrm{int}}$$

$$\mathcal{L}_{\mathrm{int}} = N_c g\ \mathrm{Tr}[\frac{g}{4}\{\phi_i^\dagger,\phi^i\}\{\phi_j^\dagger,\phi^j\} - g\, e^{-i\epsilon^{ijk}\gamma_k}\phi_i^\dagger\phi_j^\dagger\phi^i\phi^j$$

$$-e^{-\frac{i}{2}\gamma_j^-}\bar\psi_j\phi^j\bar\psi_4 + e^{+\frac{i}{2}\gamma_j^-}\bar\psi_4\phi^j\bar\psi_j + i\epsilon_{ijk}e^{\frac{i}{2}\epsilon_{jkm}\gamma_m^+}\psi^k\phi^i\psi^j$$

$$-e^{+\frac{i}{2}\gamma_j^-}\psi_4\phi_j^\dagger\psi_j + e^{-\frac{i}{2}\gamma_j^-}\psi_j\phi_j^\dagger\psi_4 + i\epsilon^{ijk}e^{\frac{i}{2}\epsilon_{jkm}\gamma_m^+}\bar\psi_k\phi_i^\dagger\bar\psi]\,.$$

- We proposed a double scaling limit: strong twist, weak coupling

$$g \to 0, \qquad e^{-i\gamma_j/2} \to \infty, \qquad \xi_j = g\, e^{-i\gamma_j/2} - \text{fixed}, \qquad (j = 1,2,3.)$$

- Gauge fields and some 4-scalar and Yukawa interactions get discarded and we are left with a new Lagrangian of presumably integrable QFT

$$\mathcal{L}_{\mathrm{int}} = N_c\,\mathrm{Tr}[\xi_1^2\,\phi_2^\dagger\phi_3^\dagger\phi_2\phi_3 + \xi_2^2\,\phi_3^\dagger\phi_1^\dagger\phi_3\phi_1 + \xi_3^2\,\phi_1^\dagger\phi_2^\dagger\phi_1\phi_2 +$$

$$+ i\sqrt{\xi_2\xi_3}(\psi^3\phi^1\psi^2 + \bar\psi_3\phi_1^\dagger\bar\psi_2) + i\sqrt{\xi_1\xi_3}(\psi^1\phi^2\psi^3 + \bar\psi_1\phi_2^\dagger\bar\psi_3) + i\sqrt{\xi_1\xi_2}(\psi^2\phi^3\psi^1 + \bar\psi_2\phi_3^\dagger\bar\psi_1)]\,.$$

Gurdogan, V.K. 2015

Special Case: Planar bi-Scalar QFT

- Special case $\xi := \xi_3$ – fixed, $\xi_1 = \xi_2 = 0$
- We are left with simple bi-scalar theory, integrable in planar limit

$$\mathcal{L}[\phi_1, \phi_2] = \frac{N_c}{2} \mathrm{Tr} \left(\partial^\mu \phi_1^\dagger \partial_\mu \phi_1 + \partial^\mu \phi_2^\dagger \partial_\mu \phi_2 + 2\xi^2 \, \phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2 \right) .$$

- Notice that Lagrangian is "chiral": the term $2\xi^{*2} \, \mathrm{Tr} \, (\phi_2^\dagger \phi_1^\dagger \phi_2 \phi_1)$ missing
- Wilson renormalization leads to new counter-terms of double trace type

$$\eta_{ij} \mathrm{Tr}(\phi_j^\dagger \phi_j) \mathrm{Tr}(\phi_j^\dagger \phi_j) \quad \text{and} \quad \tilde{\eta}_{ij} \mathrm{Tr}(\phi_i^\dagger \phi_j^\dagger) \mathrm{Tr}(\phi_i \phi_j)$$

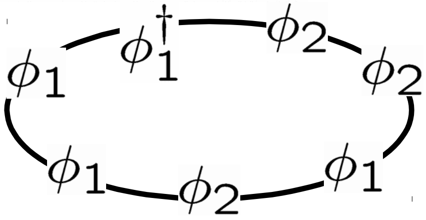
Zarembo, Tseytlin 2002
Klebanov et al
Fokken,Sieg, Wilhelm,2013, 2015

- These couplings are running even in planar limit, is not really a CFT.
- But the coupling ξ is not running in planar limit!
  The correlators not containing length=2 in initial or intermediate states are conformal! We can study them at $\eta_{ij} = \tilde{\eta}_{ij} = 0$ (for any i, j)

Correlation Functions of Conformal QFT

- Main physical quantities - local operators:

$$\mathcal{O}(x) = \mathrm{Tr}\left[(\phi_1)^{L_1}(\phi_2)^{L_2}(\phi_1)^{\dagger L_1'}(\phi_2)^{\dagger L_2'}\right](x) + \text{permutations}$$



- 4D Correlators:

scaling dimensions

structure constants

non-trivial functions of coupling ξ

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \frac{\delta_{ij}}{|x|^{2\Delta_j(\xi)}}$$

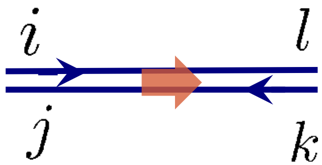
$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\mathcal{O}_k(x_3)\rangle = \frac{C_{ijk}(\xi)}{|x_{12}|^{\Delta_i+\Delta_j-\Delta_k}|x_{23}|^{\Delta_j+\Delta_k-\Delta_i}|x_{31}|^{\Delta_i+\Delta_k-\Delta_j}}$$

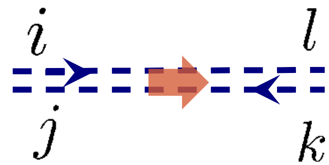
- They describe the whole conformal theory via operator product expansion

Feynman Rules for our bi-Scalar QFT

- Perturbation theory w.r.t. coupling ξ in terms of double-line Feynman graphs

- Propagators

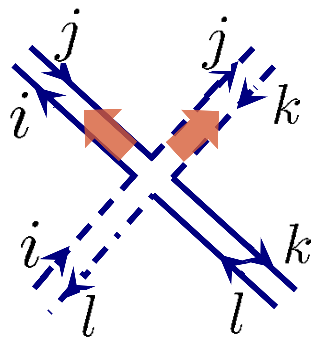




$i, j, k, l = 1 \dots N_c$

$$\left\langle \phi_1^{*ij}(y)\phi_1^{kl}(x) \right\rangle_0 = \left\langle \phi_2^{*ij}(y)\phi_2^{kl}(x) \right\rangle_0 = \delta^{il}\delta^{jk}\frac{1}{(x-y)^2}$$

- Vertex: $\xi^2 \mathrm{Tr}\,(\phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2)$



- Vertex with opposite orientation (chirality) is absent.
  We could pick it in the opposite DS limit:

$$g \to 0, \qquad e^{-i\gamma_j/2} \to 0, \qquad \xi_j = \frac{g}{e^{-i\gamma_j/2}} - \text{fixed}, \qquad (j = 1, 2, 3.)$$

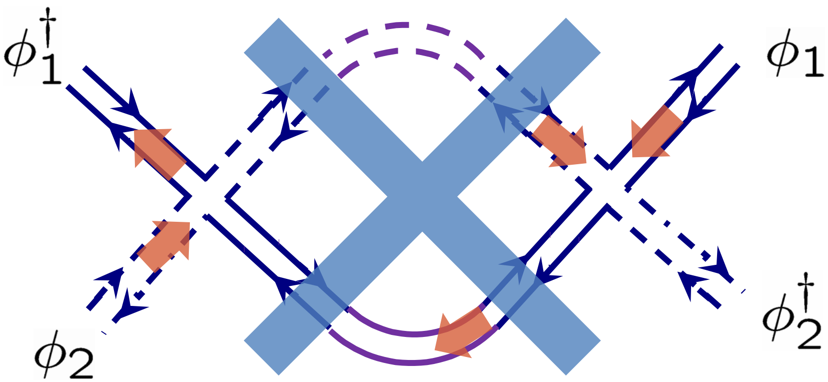
Gurdogan, V.K. 2015

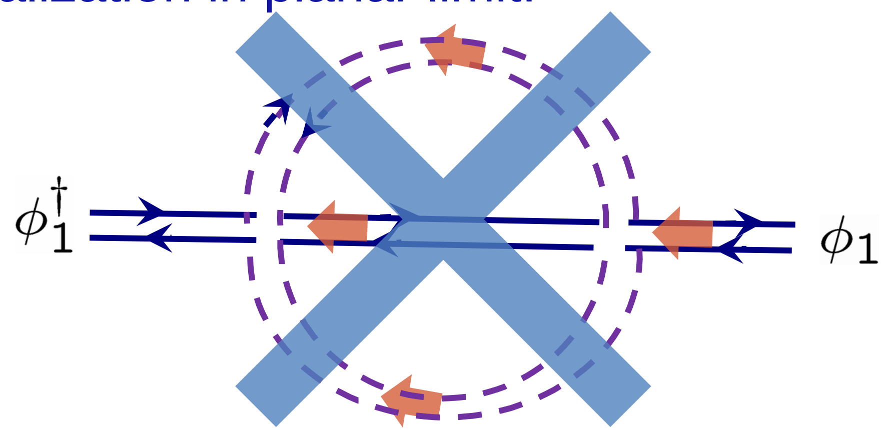
Planar Feynman Graphs for bi-Scalar QFT

- The number of planar graphs in chiral bi-scalar QFT is very limited: essentially – one (or none) Feynman graph at each order of perturbation theory for a given physical quantity.

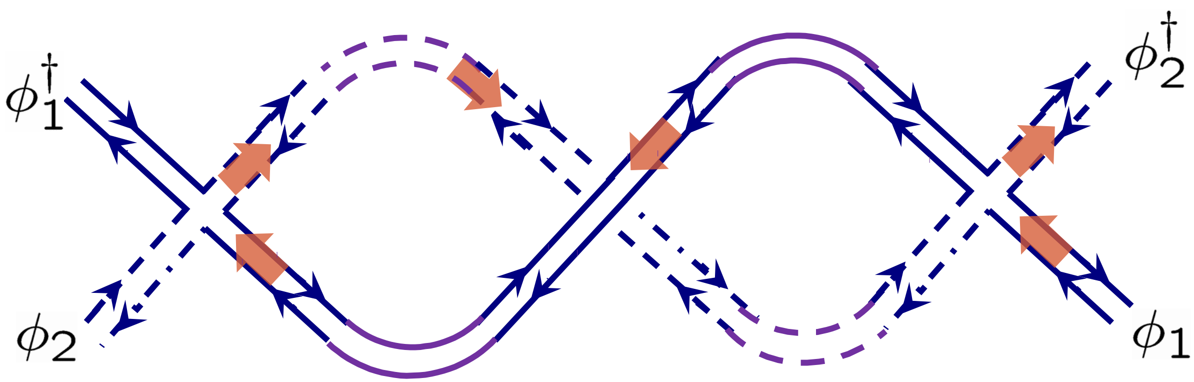
Zero dimensional analogue:
Kostov, Staudacher 1995

- Example: no mass or vertex renormalization in planar limit:





- Double-trace coupling is generated by the graph:



$$\mathrm{Tr}(\phi_1^\dagger \phi_2)\mathrm{Tr}(\phi_2^\dagger \phi_1)$$

Still leading order in 1/N !

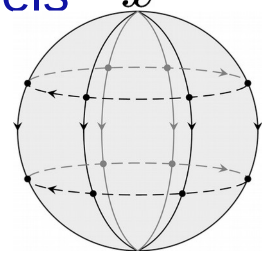
Gurdogan, V.K. 2015

BMN vacuum dimension, “Wheel” and “Fishnet” Graphs

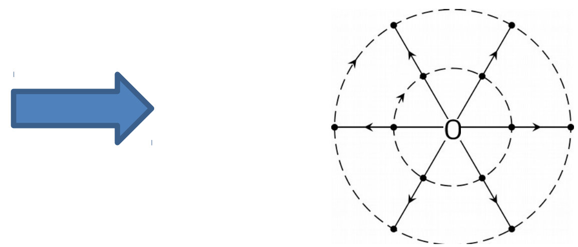
- Pair correlator for simplest operators (called BMN vacuum) $\mathrm{Tr}[\phi_1(x)]^L$

$$K_L(x,0) = \langle \mathrm{Tr}[\phi_1(x)]^{\dagger L} \mathrm{Tr}[\phi_1(0)]^L \rangle$$

- Admissible graphs look like a “globe”, with ϕ_1 “meridians” and ϕ_2 “parallels”



Gurdogan, V.K. 2015
(and discussion with Sieg, Wilhelm)



$$= \sum_{k=0}^{M} C_{L,k}^{(M)} (\log \Lambda)^k$$

- If $x \to \infty$ (IR domain) then the integration domains, due to logarithmic UV divergences, will be concentrated at the south pole.
  Amputated “wheel” graphs with L spokes and M frames
- Then the propagators at north pole factor out as $|x|^{-2L}$ and we can read off the anomalous dimension from the formula

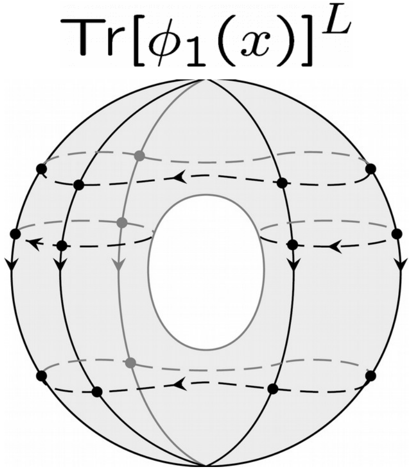
$$\gamma_L = \sum_{r=1}^{\infty} \xi^{2LM} \gamma_L^{(M)} = \frac{1}{2\log\Lambda} \log\left(1 + \sum_{M=1}^{\infty} \xi^{2LM} \sum_{k=0}^{M} C_{L,k}^{(M)} (\log\Lambda)^k\right)$$

The higher powers of logarithms cancel in CFT !

Gurdogan, V.K. 2015

1/N expansion: failure of conformality

- The 1/N corrections will inevitably contain singular L=2 states running around non-trivial cycles of Feynman graphs of higher topologies:

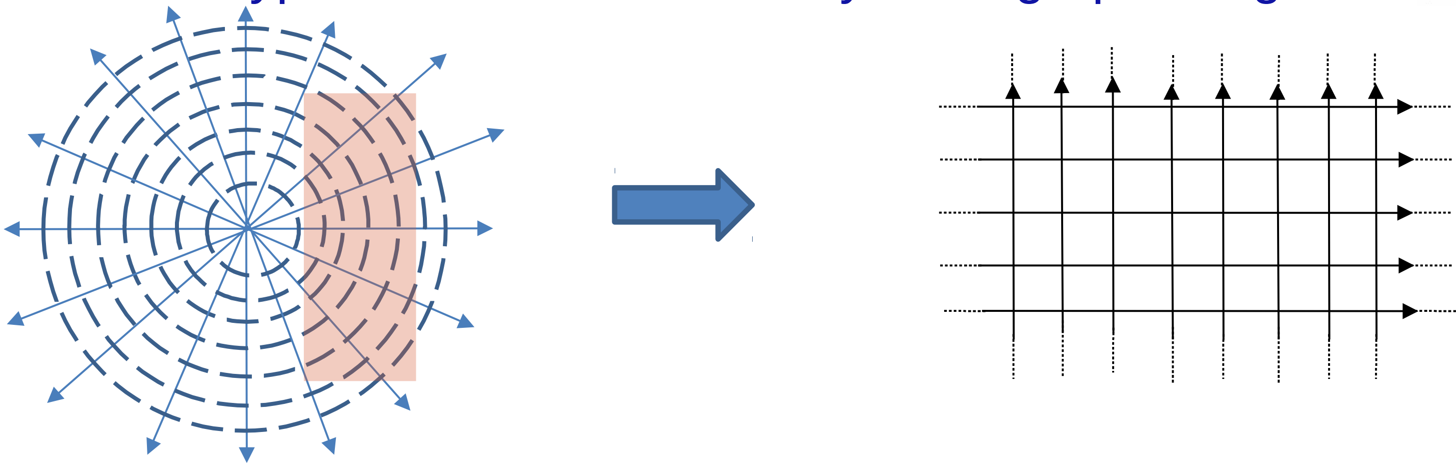


- States of length L=2 probably play the role of tachyons in the 4-dimensional dual string theory, which would be interesting to find.

- This string theory should have non-interacting chiral and anti-chiral components for worldsheet fields, corresponding to large and small twist DS limits of SYM.

Integrability of "Fishnet" Graphs

- "Fishnet" is a typical bulk element of Feynman graphs, e.g. for $\mathrm{Tr}[\phi_1(x)]^L$



- Fishnet graphs are integrable, i.e. in principal calculable.

A.Zamolodchikov 1980

- Then the whole bi-scalar theory looks integrable in the large N, planar limit!

- Let us try to precise and exploit this integrability

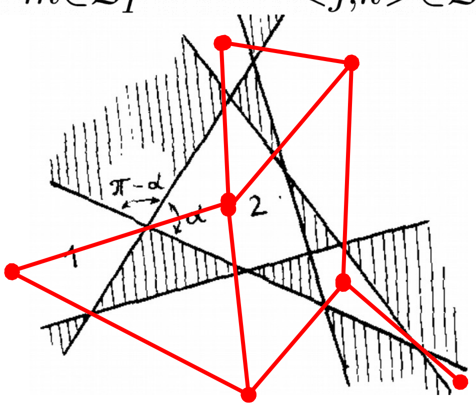
A.Zamolodchikov 1980

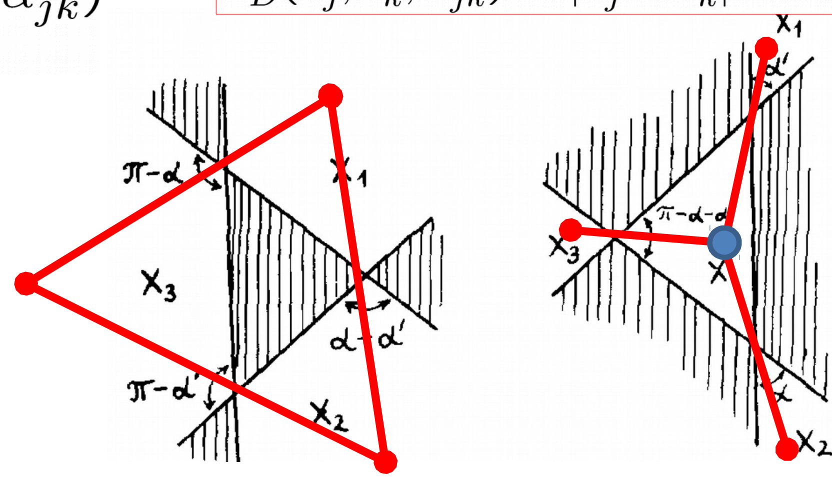
Review of Zamolodchikov "Fishnet" graph Integrability

- Construct a Feynman graph on Baxter lattice (system of arbitrary intersecting straight lines on the plane) by the following rules. Dash all the faces connected through the common vertices forming a sublattice type I, leaving blank the similar complimentary sub-lattice of type II.
- Place a 4D coordinate x_j in the middle of each blank face and connect the neigbours
- Integrate over all 4D coordinates the product of propagators over neighbors

$$\mathcal{Z}_B = \int \prod_{m \in \mathcal{L}_I} d^D x_m \prod_{<j,k> \in \mathcal{L}_I} G_D(x_j, x_k, \alpha_{jk})$$

$$G_D(x_j, x_k, \alpha_{jk}) = |x_j - x_k|^{\frac{D}{\pi}(\alpha_{jk} - \pi)}$$





- Integrability amounts to famous Baxter's star-triangle relation (a version of Yang-Baxter relation): it allows to move vertical line to the left, replacing the

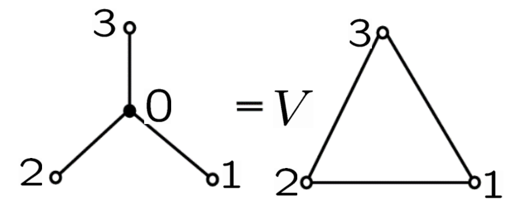
dashed triangle by a blank triangle, i.e. integrate over the associated

Star-Triangle Relation and 4D fishnet

- To make it possible, the following star-triangle identity is applied:

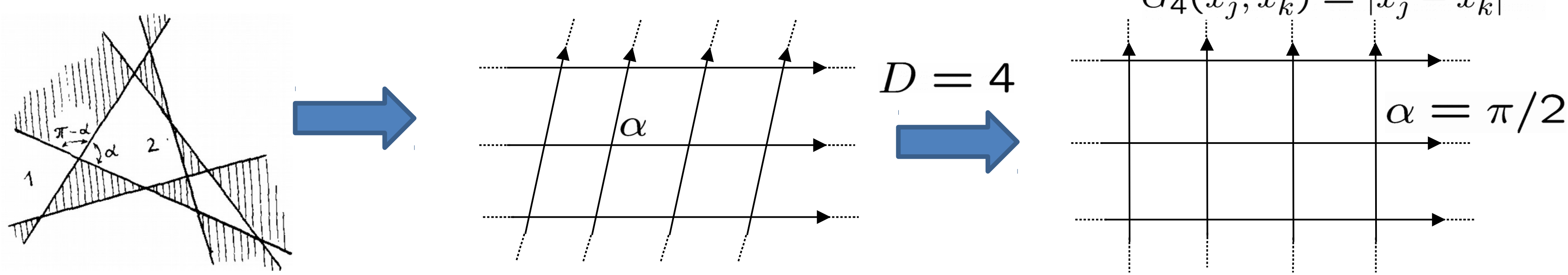
$$\int \frac{d^D x_0}{|x_{10}|^{D\frac{\alpha}{\pi}} |x_{20}|^{D\frac{\beta}{\pi}} |x_{30}|^{D\frac{\pi-\alpha-\beta}{\pi}}} = \frac{V(\alpha,\beta)}{|x_{12}|^{D\frac{\alpha+\beta}{\pi}} |x_{23}|^{D\frac{\pi-\alpha}{\pi}} |x_{31}|^{D\frac{\pi-\beta}{\pi}}}, \qquad x_{ij} = x_i - x_j$$

where $V(\alpha,\beta) = \pi^{D/2} \frac{\Gamma(D\frac{\alpha+\beta}{\pi})\Gamma(D\frac{\pi-\alpha}{\pi})\Gamma(D\frac{\pi-\beta}{\pi})}{\Gamma(D\frac{\pi-\alpha-\beta}{\pi})\Gamma(D\frac{\alpha}{\pi})\Gamma(D\frac{\beta}{\pi})}$



- Particular case: from Baxter to parallelogram and square lattice

$G_4(x_j, x_k) = |x_j - x_k|^{-2}$



- Singular limit $\alpha + \beta - D\pi/2 = \epsilon \to 0$ based on identites:

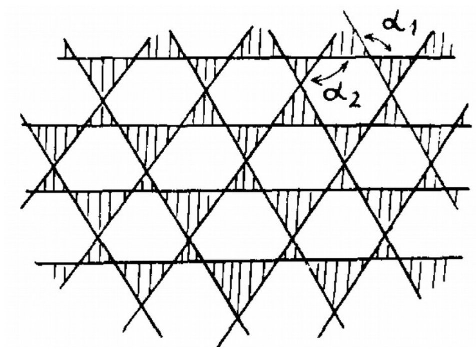
$$\frac{\epsilon}{(x_{30}^2)^{D/2-\epsilon}} = C_D\, \delta^{(D)}(x_{30}) \qquad \int \frac{d^D x_0}{|x_{10}|^{D\frac{\alpha}{\pi}} |x_{20}|^{D\frac{\beta}{\pi}}} = \frac{V(\alpha,\beta)}{|x_{12}|^{D\frac{\alpha+\beta}{\pi}-D}}$$

- We get precisely the massless fishnet graph for $\alpha = \beta = \pi/2, \quad D = 4$

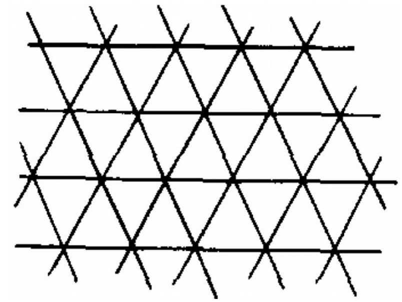
A.Zamolodchikov 1980

Integrable Kagome graphs, 3D and 6D

- We can choose three systems of lines with angles $\alpha = 0, \frac{\pi}{3}, \frac{2\pi}{3}$



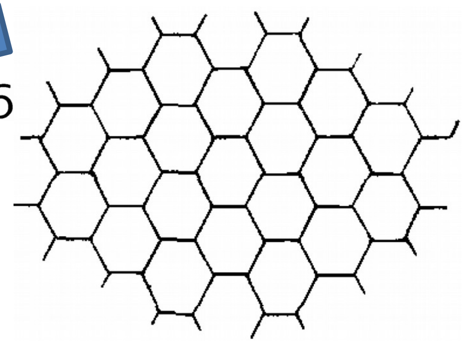
$D = 3$



$G_3(x_j, x_k) = |x_j - x_k|^{-1}$

$\mathrm{Tr}\,(\phi_1^\dagger \phi_2^\dagger \phi_3^\dagger \phi_1 \phi_2 \phi_3)$

$D = 6$



$G_6(x_j, x_k) = |x_j - x_k|^{-4}$

$\mathrm{Tr}\,(\phi_1^\dagger \phi_2^\dagger \phi_3)$

- The triangulated graph corresponds to integrable 8-scalar theory following from the twisted ABJM model. 6D case: is it the famous “Little String Theory”?
- We also checked this integrability by introducing the row transfer matrix building the wheel graph and showing its commutativity with standard transfer-matrix of SL(4) non-compact spin chain, i.e. for spins on conformal 4D group.

V.K , S.Negro, G. Korchemsky (in work)

Caetano, Gurdogan, V.K.
(to be published)

Strongly twisted ABJM in double scaling limit

- Gamma-twisted ABJM appears to depend on a single twist parameter.
- Double scaling limit:

$$k \to \infty, \qquad e^{-i\gamma_j/2} \to \infty, \qquad \xi = k^{-1}\, e^{-i\gamma_3/2} - \text{fixed}, \qquad (j = 1, 2, 3.)$$

- Lagrangian in this limit

$$\mathcal{L}^{(DS)}_{\text{ABJM}} = \text{Tr}[-\frac{1}{2}\partial^\mu Y_i^\dagger \partial_\mu Y^i + i\bar{\Psi}\Gamma^\mu \partial_\mu \Psi] + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}^{\Psi}_{\text{int}} = -\frac{i\xi}{4\pi}\text{Tr}[2Y^4\bar{\Psi}^1\Psi_4\bar{Y}_1 - Y^4\bar{Y}_2\Psi_4\bar{\Psi}^2 - Y^3\bar{Y}_1\Psi_3\bar{\Psi}^1 + 2Y^3\bar{\Psi}^2\Psi_3\bar{Y}_2 + 2Y^2\bar{\Psi}^4\Psi_2\bar{Y}_4 - Y^2\bar{Y}_3\Psi_2\bar{\Psi}^3 + 2Y^1\bar{\Psi}^3\Psi_1\bar{Y}_3 - Y^1\bar{Y}_4\Psi_1\bar{\Psi}^4)\ ].$$

$$\mathcal{L}^{Y}_{\text{int}} = \frac{\xi^2}{4\pi}\text{Tr}\left(Y^2\bar{Y}_3Y^4\bar{Y}_2Y^3\bar{Y}_4 + Y^3\bar{Y}_1Y^4\bar{Y}_3Y^1\bar{Y}_4 + Y^2\bar{Y}_1Y^4\bar{Y}_2Y^1\bar{Y}_4 + Y^2\bar{Y}_3Y^1\bar{Y}_2Y^3\bar{Y}_1\right)$$

- Gauge fields, some 6-scalar and some “Yukawa” interactions discarded
- Typical 3D Feynman graphs contain a regular triangular “fishnet” bulk part
- One could think of integrable 6D QFT realizing honeycomb “fishnet” graphs

$$\mathcal{L}_{6D} = \text{Tr}[-\frac{1}{2}\partial^\mu \phi_i^\dagger \partial_\mu \phi^i + \xi\phi_1^\dagger\phi_2\phi_3 + \xi\phi_1\phi_2^\dagger\phi_3^\dagger] + \dots$$

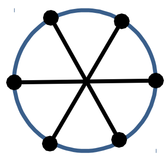
- Do we have an untwisted analogue of it?
  Candidates: (0,2) and (1,1) 6D gauge theories? Little string theory?

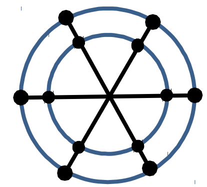
Wheel graphs and dimension of BMN vacuum

- Single wrapped graph was computed directly long ago

Broadherst 1980
Fokken, Sieg, Wilhelm 2013

- Double wrapped graph can be extracted from integrability (TBA) result of Ahn et al. (in terms of complicated double sums and integrals)
- We brought this result to explicit form:





Ahn, Bajnok, Bombardelli, Nepomechie 2013
Gurdogan, V.K , 2015

$$\gamma_{\text{vac}}(L) = -2\binom{2L-2}{L-1}\zeta_{2L-3}\xi^{2L} + \left[A(L) + 2\binom{4L-2}{2L-1}\zeta_{4L-5}\right]\xi^{4L} + \mathcal{O}(\xi^{6L})$$

$$A(3) = -\frac{63}{4096}\zeta_7 - \frac{9}{256}\zeta_3^2 .$$

(confirmed by direct graph computation!) E.Panzer, 2015

$$A(4) = \frac{1}{2048}\left[-30\zeta_{11} + 4\zeta_{3,8} + 5\zeta_{5,6} - \zeta_{6,5} + 10\zeta_{8,3} - 2\zeta_{3,3,5} + 10(\zeta_{3,5,3} + \zeta_{5,3,3})\right] - \frac{25}{1024}\zeta_5^2 .$$

Rieman multi-zeta numbers

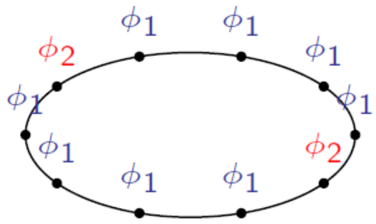
$$\begin{aligned}
A(L) = \frac{2^{3-4L}}{\Gamma(L)^2}\Bigg\{ & \sum_{\substack{j_1,j_2>0 \\ j_1+j_2<2L-3}} \frac{\Gamma(2L-j_1-j_2-1)\Gamma(L+j_1)\Gamma(L+j_2)}{\Gamma(j_1+1)\Gamma(j_2+1)\Gamma(L-j_1)\Gamma(L-j_2)}\left[\sum_{k=1}^{L+j_1-2}\left(\binom{L+k+j_2-4}{k-1}\left(2\zeta_{\substack{L+j_1-k-1, \\ L+j_2+k-3, \\ 2L-j_1-j_2-1}} + \zeta_{\substack{L+j_1-k-1, \\ 3L-j_1+k-4}}\right)\right.\right. \\
& \left.\left. - \binom{L+k+j_2-2}{k}\left(2\zeta_{\substack{L+j_1-k-1, \\ L+j_2+k-2, \\ 2L-j_1-j_2-2}} - 2\zeta_{\substack{L+j_1-k-1, \\ L+j_2+k-1, \\ 2L-j_1-j_2-3}}\right)\right) + 2\zeta_{\substack{2L+j_1+j_2-3, \\ 2L-j_1-j_2-2}} + 2\zeta_{\substack{2L+j_2-1, \\ L+j_1-1, \\ 2L-j_1-j_2-3}} \zeta_{\substack{L+j_1-2, \\ 3L-j_1-3}} - \zeta_{\substack{2L+j_1+j_2-2, \\ 2L-j_1-j_2-3}}\right] \\
& - \sum_{j_1=0}^{L-1}\frac{(2L-2)!}{L-j_1}\binom{L+j_1-1}{j_1}\zeta_{2L-3}\zeta_{\substack{L+j_1-2, \\ L-j_1}} \\
& + \frac{\Gamma^2(2L-1)}{2\Gamma^2(L)}\left[\sum_{k=1}^{2L-4}\left(\binom{2L+k-4}{k}\left(3\zeta_{\substack{2L-k-3, \\ 2L+k-2}} + 2\zeta_{\substack{2L-k-3, \\ 2L+k-4, \\ 2}}\right) - 2\left(\binom{2L+k-4}{k-1} + \binom{2L+k-4}{k+1}\right)\zeta_{\substack{2L-k-3, \\ 2L+k-3, \\ 1}}\right)\right. \\
& \left. + 2\zeta_{\substack{2L-3, \\ 2L-4, \\ 2}} - 4(L-3)\zeta_{\substack{2L-3, \\ 2L-3, \\ 1}} + \zeta_{\substack{2L-4, \\ 2L-1}} - 4\zeta_{\substack{2L-2, \\ 2L-3}} + 4\zeta_{\substack{4L-6, \\ 1}} + 2\zeta_{\substack{3L-4, \\ 2L-2, \\ 1}} + \frac{2L+1}{L}\zeta_{2L-3}\zeta_{2L-2} - 5\zeta_{4L-5} - \zeta_{2L-3}^2\right]\Bigg\}
\end{aligned}$$

Caetano, Gurdogan, V.K , 2016
(to be published)

Operators with magnon and spiral graphs

- Operators with M magnons ϕ_2 in the “vacuum” of ϕ_1

$$\mathcal{O}_{L,M}(x) = \text{tr}\left(\underbrace{\phi_2\phi_1\phi_1\phi_1\phi_2\ldots\phi_1}_{L\text{ fields}}\right)(x) + \text{permutations}$$

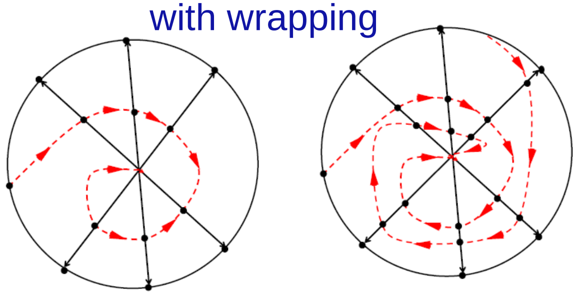


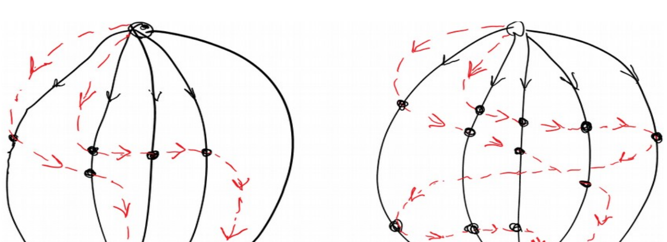
- Graphs with magnons generalize wheel graphs. They look like multi-spiral (dotted lines) winding around collinear (solid) lines. Still look like “fishnet” in the bulk.

“spiral” 2-magnon graphs

amputated graphs

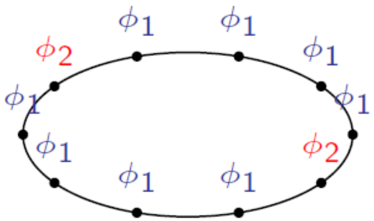
1- and 2-magnon graphs with wrapping





Mixing matrix of multi-magnon states

- Example: two magnons

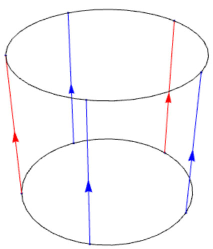
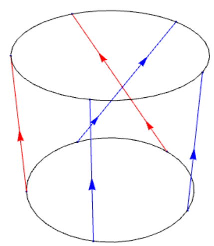
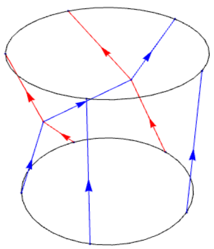


$$\mathcal{O}_{2-magnon}(x) = \mathrm{tr}\left(\underbrace{\phi_2\phi_1\phi_1\phi_1\phi_2\ldots\phi_1}_{L \text{ fields}}\right)(x) + \text{permutations}$$

- Graphs for correlation function with two magnons, length L=5

$\mathcal{O}_j(x)$

$\mathcal{O}_k(0)$

 $+\xi^2$  $+\xi^4$  $+\ldots$

- Anomalous dimensions – eigenvalues of mixing matrix $\widehat{D}_{jk}$

$$\mathcal{O}_j^{ren}(\Lambda x) = \left(\Lambda^{\widehat{D}}\right)_{jk}\mathcal{O}_k(x) \qquad \widehat{D}_{jk}\mathcal{O}_k = \Delta\mathcal{O}_j$$

- Perturbation theory:

$$\widehat{D} = \widehat{D}^{(0)} + \xi^2\widehat{D}^{(1)} + \xi^4\widehat{D}^{(2)} + \ldots \quad \Rightarrow \quad \Delta = \Delta^{(0)} + \xi^2\Delta^{(1)} + \xi^4\Delta^{(2)} + \ldots$$

Mixing matrix via dimensional regularization

- "Dimreg" is the most popular scheme. Here we review it for computation of mixing matrix and anomalous dimensions
- We compute 2-point correlator of bare operators by perturbation theory

$$\langle \mathcal{O}_\alpha^{\text{bare}}(x)\mathcal{O}_\beta^{\text{bare}}(0)\rangle = \frac{1}{x^{2\Delta_0(1-\epsilon)}}\left(\mathcal{N}_{\alpha\beta} + \xi^2(x\mu)^{2\epsilon}I_{\alpha\beta}^{(2)}(\epsilon) + \xi^4(x\mu)^{4\epsilon}I_{\alpha\beta}^{(4)}(\epsilon)\right)$$

where we evaluate Feynman graphs in Laurent series in ε = (4 – D)/2

$$I_{\alpha\beta}^{(n)}(\epsilon) = \frac{c_{\alpha\beta}^{(n,n)}}{\epsilon^n} + \frac{c_{\alpha\beta}^{(n,n-1)}}{\epsilon^{n-1}} + \dots .$$

- Notice that the highest pole has same power as the order of PT since it corresponds to the number of subdivergencies
- The bare operator in CFT gets renormalized by a multiplicative factor

$$\mathcal{O}_\alpha^{ren}(x) = Z_{\alpha\beta}(\epsilon)\mathcal{O}_\beta^{\text{bare}}(x,\epsilon)$$ normalized as $Z^*_{\alpha\gamma}Z_{\gamma\delta} = \delta_{\alpha\beta}$

so that the renormalized correlator is finite at $\epsilon \to 0$

$$Z^*_{\alpha\gamma}\langle \mathcal{O}_\gamma^{\text{bare}}(x)\mathcal{O}_\delta^{\text{bare}}(0)\rangle Z_{\delta\beta} = \mathsf{C}_{\alpha\beta}(x) + \mathsf{O}(\epsilon)$$

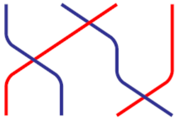
- The mixing matrix is then given by

$$\hat{D}_{\alpha\beta} = 2g^2 \lim_{\epsilon\to 0} \epsilon\, Z^{-1}_{\alpha\gamma}\frac{\partial Z_{\gamma\beta}}{\partial \xi^2} .$$

Caetano, Gurdogan, V.K , 2016
(to be published)

Magnon graphs from AdS/CFT integrability

- AdS/CFT integrability is good for computing anomalous dimensions given by sums of planar graphs, avoiding computation of graphs
- In our model, at each loop order we have a single graph for a given configuration of in- and out-fields (element of mixing matrix)
- Hence we can try to reverse the logic and use integrability is a tool for computing these, in general very complicated, Feynman graphs.
- For “unwrapped” graphs, we use asymptotic Bethe ansatz (ABA), for wrapped graphs -- Quantum Spectral Curve (QSC) or TBA
- Sequence of actions:



1. List the entries of mixing matrix with unknown coefficients
2. Find eignevalues (dimensions) as functions of these coefficients
3. Compute same dimensions explicitly from ABA and fix as many coefficients of mixing matrix as you can. Unknown coefficients are due to invariance of spectrum w.r.t. rotations $\hat{D} \to \Omega \hat{D} \Omega^{-1}$
4. Fix the remaining coefficients by computing a few simplest graphs explicitly. They will depend on the renormalization scheme

Dimensions from ABA

Eden, Beisert, Staudacher, 2005

- Magnons in N=4 SYM have are characterized by rapidities $u_1, u_2, \ldots$ which satisfy the asymptotic Bethe ansatz (ABA)
- In our DS limit, rapidities live in "mirror" and the double-scaled ABA becomes (in SU(2) subsector of operators with two scalars)

$$(u_j^2 + 1/4)^L = \xi^{2L} \prod_{k \neq j} \frac{u_j - u_k - i}{u_j - u_k + i} \sigma^2(u_j, u_k),$$

Caetano, Gurdogan, V.K (to be published)

where the "dressing phase" is

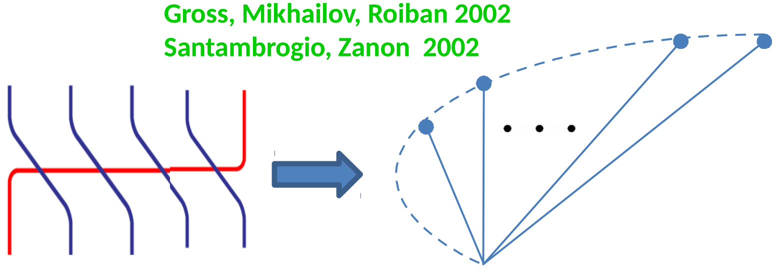
$$\sigma(u,v) = \frac{(1+4u^2)\,\Gamma\left(\frac{1}{2} - iu\right)\Gamma\left(\frac{3}{2} - iu\right)\Gamma(1+iu-iv)^2\,\Gamma\left(\frac{1}{2}+iv\right)\Gamma\left(\frac{3}{2}+iv\right)}{(1+4v^2)\,\Gamma\left(\frac{1}{2} - iv\right)\Gamma\left(\frac{3}{2} - iv\right)\Gamma(1+iv-iu)^2\,\Gamma\left(\frac{1}{2}+iu\right)\Gamma\left(\frac{3}{2}+iu\right)}.$$

- Momentum conservation: $\prod_{j=1}^{M} (u_j^2 + 1/4) = \xi^{2M}$
- The anomalous dimension are given by $\Delta = L - M + 2i \sum_{j=1}^{M} u_j$
- One magnon: in DS limit, only one chirality left: $\xi^2 = g^2 e^{ip}$ fixed and $g^2 e^{-ip} \to 0$ in standard SYM dispersion relation $\Delta_{1\text{m}} - L = -1 + \sqrt{1 + 4g^2 \cos^2 \frac{p}{2}}$

$$\Delta_{1\text{m}} - L = -1 + \sqrt{1+4\xi^2} = \sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2(1-2n)} \xi^{2n}$$

Gross, Mikhailov, Roiban 2002
Santambrogio, Zanon 2002

- This reproduces results for relatively complicated Feynman graphs of the type



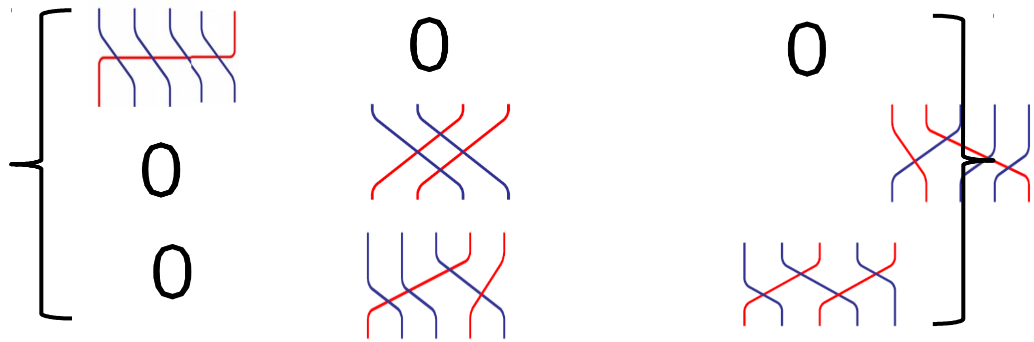
Example: 2 magnons, 4 loops…

- From ABA we find the following perturbative expansion of anomalous dimensions of operators for length L=5 for N=2 magnons up to 4 loops

$$\gamma_{L=5,M=2} = (-1\pm\sqrt{5})\xi^2+\frac{1}{5}(-15\pm\sqrt{5})\xi^4+\left(\frac{4}{5}(-5\pm\sqrt{5})\zeta_3 \mp \frac{81}{125}\sqrt{5}+3\right)\xi^8$$

- Using this data we can calculate the mixing matrix, and hence the corresponding individual Feynman graphs, up to a single constant C

$$\hat{D}_{4-\text{loop}} =$$



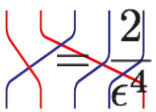
- Diagonalizing it and comparing the eigenvalues with ABA we get

$$\hat{D}^{(4-loops)}_{jk} = \begin{pmatrix} -10\xi^8-4\xi^6-2\xi^4-2\xi^2 & 0 & 0 \\ 0 & \mathcal{C}\xi^8-4\xi^6-2\xi^4 & \frac{1}{2}(\mathcal{C}+8\zeta_3-4)\xi^8-2\xi^4-2\xi^2 \\ 0 & \frac{1}{2}(\mathcal{C}+8\zeta_3-4)\xi^8-2\xi^4-2\xi^2 & -(\mathcal{C}+8\zeta_3+6))\xi^8-2\xi^2 \end{pmatrix}$$

- To fix the remaining constant C, one has to compute explicitly a single graph. More ambiguity at higher loops (in progress).

Fixing 4-loop 2-magnon Graphs

- Using this data we can calculate the mixing matrix, and hence the corresponding individual Feynman graphs, up to a single constant C
- We can compute explicitly the simplest, minimally connected graph

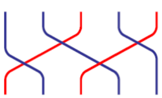


$$= \frac{2}{\epsilon^4} + \frac{29}{6\epsilon^3} - \frac{2\left(3+\pi^2\right)}{3\epsilon^2} + \frac{-\frac{56\zeta(3)}{3} - \frac{20}{3} - \frac{29\pi^2}{18}}{\epsilon}$$

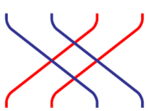
- The scheme dependent constant C is fixed then as

$$\mathcal{C} = -2(4\zeta(3) - 1)$$

- It fixes, without computation the following more complicated graphs



$$= \frac{10}{3\epsilon^4} + \frac{29}{3\epsilon^3} - \frac{\frac{10\pi^2}{9} - \frac{13}{6}}{\epsilon^2} - \frac{512\zeta(3) + 58\pi^2 + 213}{18\epsilon}$$



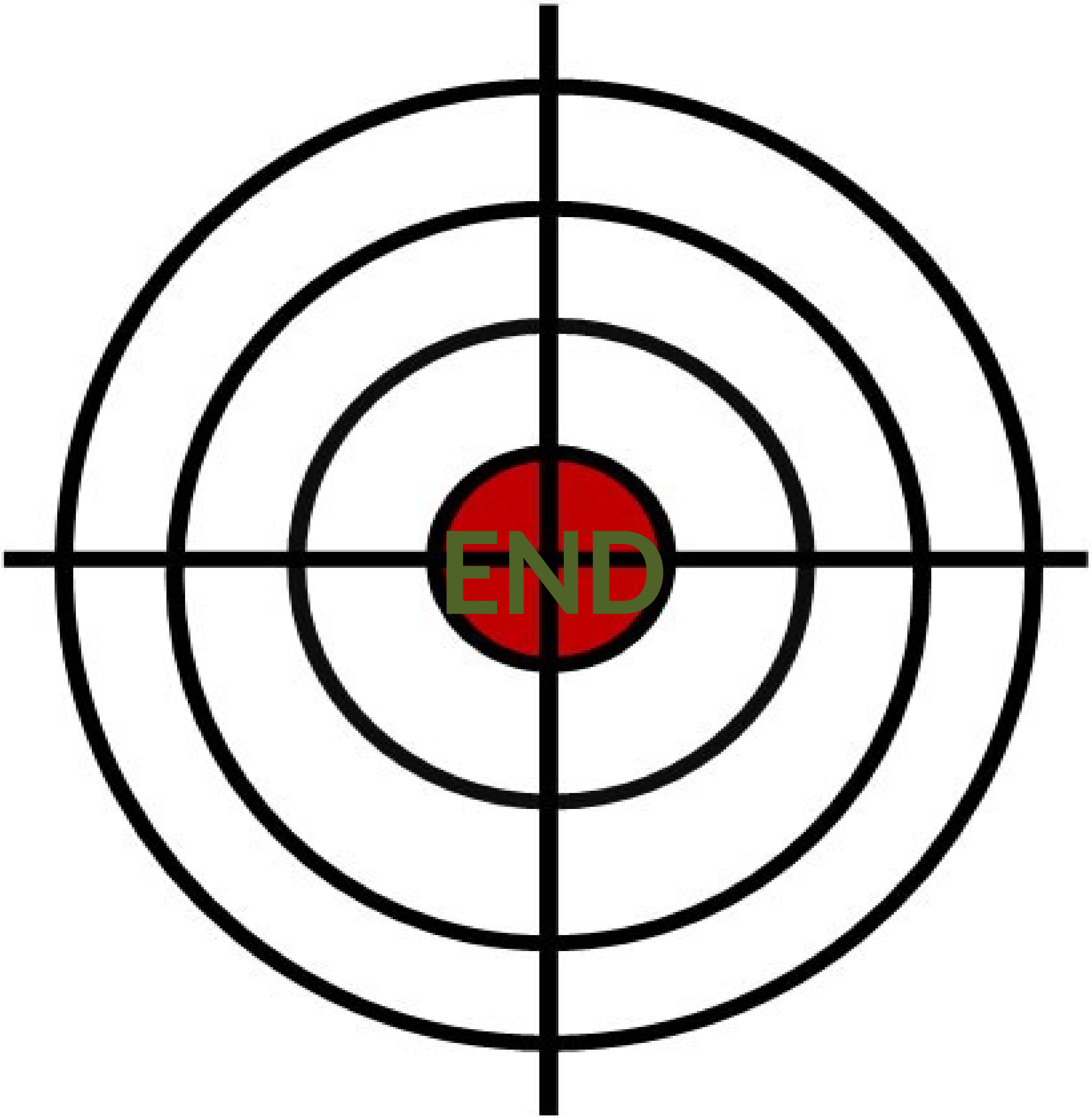
$$= \int \frac{d^D x_{b_1} d^D x_{b_2} d^D x_{b_3} d^D x_{b_4}}{(x_{b_1 1}^2)^2 x_{b_2 1}^2 x_{b_2 2}^2 (x_{b_3 2}^2)^2 x_{b_4 1}^2 x_{b_4 2}^2 x_{b_1 b_2}^2 x_{b_2 b_3}^2 x_{b_3 b_4}^2 x_{b_4 b_1}^2}$$

$$= \frac{4}{3\epsilon^4} + \frac{3}{\epsilon^3} - \frac{4(6+\pi^2)}{9\epsilon^2} - \frac{7 + \pi^2 + \frac{106}{9}\zeta_3}{\epsilon}$$

- In this way we were able to fix some unknown 6-loop graphs

Conclusions and prospects

- In a special double scaling limit of twisted N=4 SYM theory, we found new 4-dimensional chiral QFTs integrable in planar limit.
- Similar observation for the DS limit of gamma deformed 3D ABJM
- We observe that the integrability of the model is related to integrablility of 4D “fishnet” graphs – the basic ingredient of Feynman expansion of any physical quantity. This demonstrates, for the first time, some deep reasons of planar N=4 SYM integrability.
- The model is conformal in certain sector and the anomalous dimensions (not necessarily real) can be computed.
- Since a generic physical correlator has only one Feynman graph at each loop order, the integrability of this model allows, in principal, to compute certain complicated multi-loop graphs in 4D (and 3D)
- Structure constants? 4-point correlators? Amplitudes? Easier to compute than for general N=4 SYM
- It is interesting to understand what is the dual 4D string theory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