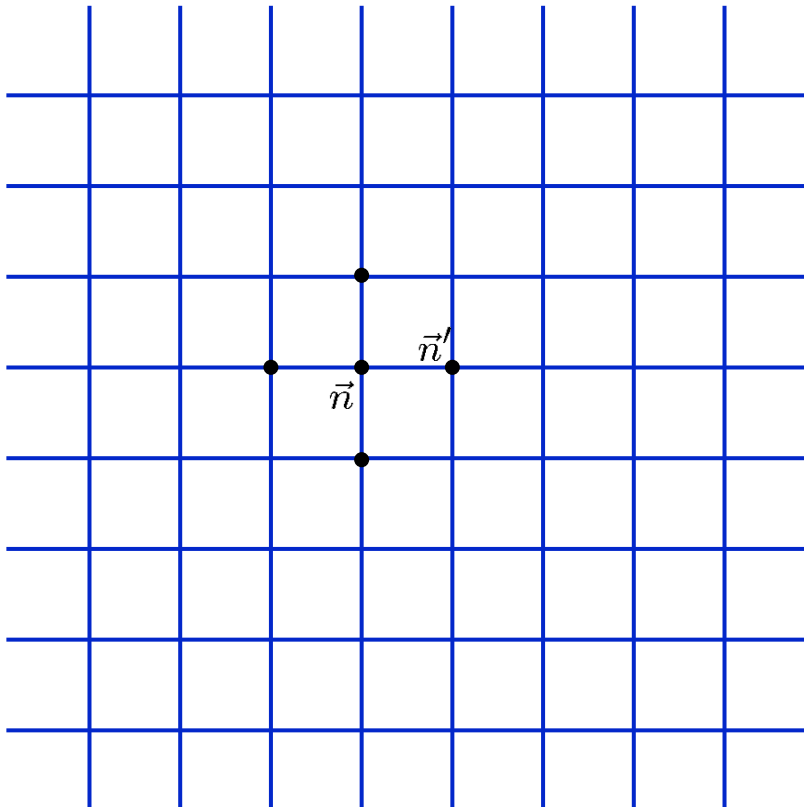


# 2D Ising model

$$E = -J \sum_{(\vec{n}, \vec{n}')} s_{\vec{n}} s_{\vec{n}'} + B \sum_{\vec{n}} s_{\vec{n}}$$

$$s_{\vec{n}} = \pm 1$$



Partition function

$$Z = \sum_{\{s\}} e^{-E/k_B T}$$

RG Fixed point

$$\frac{J}{k_B T_c} = \frac{1}{2} \operatorname{asinh}(1) = 0.4406 \dots$$

$$B_c = 0$$

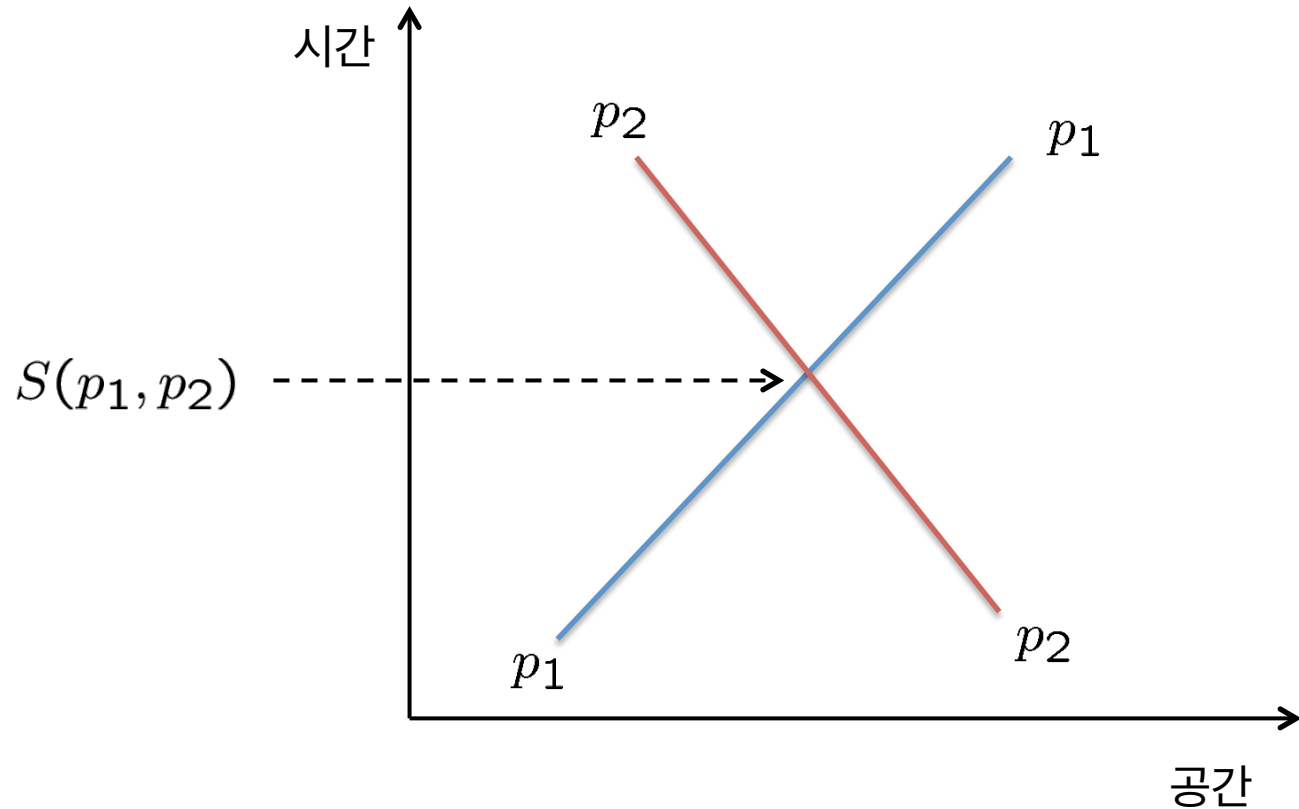
Onsager (1944)

$$\left. \vphantom{\frac{J}{k_B T_c}} \right) B = 0$$

Can we solve for  $B \neq 0$

# S-행렬

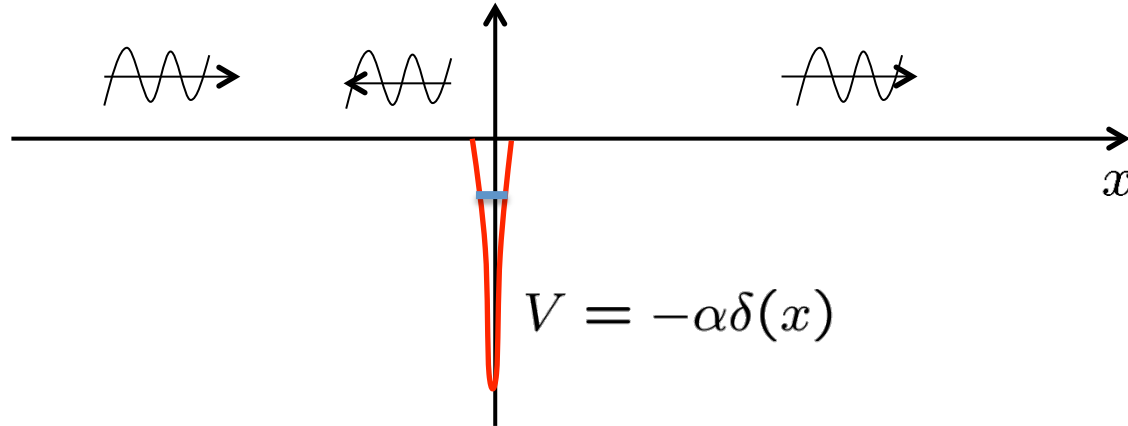
- 무한개의 보존전하  $\rightarrow$  완전탄성산란행렬



- S-행렬로부터 정확한 물리량 계산가능

- Pole of S  $\rightarrow$  bound state [구속입자]

- (ex) 양자역학의 delta함수 퍼텐셜



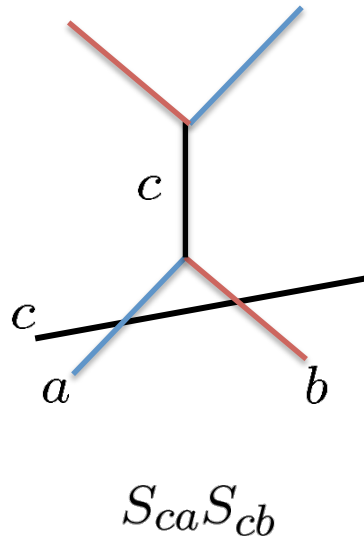
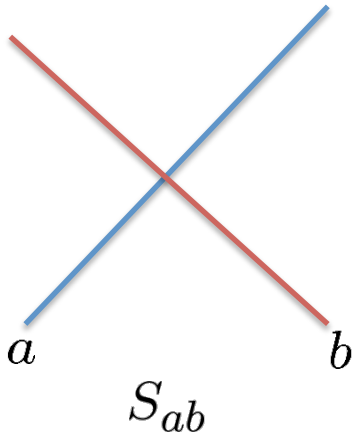
- Bound state energy:  $E = -\frac{m\alpha^2}{2\hbar^2}$  Griffiths pp.73-74

- Scattering amplitudes: ( $E > 0$ )

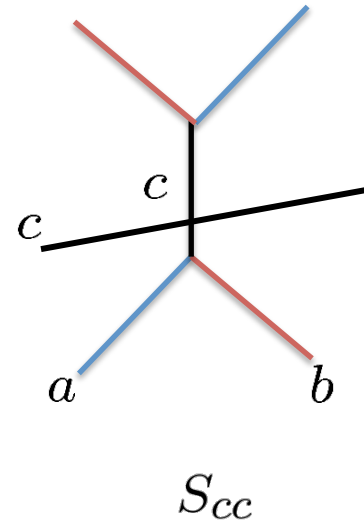
$$r(p) = \frac{i\beta}{1 - i\beta}, \quad t(p) = \frac{i\beta}{1 + i\beta}, \quad \beta \equiv \frac{m\alpha}{\hbar p}$$

Pole at  $p = i\frac{m\alpha}{\hbar} \quad \text{-----} \rightarrow \quad E = -\frac{m\alpha^2}{2\hbar^2}$

# Bootstrap



=



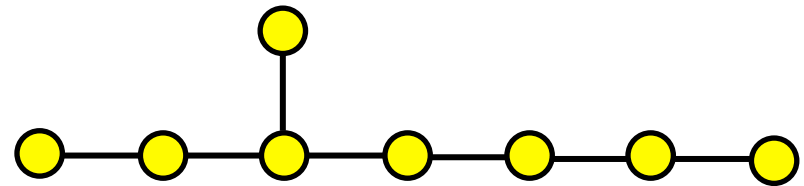
# Exact results

- S-matrices by Zamolodchikov (1988)
- Mass spectrum from poles of S-matrices

$$\begin{aligned}
 m_1 &= (4.40490857\dots)|B|^{5/13}, \\
 m_2 &= 2m_1 \cos \frac{\pi}{5}, \text{ -----} \rightarrow \frac{\sqrt{5}+1}{2} \\
 m_3 &= 2m_1 \cos \frac{\pi}{30}, \quad \text{황금비} \\
 m_4 &= 4m_1 \cos \frac{\pi}{5} \cos \frac{7\pi}{30}, \\
 m_5 &= 4m_1 \cos \frac{\pi}{5} \cos \frac{2\pi}{15}, \\
 m_6 &= 4m_1 \cos \frac{\pi}{5} \cos \frac{\pi}{30}, \\
 m_7 &= 8m_1 \cos \frac{\pi}{5} \cos \frac{\pi}{5} \cos \frac{7\pi}{30}, \\
 m_8 &= 8m_1 \cos \frac{\pi}{5} \cos \frac{\pi}{5} \cos \frac{2\pi}{15}
 \end{aligned}$$

Perron-Frobenius vector  
of Cartan matrix

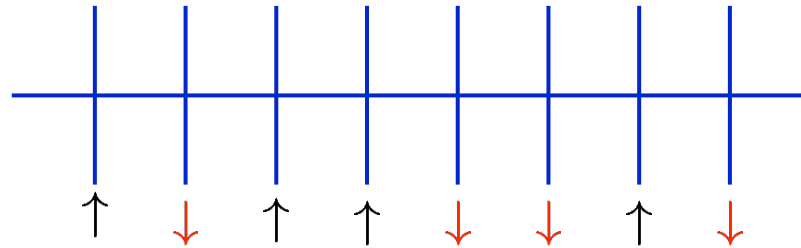
$E_8$  Lie algebra



# Experimental evidence of $E_8$

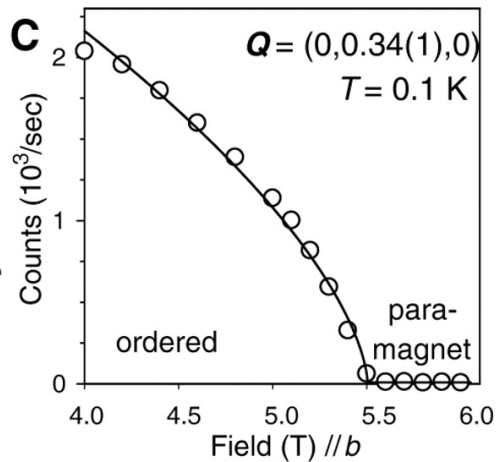
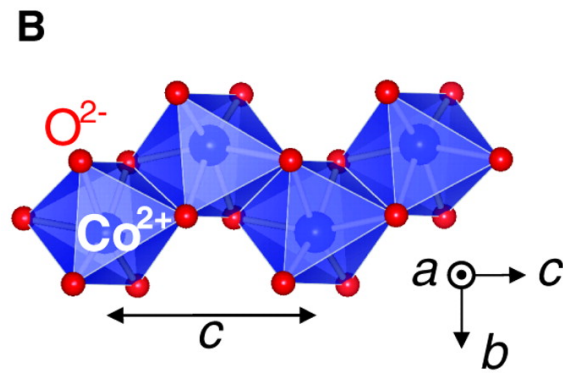
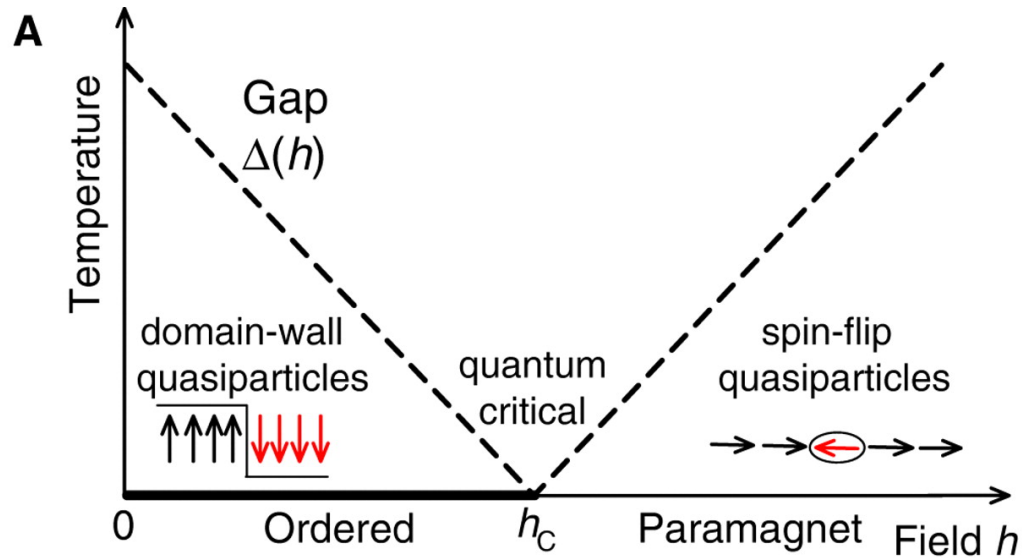
- 2D Ising model = 1D Ising spin chain

$$\hat{H} = -J \sum_{j=1} \left[ \sigma_j^z \sigma_{j+1}^z + h \sigma_j^x \right] + B \sigma_j^z$$



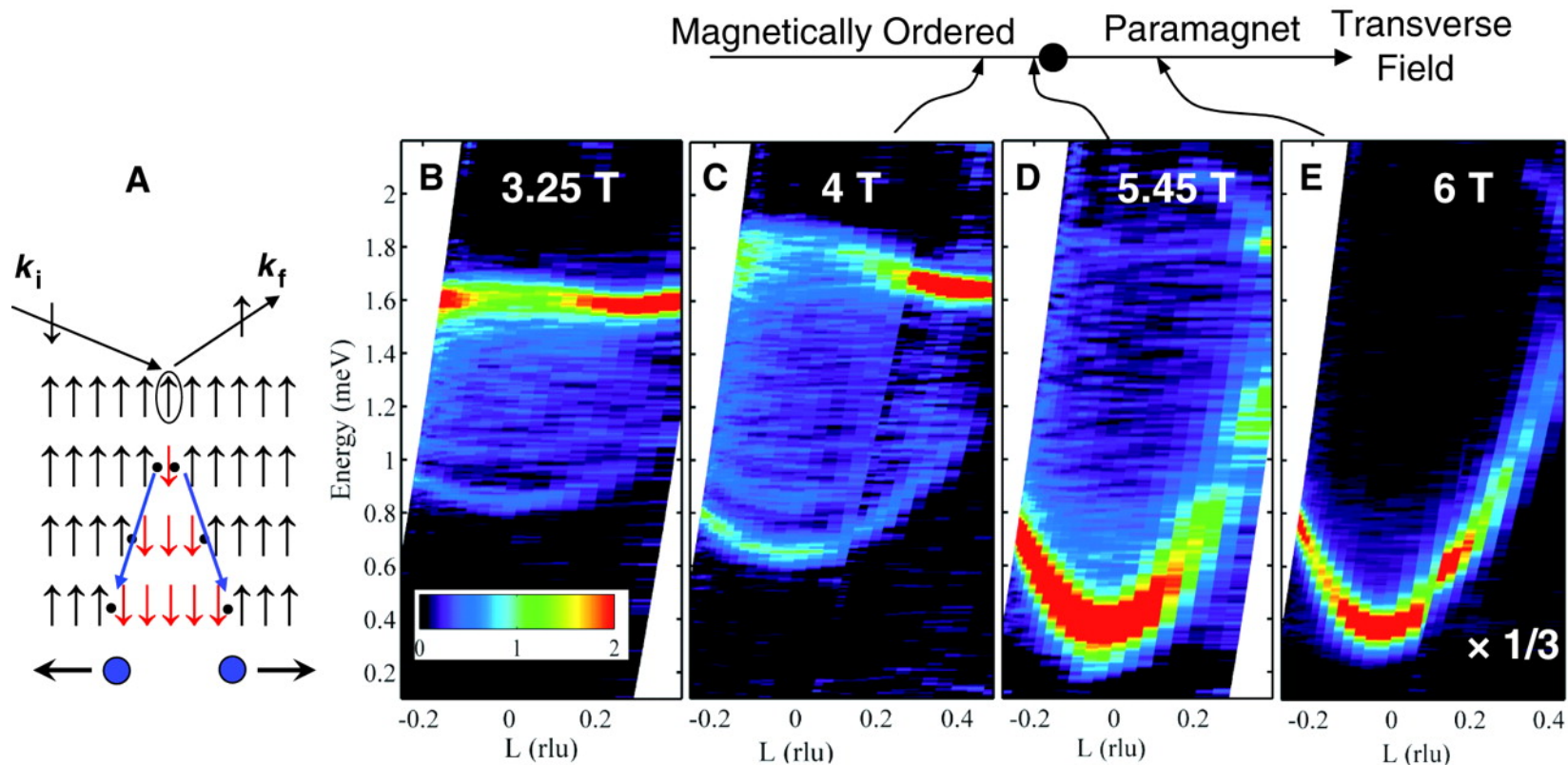
$$h = h_c = \frac{1}{2} \quad \text{Critical Ising model}$$

- Material:  $\text{CoNb}_2\text{O}_6$



R Coldea et al. Science 2010;327:177-180

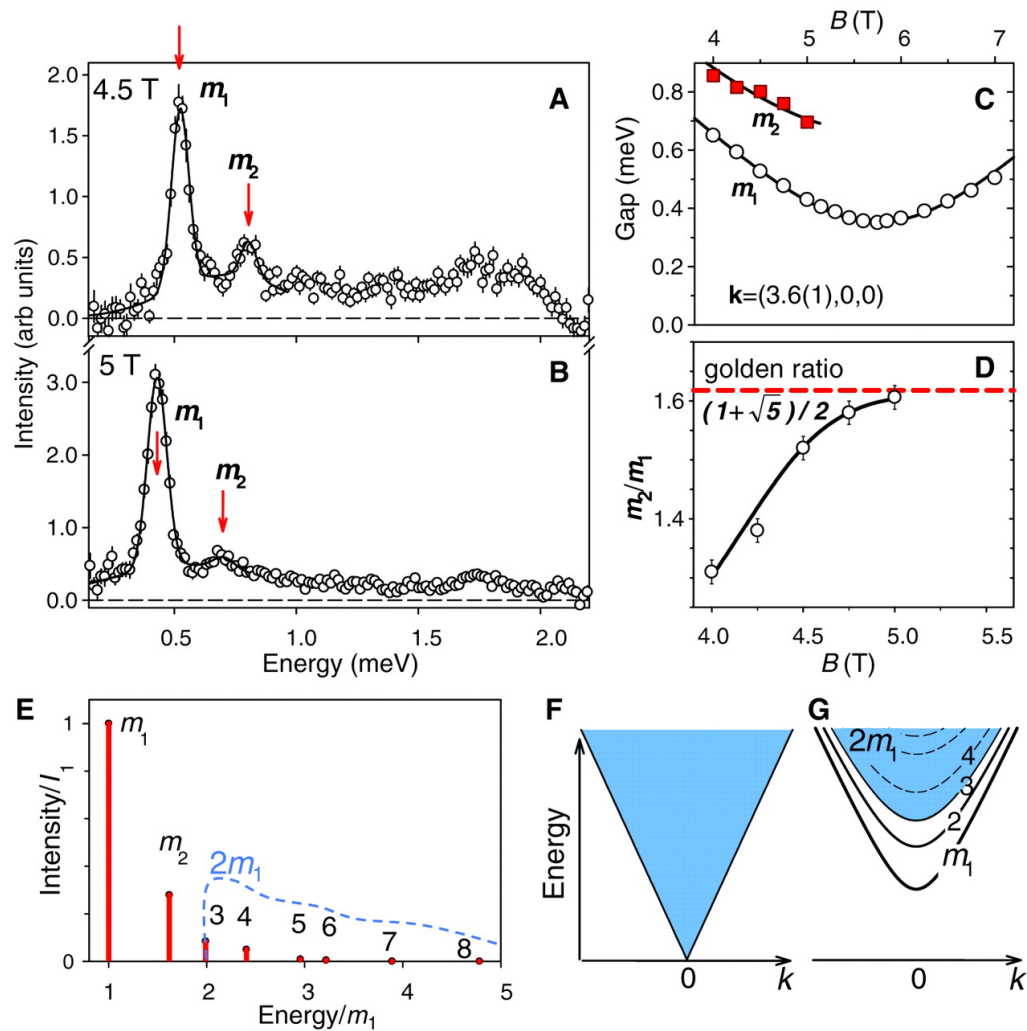
(A) Cartoon of a neutron spin-flip scattering that creates a pair of independently propagating kinks in a ferromagnetically ordered chain.



R Coldea et al. Science 2010;327:177-180



(A and B) Energy scans at the zone center at 4.5 and 5 T observing two peaks,  $m_1$  and  $m_2$ , at low energies.



R Coldea et al. Science 2010;327:177-180